FEDERAL DEPOSIT INSURANCE PREMIUMS: ISSUES, PROPOSALS, AND CRITIQUES (U)

Hwang Dar-Yeh

黃 達 業*

摘 要

本文旨在全面檢視美國聯邦存款保險費率的評價理論。由於存款保險費率的訂價模式究竟應維持現行單一費率制度（Flat－rate premiums）還是應採行風險基準的變動費率制度（Risk－based Variable rate premiums）在美國已引起長達十多年的爭辯，至今仍然是熱門的討論課題。由於我國銀行經營競爭日趨劇烈，國內的存款保險制度也須未雨綢繆而有必要藉助於先進國家的制度和經驗，因此本文將現行被提出的各種存保費率訂價模式作全面性的深入探討，並且針對各種可能的改進方案作比較分析及評論，希冀提供國人對存保費率的評價問題上能有更深一層的認識與瞭解。

Abstract

This study critically examines the issues, proposals and models for evaluating Federal Deposit Insurance Premiums from both micro and macro viewpoints. The models examined include both alternative option pricing and non-option approaches.

For the option approach, we first examine Merton's [1977] cost of deposit insurance pricing model. Then, we look at alternative pricing models following along Merton's lines. Finally, we introduce the theoretical application of the option pricing model with stochastic volatility to the valuation of FDIC deposit insurance premiums.

For the non-option approach, we first review the risk-based bank-failure contingency models. Then, we survey Sharpe's state preference model and its extensions. Finally, we investigate the pricing models which consider bank's risk-shifting incentive issues from macro viewpoints.

Finally, we review alternative proposals for reforming the insurance system. We first examine 100% deposit insurance. Then, we review and examine the variable-rate insurance system. Finally, we critically investigate the private deposit insurance system.

* Associate Professor of Finance, Department of Finance, National Taiwan University, and Adjunct Associate Professor, Department of Banking, National Chengchi University, Taipei, Taiwan, R.O.C.

*作者為本校銀行系兼任副教授

I. INTRODUCTION

This paper is motivated by the current Savings and Loans (S&L) and Commercial Bank's Crisis since it impacts the whole economy by costing taxpayers billions of dollars to solve these two industries' crisis. It is very important to know what causes these crisis and how to resolve the current thrift crisis. The purpose of this paper is to examine, integrate and introduce alternative pricing models for estimating FDIC deposit insurance premiums to supplement the current reform of federal deposit insurance system.

Reform of the financial services industry became a hotly debated issue in the 1980s and continue to rage in the 1990s. Much of the debate has been generated by a growing recognition that fundamental reforms are needed in our banking regulatory systems to respond to market-driven changes in the financial services industry. As of January 1989, the FSLIC (Federal Savings and Loan Insurance Corporation) listed approximately 340 federally insured S&Ls institutions as insolvent. Estimates of these insolvent institutions run from $90 billion to about 285 billion dollars. The FIRREA (Financial Institutions Reform, Recovery and Enforcement Act) of 1989 formally commits $159 billions of taxpayer money to resolve the thrift crisis and mandates that a study of federal deposit insurance be undertaken. So, the federal deposit insurance reform has taken the center stage in the political arena.

In this paper, we will critically examine the issues, models, and proposals for evaluating Federal Deposit Insurance premiums from both micro and macro viewpoints. The models examined here include both option pricing model and non-option pricing model approaches.

For the option approach, we first examine Merton's [1977] cost of deposit insurance pricing model. Then, we look at alternative pricing models following along Merton's lines. Finally, we introduce the theoretical application of the option pricing model with stochastic volatility to the valuation of FDIC deposit insurance premiums.

For the non-option approach, we first review the risk-based bank-failure contingency models. Then, we survey Sharpe's state preference model and its extensions. Finally, we investigate the pricing models which consider bank's risk-shifting incentive isssues from macro viewpoints.

Finally, we will review alternative proposals for reforming the deposit insurance system. We first examine 100% deposit insurance. Then, we review and examine the variable-rate insurance system. Finally, we critically investigate the private deposit insurance system.

II. ALTERNATIVE OPTION PRICING APPROACHES FOR ASSESSING THE COST OF DEPOSIT INSURANCE

II. 1. INTRODUCTION

Since Merton (1977) applied the option pricing model to the valuation of government guarantees of loans, there has been much work done in this area. Merton (1978) developed a pricing model for the cost of deposit insurance by including the surveillance cost (or auditing cost) and random auditing times. McCulloch (1985), Pyle (1983), Marcus (1984), Marcus and Shaked (1984), Kane (1986), Ronn and Verma (1986), Pennacchi (1987a, 1987b), and Acharya and Dreyfus (1989) all are along the same line with *minor modifications of assumptions.*

By following the standard assumptions made in Black-Scholes option pricing model, Merton (1977) demonstrated an isomorphic correspondence between deposit insurance and the put option. Then, by allowing the random audit times and including surveillance, Merton (1978) and Pennacchi (1987a, 1987b) extended the original model and derived a valuation formula for the FDIC liabilities. Merton (1978) also showed that the auditing cost component of the deposit insurance premium is in effect paid by the depositors, and the put option component is paid by the equity holders of the bank. So far, an increasing number of studies have suggested the option pricing approach to risk-adjusted deposit insurance premiums. Many authors have applied option pricing methods to estimate insurance values using historical market data[1].

In this section, we will examine alternative option pricing models for assessing the cost of deposit insurance.[2]. and derive the deposit insurance pricing formula based on the stochastic volatility assumption. Among all the alternative assumptions in option pricing models, we will look at how the pricing model will change

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1 See Marcus and Shaked [1984]; Ronn and Verma [1986, 1989], Pennacchi [1987a, 1987b]; King and O'Brien [1991].

2 For example, Merton [1978]; Pyle [1983]; McCulloch [1981, 1985], Marcus and Shaked [1984], Kang [1986]; Ronn and Verma [1986, 1987, 1989]; Pennacchi [1987a, 1987b]; Flannery [1991]; King and O'Brien [1991];

when (1) the company pays constant dividend (2) the interest rate is stochastic instead of constant, (3) the distribution of the stock's return is stable paretian instead of normal distribution, and (4) the exercise price is changing instead of being nonstochastic. Basides, we also look at how the option methodology could be used to establish a risk-based examination schedule under the current flat-rate insurance system. Based on this model, a riskier bank would be examined on a more frequent basis (King and O'Brien [1991]). Also, due to the fairly complicated mathematical techniques and unobservable variables, Kendall and Levonian (1991) proposed a simple and practical approach to better deposit insurance pricing by setting premium schedules to match premium payments with contingent-claim insurance values.

In most option models, we assume perfect risk assessment. However, this assumption seems implausible because many bank assets involve substantial information costs or asymmetries. Hence, any external agent will measure bank asset risk and/or bank capital with some error (Pyle [1983]; Benston et al [1986]; Flannery [1991]). In the option model, the volatility of stock's return plays a major role in determining the option's value. Based on the recent work of option literature (Hull and white [1987]; Johnson and Shanno [1987]; Sott [1987]; Wiggins [1987] and Finucane [1989]), the Black-Scholes (B-S) option model frequently undervalues deep in-and out-of-the-money options[3]. Hence, with the stochastic volatility assumption, we will have higher call option value than the standard B-S one.

In this section, we will propose an alternative pricing model for assessing the deposit insurance based on the stochastic volatility assumption. We also will examine the valuation of deposit insurance under two different scenarios: When the market value of bank's assets is uncorrelated or correlated with the stochastic volatility of the return of bank's assets. We expect that the future empirical estimates using alternative option model with stochastic volatility will provide a reasonable explanation on the S&Ls crisis, i.e., the FDIC under-charged the insurance premium in general.

In the following sections, we will examine alternative option pricing models

[3] Define C= Max [S-K, 0]. Then, if the stock price (S) is greater than the exercise price (K) (S>K), a call is in-the-money; if S=K, it is at-tht-money; if S<K, it is out-of-the-money. For a put, the definition is reversed. If S is much greater than K, then a call is said to be deep-in-the-money and a put deep-out-of-the-money. If S is much less than K, then a call is deep-out-of-the-money and a put is deep-in-the-money.

for assessing the deposit insurance in the banking industry and propose alternative model for pricing the cost of deposit insurance with stochastic volatility assumptions. We start out in section II.2 with Merton's (1977) pathbreaking model of deposit insurance pricing which applies the modern option pricing formula developed by Black and Scholes [1973]. Then, in sections II.3 to II.6., we will examine alternative pricing models for estimating deposit insurance premiums with different assumptions, including constant dividend payment (Marcus and Shaked [1984]), with stochastic interest rate (Ronn and Verma [1986]), with stable paretian distribution assumption (McCulloch [1985]), and with random examination schedule (King and O'Brien [1991]), In the sections II.7 and II.8., we will look at the pricing model considering the FDIC resolution policies (Pennacchi [19781, 1978b]) and simple pricing model with quadratic loss function (Kendall and Levonian [1991]). The other application of OPM will be explored on section II.9. where we derive the valuation model for deposit insurance by assuming stochastic volatility. For the pricing model with stochastic volatility, we derive the equation for evaluating bank's equity by considering either the stochastic volatility is uncorrelated or correlated with the market value of bank's assets. Then, by the put-call parity argument, we derive the equation for pricing the cost of deposit insurance. The summary with conclusions will be presented in the last section.

II.2 MERTON'S COST OF DEPOSIT INSURANCE PRICING MODEL

Merton (1977) proposes a systematic theory for determining the FDIC deposit insurance premium in a manner analogous to the Black-Scholes (1973) pricing of put option. Based on the isomorphic correspondence between loan guarantees and common stock put options, Merton is able to apply the well developed theory of put option pricing to evaluate the deposit insurance premium.

Merton (1978) also derives what would be a fair one time payment by banks or chartering fee for deposit insurance given that the insuring agency audits banks at random time intervals. Pennacchi (1987) analyzes additional influences (such as bank financial structure characteristics and alternative policy assumptions concerning an insuring agency's pricing of insurance and method for handling bank closing) on fair insurance pricing and bank risk taking behavior by generalizing Merton's model.

The use of the option-pricing approach to estimate appropriate premium rates offer two advantages relative to the use of historical system-wide loss experience.

First, the option pricing model allows for bank-specific estimates of the correct premium. Second, the appropriate premium can be computed using data collected over fairly short time periods and hence the data collected are still relevant to the problem at hand. Now, we start to look at how the Black-Scholes put option pricing formula can be applied to the evaluation of deposit insurance premium based on Merton's work.

II.2.1. Simple Pricing Model for Firm's Debt Insurance

(A) The Black-Scholes Put Option Pricing Model can be written as

$$p(S,\tau) = X.e^{-r\tau}N(-d_2) - S.N(-d_1) \tag{2.1}$$

$$\text{where} \begin{cases} -d_1 = [\ln(X/S) - (r+0.5\sigma^2)\tau] \,/\, [\sigma(\tau)^{1/2}] \\ -d_2 = -d_1 + \sigma(\tau)^{1/2} \end{cases}$$

(B) The Pricing Model for Firm's Debt Insurance is as follows:

(1) Define the debt Value = Min[V, B], and the equity value = Max [0, V-B] where V is the firm's market value and B is the firm's debt value.

(2) Suppose we have third-party guarantee to pay the bondholders in case of default. Then at t=T (the maturity date of debt), the payoff for all parties can be denoted as table 2-1:

Table 2-1: Payoffs of Firm's Creditors with the third-party guarantee.

Value for	V ≥ B	V < B		General Form
Bondholders:	B	B	➡	B
Equityholders:	V-B	0	➡	Max[0,V-B]
Guarantors:	0	−(B-V)	➡	Min[0, V-B)

(3) Let $G(\tau)$ denote the value of cash flow created by guarantee, then, at time t=T, the value of G(0) is

$G(0) = -\text{Min}\ [0,\ V\text{-}B] = \text{Max}\ [0,\ B\text{-}V]$ (2.2)

So, the equation (2.2) is in the form of put option.

At time τ, $G(\tau,V) = B.e^{-r\tau}N(-d_2) - V.N(-d_1)$ (2.3)

where $-d_1 = [\ln(B/V) - (r+0.5\sigma^2)\tau] / [\sigma(\tau)^{1/2}]$

$-d_2 = -d_1 + \sigma(\tau)^{1/2}$

(4) Let $B.e^{-R(\tau)\tau}$ be the market value of debt without guarantee where $R(\tau)$ is the promised yield. Also $B.e.^{-r\tau}$ be the market value of debt with guarantee. This implies that $G(\tau) + B.e^{-R(\tau)\tau} = B.e^{-r\tau}$ (2.4)

and $G(\tau) / (B.e^{-r\tau}) = 1 - e^{-[R(\tau)-r]\tau}$ (2.5)

Hence, $G(\tau) / (B.e^{-r\tau})$ is the cost of loan guarantee as a fraction of the amount of money raised.

II.2.2. Simple Pricing Model for the Bank's Deposit Insurance

(A) For any given bank, the deposit (D) is equivalent to a firm's debt (B). Let τ = the length of time until the next audit of bank's assets. Since the deposit's principal and interest are guaranteed, the insured deposits will be riskless and its value will be $B.e^{-r\tau}$.

(B) Let $g = G(\tau) / D = G(\tau) / (B{*}e^{-r\tau})$, and $d = D/V$.
Then, $G(\tau)/B{*}e^{-r\tau}$ is the cost of the guarantee per dollar of insurance deposit and D/V is current deposit to asset value ratio. Hence, by applying (2.3), (2.4), and (2.5), we can obtain the valuation equation for the insured deposit insurance as follows,

$g(d,\tau) = g(D/V,\ \tau) = N(-h_2) - (1/d)\ [N(-h_1)\ ]$ (2.6)

where $-h_1 = [\ln(d) - 0.5\sigma(\tau)^{1/2}] / [\sigma(\tau)^{1/2}]$ and

$-h_2 = -h_1 + \sigma(\tau)^{1/2}$.

(C) Let's examine the impact of change in variables on the value of per dollar deposit insurance. This can be done by taking the first derivative of g with respect to d and τ, i.e.,

$(\partial g/\partial d) = N(-h_1)\ (1/d_2) > 0 \quad (d\uparrow \rightarrow g\uparrow)$ (2.7)

$(\partial g/\partial \tau) = N'(-h_1)\ \{1/[2d\sigma(\tau)^{1/2}]\} > 0\ (\tau\uparrow \rightarrow g\uparrow)$ (2.8)

Equation (2.7) says that when the ratio of total deposits to assets value increases, the value of per dollar deposit insurance will increase. Equation (2.8) implies that when either the variance of the bank's assets return or the time to maturity increases, the FDIC will charge the bank a higher deposit insurance premium (see figure 2-1).

II.3. MARCUS AND SHAKED'S MODEL WITH DIVIDEND PAYMENT

Marcus and Shaked (hence M-S) (1984) extends Merton's model by assuming that the company makes proportional dividend payment. In their model, they also make the following assumptions: (1) The value of bank assets follows a diffusion process with log-normal return. (2) The FDIC may choose not to renew its offer of deposit insurance at the end of the period or may renegotiate the terms under which it is offered. (3) The time until the next examination, τ, is the effective maturity of the debt. (4) Each bank pays a constant, proportional dividend to the stockholders. (5) Book value of the total debt is a fairly accurate measure of market value. (6) The FDIC insures all deposits including the deposits at foreign branches.

Following Merton's derivation of the deposit insurance pricing formula by including the constant, proportional dividend payment case, the value of guaranteed cost will be $g(d,\tau) = g(D/V, \tau) = N(-h_2) - (1/d)\ [e^{-\delta\tau}N(-h_1)]$ (2.9)

where

$g(d,\tau)$ = the cost of guarantee per dollar of insurance debt.

$d = D/V$ = the ratio of bank's current debt-to-asset value.

$-h_1 = [\ln(d)-(0.5\sigma^2-\delta)\tau]/[\sigma(\tau)^{1/2}]$

$\quad = [\ln(B/V)-(r+0.5\sigma^2-\delta)\tau]/[\sigma(\tau)^{1/2}]$

$-h_2 = -h_1 + \sigma(\tau)^{1/2}$

τ = the length of time until the next bank's audit.

V = the current value of total asset.

$D = B_T e^{-r\tau}$ = the current market value of debt with guarantee.

δ = the constant, proportional dividend payment.

M-S also rewrite the pricing formula for the cost of guarantee per dollar in terms of the total deposit insurance premium at time τ, I, as follows:

$$I = D.g(d'\tau) = D.N(-h_2)-(D/d)\ e^{-\delta\tau}N\ (-h_1) \qquad (2.10)$$

$$\text{or } I = B_T e^{-r\tau}.N(-h_2)-e^{-\delta\tau}.\ V.N(-h_1)$$

$$I = B_T e^{-r\tau}[1-N(h_2)]-e^{-\delta\tau}.A_0[1-N(h_1)] \qquad (2.11)$$

where $A_0 = V$, $h_1 = [\ln(A_0/B_\tau)+(r+0.5\sigma^2-\delta)] \bullet\tau/[\sigma(\tau)^{1/2}]$,

$h_2 = h_1 - \sigma(\tau)^{1/2}$, and

I = the total deposit insurance premium

A_0 = the current value of bank's total assets = D + E − I.

Besides, M-S use the variance of the rate of retunn bank's equity to estimate the variance of the bank's asset by the following formula:

$$\sigma = \sigma_E \{1-\{[B_\tau e^{-r\tau}N(h_2)] / [e^{-\delta T}A_0N(h_1]\}\} \quad (2.12)$$

By substituting A_0 = D + E − I into (2.11) and (2.12), we can solve these two simultaneous equations with two unknowns σ and I by numerical method, and obtain the estimations for the deposit insurance premiums and the standard deviation of the return for the bank's assets.

II.4. THE PRICING MODEL WITH STABLE PARETIAN DISTRIBUTION

Since Merton (1977) applied the well-known Black-Scholes option pricing formula to the problem of evaluating bank deposit insurance, most authors (Merton [1978]; Pyle [1983]; Marcus and Shaked [1984]; Ronn and Verma [1986]; Pennacchi [1987a, 1987b] et cetera) by following his model made a very strong assumption that the random value of the bank's assets (A) is log normal. McCulloch (1985) in his model assumes that the logarithm of A is instead distributed according to the symmetric Paretian stable class of distributions and uses option pricing model to evaluate the FDIC deposit insurance, which can be viewed as put options that entitle the banking firm to "sell" the firm's assets to the insuring agency for a pre-arranged price determined by the face value of the insured liabilities.

The shape of a stable distribution is completely determined by three parameters: (1) the characteristic exponent α, which governs how fast the tails taper off, (2) the standard scale c, which is critical for the value of bank insurance since higher values of c increase the probability of failure, and (3) the location parameter, which is the mean of the distribution (McCulloch, [1985]). When $\alpha=2$, then the normal/Gaussian distribution will be obtained. When $\alpha<2$, then the distribution will have a "paretian" tails which are longer than the tails of a normal distribution. Hence, the probability of bank failure will depend crucially on the value of α. The B-S option pricing formula used by Merton and others assuming a normal distribution will greatly understate the value of deposit insurance if in fact the distribution is non-Gaussian stable McCulloch, [1985]). Now, let A be the initial market value of the bank's assets.

L be the initial market value of its liabilities.
K be the initial economic value of its capital (A-L).
q be the market value ratio of bank's assets and capital.

Then the annual rate of occurrence of discontinuities in the value of the bank's assets large enough to cause insolvency can be shown (McCulloch, [1985]) as

$$\lambda = (12/\pi)\ \Gamma(\alpha)\ \sin(\pi\alpha/2)\ (c/-\log r)^{\alpha} \tag{2.13}$$

where $r=1-q$ is the bank's leverage ratio (= D/A) (Note: when $\alpha=2$, then both $\sin(\pi\alpha/2)$ and λ are equal zero and there are no discontinuities in net worth).

The value per year of deposit insurance with continuous surveillance computed as a fraction of liabilities (π) is given by

$$\pi = \lambda\ H(1-q,\alpha)\ /\ (1-q) \tag{2.14}$$

$$\text{where } H(1-q,\alpha) = r-\alpha(-\log[1-q])^{\alpha} \int_{-\log(1-q)}^{\alpha} e^{(-x)}\ x^{(-\alpha-1)}\ dx$$

The fraction H (1-q, α) represents the expected cost of failure should a failure occur and the equation tells us how to compute the fair value of insurance from leverage ratio (q), characteristic exponent (α), and scale parameter (c).

II.5. THE PRICING MODEL WITH STOCHASTIC INTEREST RATE

In this section, we will examine the pricing model of deposit insurance with stochastic interest rate based on the Ronn and Verma [1986] model and discuss how stochastic interest rates will affect the valuation of insurance premiums. Besides the standard assumption of B-S, the assumptions made in Roon and Verma's stochastic interest rate model are as follows: (1) A single homogeneous-term debt issue (2) Assuming all pre-insurance debt to be of equal seniority. (3) Assuming that all debt is issued at the risk-free rate of interest (4) Assuming one-period model, i.e., the FDIC may choose not to renew its offer of deposit insurance at the end of the period. By using the following notations,

V = the unobserved post-insurance value of the bank's assets
V' = the unobserved pre-insurance value of the bank's assets
B_1 = the face value of the insured deposits
B_2 = the face value of all debt liabilities other than the insured deposits
$B = B_1 + B_2$ = the face value of total debt liabilities
σ_V = the instantaneous s.d. of the rate of return on the value of the bank's assets after insurance
σ_E = the instantaneous s.d. of the return on equity E
T = time until the next audit of the bank's assets
δ = dividend per dollar of value of the assets, paid n times per period

we can denote the value of insured deposits to the holders at the maturity of the debt as

$$\text{Min } [FV(B_1),\ V_T(B_1/(B_1+B_2))] \qquad (2.15)$$

where FV represents the future value operator and V_T represents the terminal value of the bank's assets. The maturity value of deposit insurance is given by:

$$\text{Max } [0,\ FV(B_1) - V_T(B_1/(B_1+B_2))] \qquad (2.16)$$

Then, following Merton's deposit insurance pricing formulas, the value of deposit insurance is equivalent to the value of a put written with a striking price equal to total debt and then scaled down by the proportion of demand deposits to total debt, B_1/B. In other words, the value of the deposit insurance

$$I = B_1.N(Y_1) - (1-\delta)^n.V(B_1/B)\ .N(Y_2) \qquad (2.17)$$

where $Y_1 = Y_2 + \sigma_V(T)^{1/2} = \ln\{B_1/(1-\delta)^n V[B_1/(B_1+B_2)]\} + 0.5\sigma^2_V\ T$
$= \{\ln\{B/[(1-\delta)^n V]\} + 0.5\sigma^2_V T\}/[\sigma_V(T)^{1/2})$

and $Y_2 = Y_1 - \sigma_V\sqrt{T}$, $B = B_1 + B_2$. Then, if we define $d = I/B_1$ as the *per dollar deposit insurance premium*, we have

$$d = N(Y_1) - (1-\delta)^n(V/B)N(Y_2) \qquad (2.18)$$

where $Y_1 = \{\ln\{B/[(1-\delta)^n V]\} + (0.5\sigma^2_V T)\} / (\sigma_V\sqrt{T})$, $Y_2 = Y_1 - \sigma_V\sqrt{T}$ and B is the present value of the striking price. Hence, we don't have risk-free rate term in (2.18).

From equation (2.18), we can see that the per dollar price of deposit insurance depends on total debt B, not on the insured debt, B_1. Also, the equity of a firm (E) can be represented as a call option on the value of the assets of the firm with the same maturity equity as a fully dividend-protected call as follow:

$$E = VN(X_1) - BN(X_2 \qquad (2.19)$$

where $X_1 = [\ln(V/B) + 0.5\sigma^2_V T] / (\sigma_V(T)^{1/2})$, $X_2 = X_1 + \sigma_V(T)^{1/2}$, and

$$\sigma_V = [(\partial E/\partial V) / (e/V)]\ (\sigma_V)$$

Ronn and Verma [1986] assumes that the FDIC will try to revive the concerned bank by direct assistance of funds infusion instead of liquidating bank's assets whenever bank's assets has fallen below the total debt. They claim that only when the bank's assets below some percentage (call k, where $k \leq 1$) of the total debt will the bank be liquidated. Otherwise, the insuring agency will infuse up to (1-k)B amount of fund to make the value of the bank equal to B. Thus, they modify the valuation formula for the value of bank's equity as follows:

$$E = V\ N(\{\ln[V/(kB)] + 0.5\sigma^2_V\ T\} / [\sigma_V(T)^{1/2}]) - kB.N(Z_1 - \sigma_V(T)^{1/2} \qquad (2.20)$$

where $\sigma_V = \sigma_E\ [E/VN\ (Z_1)]$ (2.21)

By solving equations (2.20) and (2.21), we can get the pair value of (V, σ_V). Finally, by substituting the pair (V, σ_V) into (2.18), we can get the estimation for the value of risk-adjusted deposit insurance premiums.

II.6. THE PRICING MODEL WITH RISK-BASED EXAMINATION SCHEDULE

Ronn and Verma (1989) proposes an alternative market-based approach by applying the option methodology to solve for a minimum capital ratio. This ratio would make the market's valuation of the bank's deposit insurance equal to a flat-rate premium. Following this approach, King and O'Brien (1991) built up a risk-adjusted examination schedule whereby riskier banks would be examined more frequently with the possibility of closure or regulatory action following each examination.

In the standard application of option pricing model to setting risk-adjusted deposit insurance premiums, we normally assume a one year examination schedule for all banks. But, in reality, the examination frequencies done by the regulator like FDIC vary across banks, which depend on the condition of each bank. Moreover, a one or possibly two-year interval of examination requirements may be more reasonably representative for the majority of the banks.

In considering the use of risk-adjusted exam schedule as an alternative to risk-based premium, they take the current flat-rate premium as granted and assume there exists an examination schedule such that the insurance term will make the insurer's libility equal to the fixed premium per dollar of deposits. By using a variation of the option methodology, the insurer can determine the appropriate market-based risk-adjusted examination schedule from stock market data.

The idea of the random examination schedule model is as follows. First, take the current fixed premium rate per dollar of deposits as given and calculate the total value of the insured deposits denoted as IP_t. Then, based on the formula (Marcus and Shaked [1984] and Ronn and Verma [1986]),

$$V_T = E_t + D_t - IP_t \tag{2.22}$$

we can obtain the market value of bank's assets without deposit insurance (V^t) by subtracting the total deposit insurance (IP_t)from the sum of bank's total equity (E_t) and total debt (D_t) (equation [2.22]).

Next, by setting the insurance value equal to the fixed premium, we can solve for the random variables. These random variables include (1) the market value of bank's assets (V_t), (2) one-period standard deviation of the return on bank assets (σ_A) and (3) the insurance term (T-t). This is done by using the Black-Scholes put option pricing formula and the relationship between the standard deviation of the return of bank's assets and return on equity equation (2.27). In other words, by solving the following equations (equation [2.23] to [2.27]), we can obtain the unknown values for σ_A, V_t, and T-t.

$$dV = \mu_t \, dt + \sigma_{At} \, dz_t \tag{2.23}$$

$$E_T = \max \{V_T - B_T, 0\} \tag{2.24}$$

$$IP_t = D_t \, N(d_2) - V_t \, N(d_1) \tag{2.25}$$

where $d_1 = \ln (D_t/V_t)\ (\sigma_A)^{-1}\ (T\text{-}t)^{-1/2} - (1/2)\ \sigma_A\ (T\text{-}t)^{1/2}$

$d_2 = d_1 + \sigma_A\ (T-t)^{1/2}$

By substituting (2.20) and (2.23) into (2.24), we obtain the pricing formula for the bank's equity value as follows:

$$\begin{aligned} E_t &= V_t - D_t + D_t\ N(d_2) - V_t\ N(d_1) \\ &= V_t\ N(-d_1) - D_t\ N(-d_2) \end{aligned} \tag{2.26}$$

$$\text{where } \sigma_{Et} = (V_t/E_t)\ N(-d_1)\ \sigma_A \tag{2.27}$$

II.7. PENNACCHI'S PRICING MODEL WITH FDIC RESOLUTION POLICY

By considering banks' incentives for risk taking under alternative regulatory policies of the insurance agency, Pennacchi (1987) in his paper extended Merton's cost of deposit insurance model and derived continuous time pricing formulas for a fair value deposit insurance premium and the equilibrium value of bank equity. Following Merton's model,Pennacchi allows random auditing for the government agency. He assumes that no transaction costs occur in selling or buying assets in the economy except the individual investors. Even the bank's deposits are fully insured, the banks are assumed to have some market power in the deposit market.

The other three assumptions he made are as follows:

(1) There are only two groups of assets for the bank: loans and marketable securities. the market value of bank assets (V) follows the continuous time diffusion process:

$$dV = (u_V V - C)dt + s_V V dz \tag{2.28}$$

where u_V is the instantaneous expected return on bank's assets, C is the total net payouts per unit time from the bank such as dividend payments or the premium payments for deposit insurance and s_V is the instantaneous nonstochastic standard deviation of the return on bank assets.

(2) The current value of a riskless discount bond that pays $1 at maturity date

T at t, P (t,T) follows the diffusion process $dP(T)/P = u_p(T,t)dt + S_P(T)dq$
$(P(0,t) = 1)$ (2.29)

(3) The total deposits of the bank (D) follow the diffusion process $dD/D = (u_d+n)\ dt + S_d\ dq$ (2.30)

where $S_d = S_p(T)$ and $u_d = u_P - m$

Based on these assumptions, we can write the value of the deposit insurance premium G as a function of V and D and G (V,D) satisfies the following equation

$$dG(V,D) = [G_1(u_V V - C) + G_2(u_d+n)D + (1/2)G_{11}S^2_V V^2 + G_{12}\rho S_V S_d VD + (1/2)G_{22}S^2_d D^2]dt + G_1 S_V Vdz + G_2 d_d D\ dq - I_{A\ P}(aD+\gamma G) + I_{A\ N}(V-(a+\emptyset)D-G) \quad (2.31)$$

where $I_{A\ P} = 1$ if an audit occurs and $V > \emptyset D$; otherwise $= 0$

$I_{A\ N} = 1$ if an audit occurs and $V < \emptyset D$; otherwise $= 0$, and the subscripts of G (V,D) denote the partial derivatives.

In equation (2.31), the symbol γ is used to denote the difference between the insuring agency implementing a fixed rate scheme ($\gamma=0$) and a variable rate or risk sensitive ($\gamma=1$). Suppose $\gamma=1$ in (2.31). Then, when an audit occurs and the bank has positive capital, the insuring agency's claims, G, returns to zero. This will reflect the fact that the premium charged the bank, h, is adjusted to a new fair value rate such that $G(h^*)=0$. For the case of fixed rate, i.e., $\gamma=0$, there is no premium adjustment and hence the insuring agency's liability is not changed at that moment, i.e.,

$$(1/2)G_{11}S^2_V V^2 + G_{12}\rho S_V S_d VD + (1/2)G_{22}S^2_d D^2 + (nD-\delta V)G_1 + G_2(n-m)D + hD(1-G_1) - \lambda(aD+\gamma G) + I_N\lambda(V-\emptyset D-(1-\gamma)G) = 0 \quad (2.32)$$

where $I_N = 1$ if $V < \emptyset D$;
$= 0$ otherwise,

In his model, Pennacchi defines $g = G/d$ as the government agency's claim per dollar of insured deposit and rewrites equation (2.32) as the following equation

$$x^2 g''_1 + (2/Q)\,[(m-n-\delta)x+n-h]\,g'_1 + (2/Q)\,(n-m-\gamma\lambda)g_1 + (2/Q)\,(h-\lambda a) = 0 \qquad (2.33)$$

$$x^2 g''_2 + (2/Q)\,[(m-n-\delta)x+n-h]\,g_2 + (2/Q)\,(n-m-\lambda)g_2 + (2/Q)\,[h+\lambda(x-(\varnothing+a))] = 0 \qquad (2.34)$$

where $g'=\partial g/\partial x$, $g''=\partial^2 g/\partial x^2$, and $Q=s^2_V+s^2_d-2\rho s_V s_d$

Therefore, the government agency's claim per claim of insured deposit, g, must satisfy (2.27) and (2.28) subject to the boundary conditions,

$$\lim_{x\to\infty} |\, g_1(x) \,| < \infty ,$$

and $g_1(\varnothing) = g_2(\varnothing)$, $g'_1(\varnothing) = g'_2(\varnothing)$, $g_2(0) = -\varnothing$.

Hence, the valuation formula of the value of the insuring agency's claim per dollar deposit can be represented as

$$g(x)=c_{12}x^{\tau 12}F(-\tau_{12},1+u_1,\ -2(h-n)\ /\ Qx) + (h-\lambda a)\ /\ (m+\gamma\lambda-n) \qquad (2.35)$$

with $x \geq \varnothing$ where $u_1 = [(1-(2/Q)\ (m-n-\delta))^2+8\ (m+\gamma\lambda-n)/Q)^{1/2}$,

$u_2 = [(1-(2/Q)\ (m-n-\delta))^2+8\ (m+\lambda-n)/Q]^{1/2}$,

$\tau_{11} = (1/2)\ [1-(2/Q)\ (m-n-\delta)+u_1]$

Pennacchi (1987a, 1987b) also shows that if the insuring agency charged at least a fair premium for deposit insurance, then banks would always prefer greater leverage. However, under a direct payment policy of resolving bank failure, bank would prefer a low leverage/risk strategy for sufficiently high interest rate margins and a high leverage/risk strategy for low margins. The policy implication of this result is as follows. Financial market deregulation will increase the entry into deposit markets and hence lower monopoly rents. Then, more banks will pursue high risk strategies given the current flat-rate premium system.

II.8. THE EVALUATION OF DEPOSIT INSURANCE WITH QUADRATIC LOSS FUNCTION APPROACH

Kendall and Levonian (1991) propose a simple insurance pricing schedules to match insurance premiums with the values derived from an option pricing model of deposit insurance. The method he uses is minimization of quadratic loss function, which compares a flat-rate pricing system with alternative pricing schedules incorporating measures of risk.

The argument of their model is as follows. Suppose the premiums paid is not exactly the same as the value of the deposit insurance and hence mispricing occurs, then in order to assess the social losses, large dollar differences between the value of deposit insurance and the premiums paid should be weighted more heavily than small dollar differences. Since both underpricing and overpricing reduce allocational efficiency, either type of mispricing should be treated in the same way. This suggest the use of a quadratic loss function defined as the sum across all banks of the squared differences between the fair-priced value of insurance and the premium paid. Let $L(\beta)$ be the quadratic loss function where β is the vector of parameters.

Let IP_i be the contingent-claim value of deposit insurance premium determined by

$$IP_t = D_t N(d_2) - V_t N(d_1) \qquad (2.36)$$

where $d_1 = \ln(D_t/V_t)\ (\sigma_A)^{-1}\ (T-t)^{-1/2} - (1/2)\sigma_A(T-t)^{1/2}$

$d_2 = d_1 + \sigma_A\ (T-t)^{1/2}$

$P(V^i;\beta)$ be the paid deposit insurance premium schedule depending on the vector of pricing variables (V_i) and vector of parameters (β)[3].

N be the number of insured banks and m be the number of elements of the parameter vecotr β. Then, the quadratic loss function will be

$$L(\beta) = \sum_{i=1}^{N} [IP_i - P(V_i;\beta)]^2 \qquad (2.37)$$

The first-order conditions for an i=1 interior minimum of L with respect to β are

$$\sum_{i=1}^{N} [IP_i - P(V_i;\ \beta)]\ \partial P/\partial\beta_j = 0 \text{ for all } j=1, \ldots, m. \qquad (2.38)$$

[3] Under the current flat-rate system, premiums are a constant fraction of total domestic deposits, i.e., $P(V_i;\beta)=\beta D$ where β was set at 0.0833 % of deposits or 8.33 basis points [bp] prior to 1990 and under FIRREA, it was set at 12 bp for 1990 and scheduled to rise to 15 bp in January 1991.)

Hence, the decision policies are determined through a two-level process. The first level policy choices are to determine the variables included in V and the shape of the premium function P. The second level policy choices are to determine the exact values of the parameter vector β. For each choice of P and the elements of V, it will have a different solution for β and hence a different minimized value of $L(\beta)$. For example, the first level policy constraint is given (i.e., the premium be a constant fraction of total domestic deposits) under the current flat-rate system, then the optimal value of β can be derived by solving the first-order condition.

By incorporating the market-value equity capital ratio as an indicator of bank solvency, we can eliminate much of the mispricing of deposit insurance arising from the flat-rate system. Hence, given the flat-rate policy as an optimal standard, we can determine the optimal pricing model which has the minimum loss function (Kendall and Levonian [1991]).

II.9. THE VALUATION OF FDIC DEPOSIT INSURANCE PREMIUMS WITH STOCHASTIC VOLATILITY

A model with stochastically changing variance could be helpful in testing whether the market prices option values correctly and in explaining both the stock return data and the options data (Johnson and Shanno [1987], PP.144). The reason is, as Johnson & Shanno pointed out, because it is difficult to know which value of the variance should be used in the Black-Scholes formula. Hull and White (1987) in their model derived a pricing formula for the call option by assuming that the volatility is independent of the stock price. In this section, we are going to apply their model to the valuation of FDIC deposit insurance based on the isomorphic correspondence between stock call option and bank equity, as well as between stock put option and deposit insurance premiums. In this valuation model, we are not only look at the case when the volatility of bank assets is uncorrelated with the market value of bank's assets, but also examine the case when the volatility is correlated with the market value of bank's assets.

II.9.1. The Evaluation of Bank Equity and the Cost of FDIC Deposit Insurance Premiums

A. Assumptions and Notations of the Model

(A). Notations:

A: the market value of bank assets.
B: the total debt of the bank at the time of next auditing.
C: the aggregate deposits of the bank at current time.
g: a known constant percentage growth rate in deposits.
R: the interest rate on banks deposits.
s: the interest rate paid in the form of services.
α: the instantaneous expected rate of return on the bank assets per unit time.
V: the instantaneous variance of the return on the bank assets per unit time ($V=\sigma^2$).
$\overline{V}$: the mean variance of bank assets return over the period between two consecutive auditing time.
μ_A: the drift parameter of A depending on A, V, and t.
μ_V: the drift parameter of V depending on V and t.
α: the drift parameter of V depending on V and t.
ξ: the parameter that may depend on V and t.
E: the market value of bank equity where $E_T = Max(V_T-B, 0)$ and $E_t=E(A_t, V_t, T\text{-}t; B)$.
ρ: the correlation coefficient between the stochastic volatility and the market value of bank's assets.
Dz_A, dz_V: standard Gauss-Wiener processes.
dw_A, dw_V: standard Gauss-Wiener Processes.
$G(A_t, V_t, T\text{-}t; B)$: the bank's FDIC deposit insurance value.

(B). Assumptions of the Basic Model

(1) The dynamics for the value of a given bank assets follow the diffusion-type stochastic process with stochastic differential equation (sde):

$$dA = [\alpha A - (R+s) D] dt + dD + \sigma A dz_A, \text{ or}$$
$$dA = [\alpha A - (R+s-g) D] dt + \sigma A dz_A. \tag{2.39}$$

Let $\mu_A = \alpha - (R+s-g)(D/A)$, then we can rewrite the diffusion process for the bank assets as follows:

$$dA = \mu_A A dt + V^{1/2} A dz_A \tag{2.40}$$

where $\mu_A = \mu_A(A,V,t)$ and $v^{1/2} = \sigma$.

(2) The variance of the bank's assets' return is stochastic and follows the stochastic

differential equation as $dV = \mu_V V dt + \xi V dz_V$ (2.41)
where $\mu_V = \mu_V(V,t)$, $\xi = \xi(V,t)$ and both do not depend on A. We also assume that the Guss-Wiener processes dz_A and dz_V are correlated with the correlation coefficient ρ, i.e., $E(dz_A dz_V) = \rho dt$ (2.41)'

(3) The dynamics for bank's aggregate deposits D is nonstochastic and can be described by $dD/dt = gD$. So, the amount of deposits at the next auditing time is $B = (1+g)D$.
(4) The market for the banking industry is competitive, i.e., free entry without barriers.
(5) The FDIC charges the bank a one-time premium to insure all the deposits.
(6) The deposits are guaranteed by the government or one of its agencies (e.g., FDIC) and there is no question on the FDIC's capability and willingness to meet its obligations.
(7) Trading in securities takes place continuously in time
(8) The securities in exchange markets are "sufficiently perfect" and asset-return dynamics follows continuous-time version of Capital Asset Pricing Model.
(9) The agents in the economy can borrow or lend at the same risk-free rate denoted by r, which must be constant or at least deterministic.
(10) the total deposits comprise of bank's total debt.
(11) There is no transaction costs and no surveillance or auditing costs in this model.

(C). Additional Assumptions for later development

(1) Both the regulator and the Banks are risk-neutral.
(2) The volatility has zero systematic risk and hence is uncorrelated with the aggregate consumption.

B. The Evaluation of Bank Equity and the Cost of FDIC Deposit Insurance Premiums

Merton (1977) in his paper presented a systematic theory for determining the cost of deposit insurance. As it is well-known that the Black-Scholes techniques can be applied to the pricing of corporate liabilities, we can use option as an appraisal tool for evaluating non-tradeable securities, such as insurance contracts. The foundation for such pricing model of the deposit insurance is the isomorphic relationship between deposit insurance and common stock put option.

Suppose there is no insurance, then the value of the bank equity at the maturity date will be Max [0, A−B] where A represents the market value of the bank assets and B is the value of the deposits at maturity date. If the banks buy the insurance for their deposits from the FDIC, then the value of this deposit insurance from the bank viewpoint is precisely a put option with the length of time until the next audit of the bank's assets as the length of time to maturity in regular put option and the value of the deposits at maturity date as their exercise price. That is, when the market value of bank's assets is less than the bank's deposit at the next auditing time, then the guarantor (FDIC) must pays off the deposits by the amount of $(B\text{-}V_T)$. Hence, we can value the deposit insurance as a put option, i.e., $G_T(.,.,.,.,.) =$ Max $(0, B-A_T)$ and G_t (.,.,.) will depend on the market value of bank's assets (A_t), the exercise price (B), the volatility of bank's assets (V_t), and the time length until the next auditing time.

Based on the above notations and assumptions, we can see that the only state variables affecting the bank equity value (E) is bank assets (A) and the volatility (V). Hence, E must satisfy the following differential equation.[4]

$$(\partial E/\partial t) + (1/2)\ [\sigma^2 A^2(\partial^2 E/\partial A^2) + 2\rho\sigma^3\xi A(\partial^2 E/\partial A\partial V) + \xi^2 V^2(\partial^2 E/\partial V^2) - rE$$
$$= -rA\ (\partial E/\partial A) - \mu_V\ \sigma^2\ (\partial E/\partial V) \qquad (2.42)$$

In order to solve (2.42) for the bank equity value E, we can apply the risk-neutral valuation method. As we assume that the volatility is uncorrelated with aggregate consumption, i.e., the volatility has zero systematic risk, the bank equity value will not depend on risk preference. Hence, we will confine our model in a risk-neutral world and the bank equity value is just the present value of the expected terminal residual value of bank equity discounted at the risk-free rate. Hence, we can write the bank equity value as

$$E(A_t, V_t, t) = e^{-r(T-t)} \int E(A_T, V_T, T)\ \Phi\ (A_T \mid A_t, V_t)dA_T \qquad (2.43)$$

where $\Phi(A_T \mid A_t, V_t)$ is the conditional distribution of A_T given the current market value of bank's assets A_T and variance V_t at time t[5]. Under the risk neutral world, at any time s, the bank assets value A_s will be given by $E(A_s \mid A_t) = A_t)$ $e^{r(s-t)}$. Let $\overline{V}$ represents the mean volatility over the two consectuive auditing time

[4] Any security f with a price that depends on state variable Θ_i must satisfy the differential equation $(\partial f/\partial t) + (1/2)\ \Sigma_{ij}\ \rho_{ij}\ \sigma_i\ \sigma_j\ (\partial^2 f/\partial\Theta_i\ \partial\Theta_j) - rf = \Sigma_i\ \Theta_i$ $(\partial f/\partial\Theta_i)\ [-\mu_i + \beta_i\ (\mu^* - r)]$ (see Hull & White [1987]).

[5] Equation (2.43) is comparable to equation (6) in Hull and White (1986).

periods which can be written

as $$\overline{V} = [1/(T\text{-}t)] \int_t^T \sigma_s^2 \, ds \qquad (2.44)$$

Then, using the property of conditional density function, the conditional probability $\Phi\ (A_T | A_t,\ V_t)$ can be denoted by

$$\Phi\ (A_T \mid \sigma_t^2,\ A_t) = \int h(A_T \mid \overline{V})\ g(\overline{V} \mid \sigma_t^2,\ A_t)\ d\overline{V} \qquad (2.45) \text{ or}$$

$$\Phi\ (A_T \mid \sigma_t^2) = \int h(A_T \mid \overline{V})\ g(\overline{V} \mid \sigma_t^2)\ d\overline{V} \qquad (2.45)'$$

where h(.) and g(.) are conditional probability density functions[6]. If we plug (2.45) or (2.45)' into (2.43), then we can write the current value of bank equity as either

$$E(A_t,\ V_t,\ t) = e^{-r(T-t)} \iint E(A_T)\ h\ (A_T \mid \overline{V})\ g(\overline{V} \mid \sigma_t^2)\ dA_T\ d\overline{V} \qquad (2.46)$$

$$\text{or } E(A_t,\ V_t,\ t) = \int \{e^{-r(T-t)} \int E(A_T)\ h\ (A_T \mid \overline{V})\ dA_T\}\ g(\overline{V} \mid \sigma_t^2)\ d\overline{V} \qquad (2.47)$$

Now, let's examine the property of equation (2.47) by considering the correlation coefficient between the volatility and the market value of bank's assets ρ in some special cases.

(A) When the Volatility is Uncorrelated with the Market Value of Bank Assets (ρ=0)

When the volatility is uncorrelated with the bank equity value, then the bank equity value E should satisfy the following stochastic differential equation

$$(\partial E/\partial t) + (1/2)\ [\sigma^2 A^2 (\partial^2 E/\partial A^2) + \xi^2\ V^2\ (\partial^2 E/\partial V^2) - rE$$
$$= -rA\ (\partial E/\partial A) - \mu_V\ \sigma^2\ (\partial E/\partial V) \qquad (2.48)$$

Also, under the risk-neutral world, the market value of bank assets and its instantaneous variance V will follow the stochastic processes

$$dA = r\ A\ dt + \sigma\ A\ dw_A$$
$$dV = \sigma\ V\ dt + \xi\ V\ dw_V.$$

[6] The A_T in (2.45)' is suppressed to simplify the notation.

And the distribution of $\{A_T/A_0\}$ conditional on $\overline{V}$ is lognormal with mean $\exp^{rT-(\overline{V}T/2)}$ and variance exp $\overline{V}T$. (Note that the distribution of log $\{A_T/A_0\}$ is not normal). The reason behind this is as follows. Since the parameters of the lognormal distribution depend only on the initial bank assets value, the risk-free rate, and the mean variance over the insurance contract period, any path that the variance V may follow and that has the same value of mean variance V will produce the same lognormal distribution. (see Hull and White [1987]). This is also true even the variance is stochastic. Although there are an infinite number of paths that give the same mean variance, all of these paths still produce the same terminal distribution of bank assets value. Hence, the terminal distribution of the bank assets value given the mean variance is lognormal no matter whether the σ^2 is deterministic or stochastic.

Hence, the term inside the bracket in equation (2.47)

$$E(A_t, V_t, t) = \int [e^{-r(T-t)} \int E(A_T)\ h(A_T, V_T \mid \overline{V})\ dA_T]\ g\ (\overline{V} \mid \sigma^2) dV$$

is the Black-Scholes price for the bank equity value when the bank assets has a mean variance V. Hence. it can be represented by $C(\overline{V})$ as follows:

$$C(\overline{V}) = A_t\ N(d_1) - B\ e^{-r(T-t)}\ N(d_2) \qquad (2.49)$$

where $d_1 = \log(A_t/B) + (r + (\overline{V}/2))\ (T\text{-}t)$

and $d_2 = \log(A_t/B) + (r - (\overline{V}/2))\ (T\text{-}t)$.

(Recall that $B=(1+g)D$ is the exercise price in this model) So, the value of bank equity is given by the following equation

$$E(A_t, V_t, t) = \int C(\overline{V})\ g(\overline{V} \mid \sigma^2)\ dV \qquad (2.50)$$

the pricing equation of (2.50) is always true in a risk-neutral world when the volatility and bank equity value are instantaneous uncorrelated. Under our strong assumption that the volatility is uncorrelated with the aggregate consumption, this pricing equation is also true in a risky world. If we can derive the analytic form for the distribution of $\overline{V}$ with reasonable assumptions about the process driving V, then we can comput the bank equity value in equation (H) which is just the Black-Scholes price integrated over the distribution of the mean volatility.

Finally, based on the put-call parity, we can compute the cost of FDIC deposit insurance value, $G(A_t, \sigma^2, T\text{-}t;B)$, as follows.

$$G(A_t, V^t, T\text{-}t; B) = E(A_t, V_t, T\text{-}t; B) - A_t + Be^{-r(T-t)} \quad (2.51)$$

where $E(A_t, V_t, T\text{-}t; B) = \int C(\bar{V})\, g(\bar{V}) \mid \sigma^2)\, d\bar{V}$ and

$$C(V\bar{V}) = A_t\, N(d_1) - B\, e^{-r(T-t)}\, N(d_2)$$

$$d_1 = \log(A_t/B) + (r + (\bar{V}/2))\, (T-t)$$

$$d_2 = \log(A_t/B) + (r - (\bar{V}/2))\, (T-t).$$

Equation (2.51) gives us the value of FDIC deposit insurance Premiums derived by the option pricing model under the assumption that the volatility of the bank assets is stochastic.

(B). When the Volatility is Correlated with the Market Value of Bank Assets ($\rho \neq 0$)

After deriving the valuation formula for the bank equity and deposit insurance in the case of zero correlation between the volatility and the market value of bank assets, let's relax the assumption by allowing the volatility to be correlated with the bank assets value. From the previous derivation, the value of bank equity should satisfy the following differential equation given that the volatility is uncorrelated with aggregate consumption.

$$(\partial E/\partial t) + (1/2)\, [\sigma^2 A^2(\partial^2 E/\partial A^2) + 2\rho\sigma^3 \xi A\, (\partial^2 E/\partial A \partial V) + \xi^2 V^2\, (\partial^2 E/\partial V^2)$$
$$-rE = -rA\, (\partial E/\partial A) - \mu_V\, \sigma^2\, (\partial E/\partial V) \quad (2.42)$$

Since the volatility V and the bank assets value are now instantaneously correlated. The property of the log $\{A_T/A_0\}$ can not hold anymore. To see this, let A_j be the bank assets value at the end of ith period. Assume the variance changes at only n equally spaced times in the interval of two consecutive auditing time. Let V_{j-1} be the volatility during the i period. Then, log $\{A_j/A_{j-1}\}$ and log $\{V_j/V_{j-1}\}$ are normal distributions that in the limit have correlation ρ. So, the distribution of log $\{A_T/A_0\}$ conditional on the path followed by V has a normal distribution with mean and variance depending on the attributes of the path followed by V other than $\bar{V}$. Thus, we can not determine the term inside the bracket of equation (2.47)

$$E(A_t,\ V_t,\ t)=\int [e^{-r(T-t)} \int E(A_T)\ h(A_T,\ V_T \mid \overline{V})\ dA_T]g\ (\overline{V} \mid \sigma^2)\ d\overline{V}$$

and there will not be analytic form for the solution of the bank equity value. By the same argument as before, we can apply the put-call parity to compute the value of FDIC deposit insurance based on the bank equity value, i.e., the value of FDIC deposit insurance, $G(A_t,\ \sigma^2,\ T-t;B)$, will be

$$G(A_t,\ V^t,\ T-t;B)=\ E(A_t,\ V_t,\ T-t;B)\ -\ A_t\ +\ Be^{-r(T-t)} \tag{2.52}$$

where $E(A_t,\ V_t,\ t) =\int [e^{-r(T-t)} \int E(A_T)\ h(A_T)\ h(A_T,\ V_T \mid \overline{V})\ dA_T]g\ (\overline{V} \mid \sigma^2)\ d\overline{V}$

and $\quad E(A_T) = Max(V_T - B,\ 0)$

Now, let's look at one special case when the volatility is perfectly correlated with the bank equity value.

Case 1: $\rho=1$ (The volatility is perfectly positively correlated with the market value of bank's assets)

In this, the bank equity value should satisfy the following differential equation

$$(\partial E/\partial t)\ +\ (1/2)\ [\sigma^2 A^2\ (\partial^2 E/\partial A^2)\ +\ 2\sigma^3\xi A\ (\partial^2 E/\partial A\partial V)\ +\ \xi^2 V^2(\partial^2 E/\partial V^2)$$
$$-rE\ =\ -rA\ (\partial E/\partial A)\ -\ \mu_V\ \sigma^2\ (\partial E/\partial V) \tag{2.53}$$

Case 2: $\rho=-1$ (The volatility is perfectly negatively correlated with the market value of bank's assets)

$$(\partial E/\partial t)\ +\ (1/2)\ [\sigma^2 A^2\ (\partial^2 E/\partial A^2)\ -\ 2\sigma^3\xi A\ (\partial^2 E/\partial A\partial V)\ +\ \xi^2 V^2(\partial^2 E/\partial V^2)$$
$$-rE\ =\ -rA\ (\partial E/\partial A)\ -\ \mu_V\ \sigma^2\ (\partial E/\partial V) \tag{2.54}$$

In case 1, the bank equity value will be solved by the following formula (2.55) in addition to satisfying equations (2.53). In case 2, the bank equity value will be solved by the formula (2.55) in addition to satisfying equation (2.54).

$$E(A_t,\ V_t \mid t)=\int [e^{-r(T-t)} \int E(A_T)\ h\ (A_T,\ V_T \mid \overline{V})\ dA_T]\,g\ (\overline{V} \mid \sigma^2)\ d\overline{V} \tag{2.55}$$

where $\quad E(A_T) = Max(V_T - B,\ 0).$

Again, the value of deposit insurance will be

$$G(A_T, V^t, T-t;B) = E(A_t, V_t, T-t;B) - A_t + Be^{-r(T-t)}. \qquad (2.56)$$

II.10. SUMMARY AND CONCLUSION

In this section, we have examined and integrated alternative option pricing models for assessing the deposit insurance in the banking industry. We also proposed an alternative model for pricing the cost of deposit insurance with stochastic volatility assumptions. We started out with Merton's (1977) pathbreaking model of deposit insurance pricing. This model first applies the modern option pricing formula developed by Black and Scholes [1973] to the banking area. Then, we examined alternative pricing models for estimating deposit insurance premiums with different assumptions. These include (1) constant dividend payment (Marcus and Shaked [1984]), (2) stochastic interest rate (Ronn and Verma [1986]), (3) stable paretian distribution (McCulloch [1985]), (4) random examination schedule (King and O'Brien [1991]). We also examined Pennacchi's cost of deposit insurance model by considering the FDIC's resolution policies. Furthermore, we looked at Kendall and Levonian's simple pricing approach, which bases upon the quadratic loss function.

Finally, we derived the valuation model for deposit insurance with stochastic volatility assumption. This is done by applying the recent development of option pricing model with stochastic volatility to the valuation of deposit insurance premiums. Based on Hull and White's (1987) findings, the Black-Scholes price always overprices at-the-money options, but underprices options that are sufficiently deeply in- or out-of-the-money. Their findings provide a strong support to Buser, Chen and Kane's argument that FDIC undercharged the insurance premiums. Also, these findings will present a possible counter-argument to Marcus and Shaked's findings that the FDIC overcharged the insurance premiums in general.

III. ALTERNATIVE NON-OPTION PRICING MODELS FOR ASSESSING FDIC DEPOSIT INSURANCE PREMIUMS

In this section, we will focus on the non-option pricing model for estimating deposit insurance. We will examine both the risk-based contingency model as well as the macroeconomic model of pricing deposit insurance, which considers the incentive-compatibility and risk-shifting issues.

In recent years, there have been several specific proposals made by the federal regulatory agencies for basing insurance premiums or capital requirements on the perceived risk of depository institutions. The FDIC's proposal for risk-based deposit insurance utilizes two measures for assessing bank risk-taking. The first measure is based on examiner-determined CAMEL ratings for individual commercial banks[7]. The second measure of bank risk employed is a index weighted by six ratio variables developed by the FDIC based on publicly available Call Report data[8]. The weight in this index were estimated from historical data with a probit model that predicts whether or not an individual bank is on the FDIC's problem-bank list. This index can be interpreted as providing a measure of the likelihood that a bank is a problem bank. Banks with higher index values are more likely to be problem institutions; therefore, these banks are more likely to impose higher expected costs on the FDIC.

Sharpe (1978) provided a formal setting for the analysis of the capital adequacy of a bank with deposits insured by a third-party by employing a complete market, state-preference model. He claims that the present value of FDIC's guarantee liability will depend on the risk of the bank's assets, the interest rate risk associated with the deposits, and the ratio of the economic value of the bank's assets to the default-free value of its deposits.

Goodman and Santomero (1986) and John, John and Senbet (1991) both develop a pricing model from the social planner point of view. Goodman and Santomero ar gue that appropriate deposit insurance pricing model must take into account the social costs associated with both the financial-sector's bank failure and the real-sector's firm bankruptcy. John, John and Senbet argue that the risk-shifting incentives of banks arise from the existence of limited liability and the associated convex payoff to equityholders. They claim that the incentive problem cannot be solved through a risk-based insurance premium. They then propose a solution that eliminates risk-shifting through an optimal tax structure and specify a corresponding fairly priced insurance premium.

In this section, we are going to examine the alternative non-potion pricing models for estimating the cost of deposit insurance. In section III.1., we examine

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[7] CAMEL's five items will be discussed in section III.1.3.

[8] The six ratios are (a) net income/total assets (b) net loan charge-offs/total assets (c) renegotiated loans to total assets (d) nonaccruing loans to total assets (e) loans past due over 90 days to total assets (f) primary capital to total assets.

the risk-based contingency model to predict the probability of bank failure. The statistical methods used include multiple discriminant analysis, the logit model analysis as well as profit analysis. We also look at how the regulator use so-called CAMEL ratings model to predict the probability of each bank going insolvent.In section III.2, we examine Sharpe's (1978) state-preference model to estimate each bank's risk. Buser, Chen and Kane (1981) extend Sharpe's system based on the Miller and Modigliani (MM) framework. In section III.3, we will investigate the cost of the deposit insurance model in both the real and financial sectors. Any optimal insurance scheme must incorporate social considerations into any proposal and hence the current flat-rate system may be a reasonable second-best solution. Both the incentive-compatible and risk-shifting incentive models in the banking industry are discussed in section III.4. Chan, Greenbaum and Thakor develop an incentive-compatible pricing model of deposit insurance cost. The risk-shifting with tax structure model, developed by John, John and Senbet, first brings in the corporate finance content to the banking area.

III.1. RISK-BASED BANK-FAILURE CONTINGENCY MODEL

III.1.1. Introduction

Since the establishment of the FDIC in 1933, more than 1,500 banks have been declared officially insolvent and were subsequently closed, acquired or received assistance to prevent closure (see table 4-3). Failure is defined as the event of the bank's capital account becoming zero or negative, and the probability of this event is estimated by considering period-to-period changes in the capital account as a random variable whose distribution remains stationary over time (Santomero and Vinso [1977]; Martin [1977]).

The potentially adverse consequences of bank failures include financial losses to bank stockholders and creditors, disruptions of community banking arrangements, contagious losses of confidence in other banks and widespred financial distress caused by sharp contractions in the money supply (Benston et al. [1986]). Due to the severe consequences of bank failure, a warning system is very important for the insurance agencies in order to prevent the failure of insured banks or thrift institutions.

Most earlier studies on thrift-institutions and commercial bank failures can be categorized into two groups (see table 3-1). The first group of studies tries to develop early warning systems that are capable of mimicking the regulator's evaluation process (Altman [1977]; Martin [1977]; and Sinkey [1975]). The second group of studies focused on identifying financial factors that affect the likelihood of bank's closure and attempts to explain why bank fails (Avery and Hanweck [1984]; Barth et al. [1985]; and Benston [1985])

The hypothesis of the empirical studies for the first group is that appropriately selected financial ratios, which are designed to measure CAMEL's five categories of information should be able to statistically discriminate between problem and nonproblem banks. This group authors analyze financial ratios constructed from the balance sheets and income statements that institutions file regularly with regulators and incorporate them into monitoring systems. To identify the troubled banks, they typically fit cross-sectional models for each year into their sample periods. However, their evaluation of financial condition is accurate only to the extent that book values equal market value.

The second group of studies not only attempts to explain why bank fails, but also try to determine the important financial ratios, which will affect the likelihood of bank's failure. In order to pinpoint the determinants of bank closure, they use cross-sectional data over a given sample period or cross-sectional data pooled from different years to analyze the same types of financial ratios used by the first group of studies. Although this group authors acknowledge the conceptual distinction between economic insolvency and failure, their models contain the same financial ratios used in the first group of studies.

The criterion of choosing independent variables are based on the intention to proxy different dimensions of the CAMEL rating system in both groups of studies. The normal approach starts out with either a large number of financial ratios that cover all the CAMEL categories or selected financial ratios that were found to be significant in earlier studies.

The methodologies used in these empirical studies include logit model analysis, probit model analysis, and multiple discriminant analysis (MDA) (see table 3-1). Martin (1977) first applies logit probability model to evaluate commercial bank failure. He uses 1974 data on 23 failed and 5,575 nonfailed commercial banks and analyzes 25 ratios chosen for their usefulness in previous studies. Avery and Hanweck (1984), Barth et al (1985), Benston (1985), and Gajewski (1988) also apply the logit model to evaluate the failure probability of commercial banks/thrift institutions. Sinkey (1975), Altman (1977), and others use multiple discriminant analysis to analyze problem and nonproblem banks or thrift institutions. Avery et al. (1985) apply the probit model to evaluate the probability of bank failure for the FDIC insured commercial banks. The dependent and independent variables they use, the sample and sample period they choose, and the significant financial ratios they find are summarized in tables 3-1 and 3-2.

There are two primary goals for the risk-based deposit insurance system. One is to deter excessively risk taking by banks up to the point where the marginal benefits, or revenues, from risk equal the incremental costs imposed not only on themselves, but on society as well. Thus, in deciding whether to take on additional risk, banks would be forced to include or internalize the total cost of this risk in their decision-making. The other one is to provide an equitable distribution of the costs of risk-taking across high and low risk banks. So, high risk banks should pay higher premiums than low risk banks (Avery, Hanweck & Kwast[1985]).

A risk-based deposit insurance system would explicitly price risk-taking behavior on the part of insured banks. Periodically, the FDIC would assess the risk represented by each bank and charge an insurance premium reflecting the expected social costs attributable to it. Because the banks would in principle bear the full expected cost of their actions, they would either be deterred from excessive risk-taking or would pay the full expected costs to the FDIC.

A risk-based capital standard works by setting a standard that limits the amount of risk an insured bank can impose on the FDIC, rather than by explicitly pricing risk. If a bank's risk is determined by regulators to be above the allowable standard at its current level of capital, then it would be required to raise more capital. By adjusting capital "buffers", regulators can control the size of potential losses irrespective of bank behavior.

A major difference between the risk-based capital and risk-based deposit insurance systems is the type of information used to assess bank risk-taking. The risk-based deposit insurance system focuses of bank performance such as earnings and asset quality; the risk-based capital system focuses on the type of activities in which banks are involved. The risk-based insurance premium approach is based on the statistical evidence, which suggests that performance measure provide the best forecast of future bank problems. The risk-based capital approach to measuring bank risk-taking is based on the view that certain activities are inherently more risky than other activitieis and that these more risky activities should be capitalized at higher levels.

The risk-based system is based on a two part formula, which include statistical estimates of (1) the probability of bank failure and (2) the FDIC's actual costs when banks fail. The product of the probability of failure and the cost of failure provides an estimate of the expected insurance cost to the FDIC of each bank in a given year. This expected cost is the risk-based portion of the deposit insurance premium. A fixed premium component is added to the risk-based portion in order to cover existing FDIC administration costs and yearly additions to the insurance fund. In the other words, the total gross (before rebate) revenue collected by the FDIC from the risk-based system is the same as the total revenue collected under the present fixed-rate system (Avery & Hanweck [1984]; Barth, Brumbaugh, Sauerhaft & Wang [1985]).

Both the risk-based capital and risk-based insurance premium systems require an accurate method of assessing bank risk. The procedure needed to form an index or rank ordering of banks by risk can be described by two steps. First, select variables that are good predictors of risk. The second step is to use historical data on bank failure to estimate weights in order to transform values of the predictor variables into a single-valued index of risk.

III.1.2. A Risk-Based Bank-Failure Model using Multiple Discriminant Analysis, Logit Model and Probit Model Approaches

In this section, we are going to review the designation of a risk-based deposit insurance valuation system. This system is based on the statistical estimates of both the probability of bank failure and the FDIC's actual costs when banks fail. This risk-based deposit insurance system will provide an equitable distribution of insurance premiums across high and low risk banks and hopefully will deter risk taking by banks. In other words, the high risk firms should pay higher premiums than low risk firms and banks would be forced to include or internalize the total cost of the additional risk they take on.

To compute the premiums for banks, we use the publicly available data to estimate the expected insurance cost to the FDIC by multiplying the probability of failure and the FDIC's actual costs when bank fail. However, the estimated expected insurance cost is only the risk-based portion of the deposit insurance premium, which is based on bank's riskiness and set so as to recover expected costs. To cover existing FDIC administration costs, we need to add a fixed amount premium up to get to total deposit insurance premiums. Hence, the total gross revenue collected by the FDIC will be equal to that collected under the current flat-rate system.

III.1.2.a. A Risk-Based Contingency of Bank Failure Model Using the Discriminant Analysis Approach

Sinkey's (1975) and Altman (1977) used linear multiple discriminant analysis to perform a problem-bank and troubled S&L institutions studies respectively. The data Sinkey evaluated includes 220 problem and nonproblem (mostly small) commercial banks for the period 1969-1972. Half of the sample consists of commercial banks that were listed as problem banks by the FDIC in 1972 and early 1973. He matched each problem bank with a nonproblem bank according to the following characteristics: total deposits, number of banking offices, geographic market area, and Federal Reserve membership status. He concluded that the classification accuracy of the model is low due to group overlap among the problem and nonproblem banks.

Altman (1977) examined 212 S&L associations during the period 1966-1973 by categorizing into three groups: serious problems, temporary problems, and no problems. In order to cover CAMEL categories, Altman included seven variables in his best predictor model and concluded that the operating income and its trend are the most important discriminators.

The discriminant analysis specifies a joint distribution of dependent variable (Y_i) and independent variable (X_i) This method imposes a strong assumption of the multivariate notmality. It assumes that the independent variables are multivariate normal over both the population of nonfailures and the population of failures with equal covariance matrices. When X is multivariate normal, then the discriminant analysis estimator will be the Maximum Likelihood (ML) estimator. The rule for selecting potential failures can be written as

$$Z=C_0 + C_1 X_1 + C_2 X_2 + \ldots\ldots + C_k X_k > T \qquad (3.1)$$

where the coefficients C_0 through C_k are estimated parameters, the variables X_1 through X_k are financial ratios and T is a classification threshold.

In the discriminant analysis, the classification criterion is specified first. Then, all the parameters are estimated simultaneously. Since the emphasis in discriminant analysis is placed on classification and the criterion for classification may vary, the Z in the above equation does not have a standard interpretation, and the scale of the parameters C_0 through C_k and T is not uniquely determined. If the basic assumptions of linear discriminant analysis are exactly satisfied, then the classification rule will be equivalent to that of a logit model.

III.1.2.b. Estimation of Deposit Insurance Using the Logit Model Approach

Martin (1977), and Barth et al. (1985), and used the logit model to evaluate commercial bank failures. Before analyzing the group characteristics, Martin (1977) analyzed an institution's probability of becoming insolvent in a book-value sense. He obtained his best results using 23 failed and 5575 nonfailed commercial banks for 1974.

Barth et al (1985) used 318 closed and 588 non-closed saving and loans associations to analyze thrift institution closure based on a logit probability model. The data period they used covers the period from December 1981 to June 1984. They found five out of the 12 financial ratios as significant variables to measure capital adequacy, asset quality, earnings, and liquidity.

The model specifications of the logit model is as follows. Let (Y_i) be the dependent variable and X_i be the independent variables as before. We can select potential failures based on the same rule as that in equation (3.1). For the logit model, the correspondence between Z and the estimated failure probability is as follows:

$$\text{The Estimated Probability of Failure} = \left(\frac{1}{1+e^{-Z}} \right) \quad (3.2)$$

In the logit model, the parameters Co through C_k are uniquely determined but the parameter T is chosen according to a specified classification criterion after the probability estimation formula has been derived. The logit model does not require the assumption of multivariate normality for the independent variables. But, it assumes that the probability of failure conditional on particular values of the independent variables is logistic. The logit model is preferred over discriminant analysis because the significance of independent variables can be evaluated more easily. Hence, this thesis mainly will be based on logit model, while we also use probit model and disriminant analysis to supplement the analysis.

III.1.2.c. Estimation of Deposit Insurance Using Probit Model Approach

Avery et al (1985) used probit model to evaluate commercial bank failures. They extended earlier early-warning models by using probit analysis. This statistical method yields an easily interpreted measure of the probability of failure for each banks. The sample data, which consists of all federally insured commercial banks, was obtained from the FDIC data base for the years 1979-1981. He used six financial ratios as predictor variables and found that the classification ability of all these models diminished over time.

Avery, Hanweck and Kwast (1985) used 155 failed banks in the FDIC Call and Income Reports proceeding their failure and five percent random sample from other banks, totaled 5241 observations during 1981-1984. They used eleven financial ratios, drawn from the publicly available semiannual bank Call and Income Reports, as their predictor variables. By dividing the banks into three distinct groups based on the estimated risk, they found that 85 percent of banks are estimated to pay a lower insurance premium under the proposed risk-based system. Some 14 percent of all banks would pay higher premiums ranging from 8.3 to 100 b.p. of total domestic deposits. Only one percent of all banks would pay a premium of 100 b.p. or more of total domestic deposits.

For probit model, the selecting rules for the potential failures is the same as those in logit model and discriminant analysis (see equation 3.1). The Z in equation (3.1) for a probit model is a normal Z-score corresponding to the estimated failure probability as

$$(2\pi)^{-(1/2)} \int_{-\infty}^{Z} \exp\left(-[1/2]\, t^2\right) dt \qquad (3.3)$$

For probit model, the probability of failure conditional on particular values of the independent variables is assumed to be normal for probit model. Since the normal and logistic distributions are similarly shaped, probit and logit model analysis are very similar. The main difference between the two distributions is primarily in their extreme tails. However, the normal Z-scores estimated by a probit model are easier to interpret than the logit parameter.

III.1.3. Using CAMEL Ratings as the Prediction Model of Bank Failure

Under current flat-rate premium system, the regulators try to prevent deposit institutions from taking excessive risks that could lead them to economic insolvency through periodic examinations and continuous supervision. Nowadays, the supervision and examination of depository institutions are performed by the Federal Reserve, the Office of the Comptroller of Currency (OCC), the FDIC, the Office of Thrift Supervision (OTS) and the National Credit Union Administration (NCUA). The federal examiners of depository institutions mainly focus on the adequacy or inadequacy of the firms's capital account for meeting the particular forms of risk exposure. They also review and evaluate the institution's internal control system and managerial practices.

After the on-site examination, federal examiners prepare a formal report, which points out the strengths and weaknesses in the firm's operation. This report is further summarized into a five-point CAMEL ratings. CAMEL ratings are intended to measure the bank's capital adequacy (C), asset quality (A), management skills (M), earnings (E), and liquidity (L).

The component ratings of CAMEL categories are subjectively weighted by the examiner to arrive at an overall rating for the institution. The CAMEL system grades an institution on a five-point scale with 5 representing the least healthy bank. The FDIC's problem-bank list consists of all banks with CAMEL ratings of 4 and 5.

The second measure of bank risk employed is a risk index developed by the FDIC based on publicly available Call Report data[9]. The weights in this index were estimated from historical data with a probit model[10]. Probit model can predict whether or not an individual bank is on the FDIC's problem-bank list. This index can be interpreted as providing a measure of the likelihood that a bank is a problem bank. Banks with higher index values of the index are more likely to be problem institutions and therefore more likely to impose higher expected costs on the FDIC.

Premiums would be assessed by defining two premium classes. Banks having a positive value of the risk index and a CAMEL rating of 3, 4, 5, would be classified as above-normal risk. These institutions would be charged an annual premium equal to one-sixth of one-percent of domestic deposits, or twice the current premium level. All other institutions having either a negative value for the risk index or a CAMEL rating of 1 or 2 would be classified as normal-risk banks and be charged the current premium.

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[9] This risk index is a weighted average of six different ratios variables. They are (1) ratio of net income to total assets, (2) ratio of net loan charge-offs to total assets, (3) ratio of loans renegotiated to total assets, (4) ratio of nonaccruing loans to total assets, (5) ratio of loans more than 90 days past due to total assets, and (6) ratio of primary capital to total assets.

[10] The weights can be referred in chapter one.

III.1.4. Conclusion

Although each model analyze the relative discriminant power of different CAMEL categories, it is difficult to compare the findings of one study against another, due to differences in data sets, proxies, and interpretations. In this section, we have reviewed the statistical methods of predicting the bank failures, including the discriminatn analysis model, the logit as well as probit model and CAMEL ratings model.

One drawback of the discriminant analysis is that the assumption of multivariate normality for the independent variables may not be satisfied. Logit models is preferred over discriminant analysis because the significance of independent variables can be evaluated more easily and the models do not depend on the assumption of mutivariate normality for the independent variables.

Since the normal and logistic distributions are similarly shaped, probit and logit model analysis are very similar. The main difference between the two distributions is primarily in their extreme tails. However, the normal Z-scores estimated by a probit model are easier to interpret than the logit parameter. The shortcomings of the CAMEL ratings model is that the overall ratings for the banks are subjectively weighted by the examiners.

III.2. SHARPE'S STATE-PREFERENCE MODEL AND EXTENSIONS

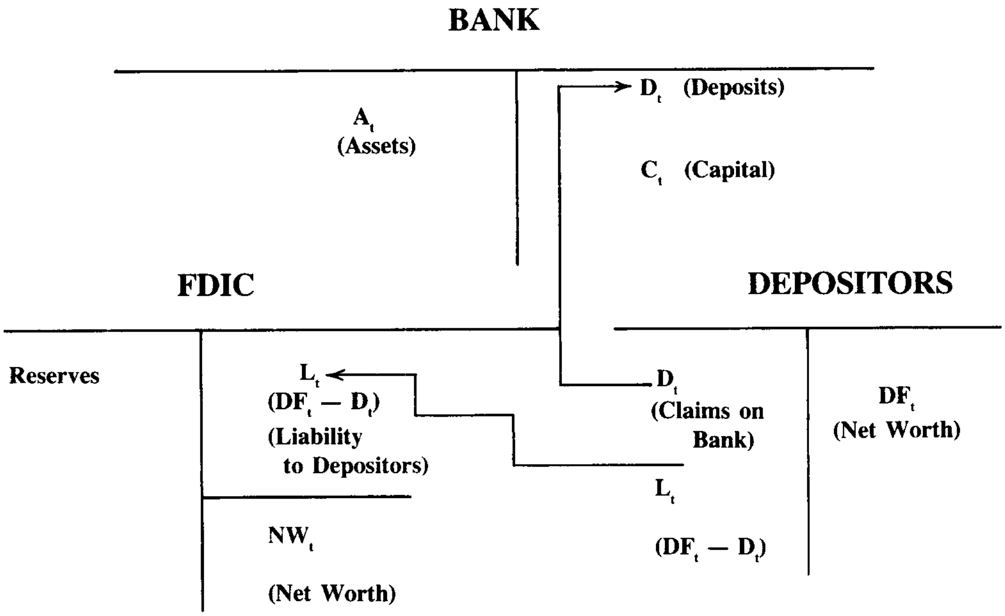
III.2.1. Sharpe's State-Preference Model

By employing a simple and powerful complete market, state-preference approach, Sharpe (1978) in his paper provided a formal setting for the analysis of the capital adequacy of an institution with deposits insured by a third party. The main goal is to describe the relationships among the values of financial institutions' assets and claims on those assets (see figure 3.1). In the bank's balance sheet, the market value of bank's assets are totally financed by the claims on assets, including those of the debtholders (total deposits) and equityholders (total equity).

Since bank might not be able to pay off its depositor's claim in full and on time, the economic value of such claims will be less than it would be if there were no such risk. When there exists the third party e.g., the FDIC, to guarantee the deposits, then the net worth of the depositors will be default-free. Then, the

Figure 3-1

The Relationship Among the Bank, Depositors & FDIC



Where DF_t : the amount the Deposit Claims would be worth at time t if they were default free
L_t : the present value of Deposits for the Bank
D_t : the economic value of Deposits for the Bank
C_t : the economic value of Bank's capital
A_t : the economic value of Bank's Asset

(Sources: Sharpe, [1978], JFQA)

insured depositors will have two claims: one on the bank and the other on the FDIC. Due to the possibility of getting insolvent, the bank's liability to the depositors is only worth the amount of depositors'claims on bank.

In order to avoid a negative net worth, the FDIC should charge such a premium that will equalize the present value of its liability ($L_t = DF_t - D_t$). Under the current flat-rate premiums system, the FDIC should charge a predetermined flat-rate premium no less than the difference between the value of the deposit claims (D_t) and the default-free value of deposit (DF_t). In mathematical terms, let p be the flat-rate premiums per dollar of deposits, then p should be charged so that it satisfies

$$p * DF_t + D_t \geq DF_t \text{ or}$$

$$p \geq (DF_t - D_t)/DF_t. \qquad (3.4)$$

In his model, Sharpe made the following assumptions:

(1) there is no transaction and information costs, (2) all deposits are insured, (3) FDIC insures a bank for one period, and (4) bank issues certificates of deposits, which promise total payments of (P_1, P_2, . . ., P_N) at time 1, 2, . ., N. Then, based on these assumptions, Sharpe showed that the greater the amount of assets covering deposits, the smaller will be the difference between the actual value of the deposits and the default-free value. The difference of these two values is exactly the value of the FDIC liability per unit of deposits. Given the relevant risks, an increase in the ratio of assets to the default-free value of deposits will reduce the per-unit value of FDIC libability. Finally, for any amount of risk, there will be some amount of capital that will make the per-unit liability equal of any preselected premium. This amount of capital is so-called the adequate amount of capital.

In general, the present value of FDIC's guarantee liability will depend on (1) the risk of bank's assets, (2) the interest rate risk associated with the deposits, (3) ratio of the economic value of the bank's assets to the default-free value of its deposits, and (4) the relationship between the risk of bank's assets and the interest rate risk associated with the deposits. The bank will have ''adequate capital'' only when the value of the FDIC's liability is no larger than the deposit insurance premium (see equation 3.4). So, from equation (3.4), we can see that determination of a bank's capital adequacy requires both an assessment of the market values of all assets and libabilities and the estimation of all relevent risks.

Sharpe also pointed out that we can use an economietric model to gain information about the capital adequacy. This model uses the change in market value of a bank's equity as dependent variable. Finally, Sharpe suggested to use the surrogates for change in asset value, in default-free liability value and in asset risk multiplied by the assets value as the predictor variables.

III.2.2. Buser, Chen and Kane's MM Pricing MOdel

By extending Sharpe's model, Buser, Chen and Kane (1981) asserted that a value-maximizing firm may reach an internally optimal leverage level with positive equity in its capital structure if it satisfies the following conditions. First,the interest expenses on debt is tax-deductible. Second, the bankruptcy is costly (see figure 3.2.1). They also analyzed what the effects of having a governmental agency (the FDIC) guarantee a bank's debt will be. They conclude that the FDIC deliberately sets its explicit insurance premium below market value in order to entice state-chartered nonmember banks to submit themselves to FDIC's regulation.

What the impact FDIC's free charge of insurance will have on the value of the firm was shown in figure 3.2 With the free insurance and costless bankruptcy, the zero-equity corner solution will arise. In order to set a fair-valued insurance premiums, FDIC should charge such a rate that the cuminsurance value of the bank net of this premium will coincide with the uninsured value at each level of deposits.

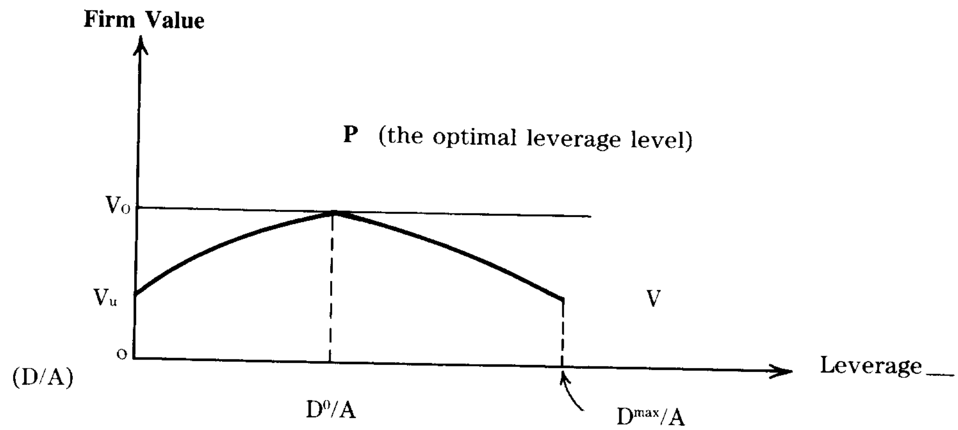
In order to persuade banks to accept its regulation, FDIC would design a mutually acceptable insurance contract. However, there is a limit for FDIC to design such a contract. The limit of the opportunity set for FDIC is shown in figure 3.2 The impact of FDIC's regulation on the insured banks's value is shown in figure 3.2 The shaded area of $(V-V_{IR})$ is the net benefit to the firm from trading bankruptcy cost without insurance for FDIC's regulation. Figure 3.2 shows that the optimal capital structure for the bank with both the implicit and explicit insurance premium. From this figure, we can see that the optimal leverage level for bank with both the explicit and implicit premiums is less than that with only explicitly fixed flat-rate insurance premiums.

Figure 3.3 shows the probabilty density function conditional on bank's current portfolio against every possible value of bank's capital on the next examination date. If a bank's capital falls into state I, then the bank will have adequate capital.

Figure 3-2

The Value of Deposit Insurance With Flat-Rate Premiums and Variable-Rate Premiums System

3-2-1. No Insurance Case



where P : the optimal leverage level
V_u: market value in unlevered state
V : market value of uninsured bank

3-2-2. With Free Charge Insurance (zero-equity corner solution)

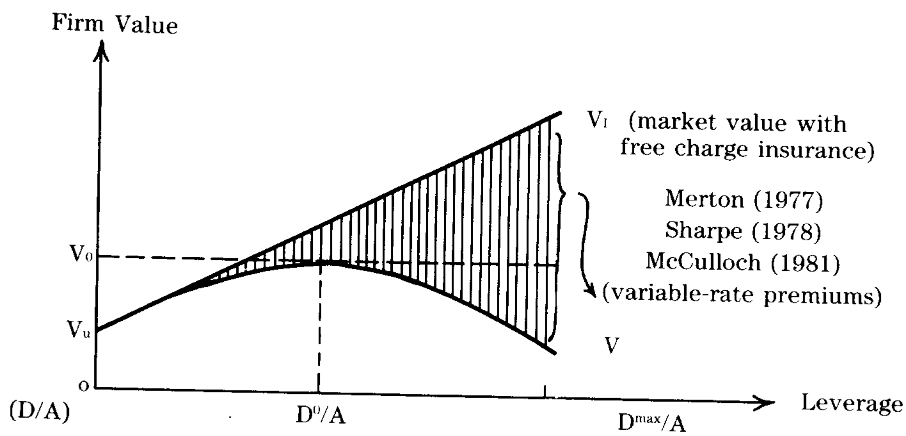
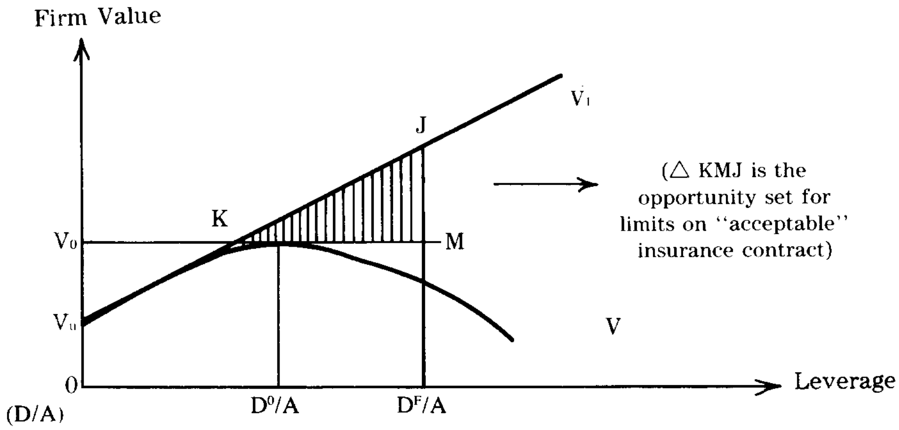


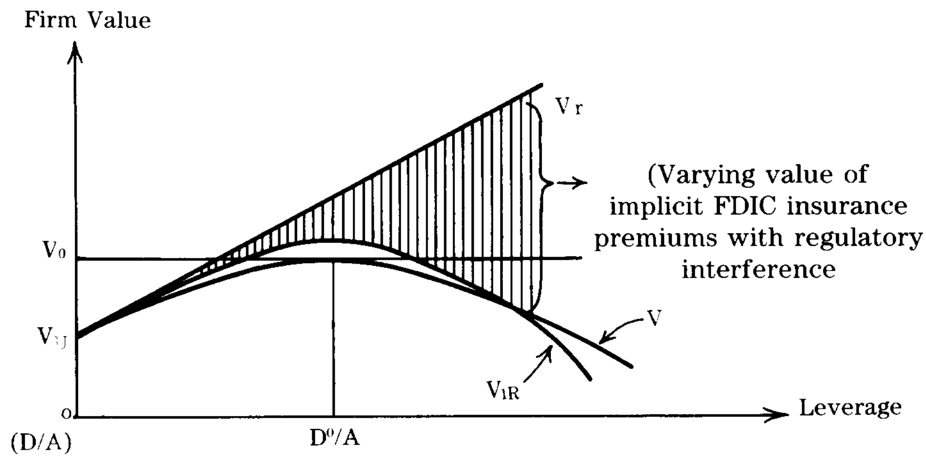
Figure 3-2 (cont'd)

3-2-3. The Acceptable Insurance Contract With Regulation



Note: D^F is FDIC's maximum acceptable level of deposit.

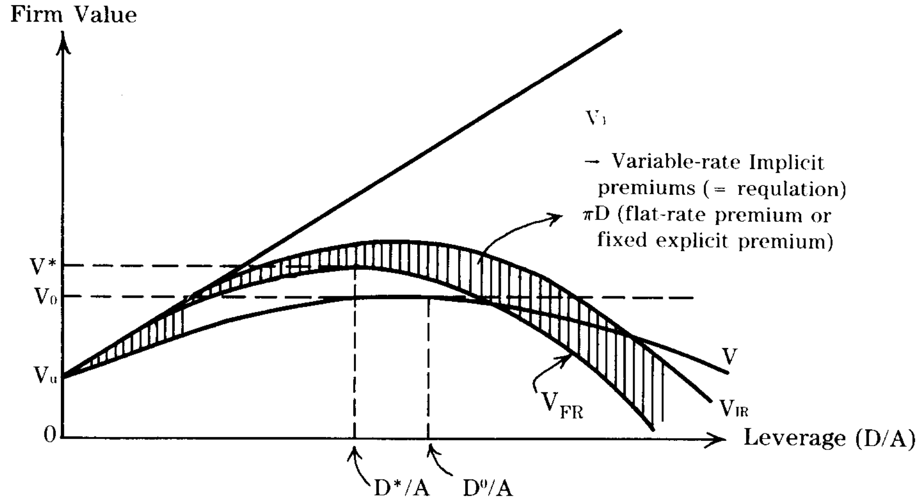
3-2-4. The Impact of Regulation on the Insured Bank's Value



Note: The shaded area of $V_I - V_{IR}$ is the net benefit to the firm from trading costly bankruptcy cost without insurance for FDIC regulation.
V_{IR} is the market value of insured bank with implicit premiums (or FDIC regulation)

Figure 3-2 (cont'd)

3-2-5. Optimal Capital Structure with Implicit and Explicit Insurance Premiums

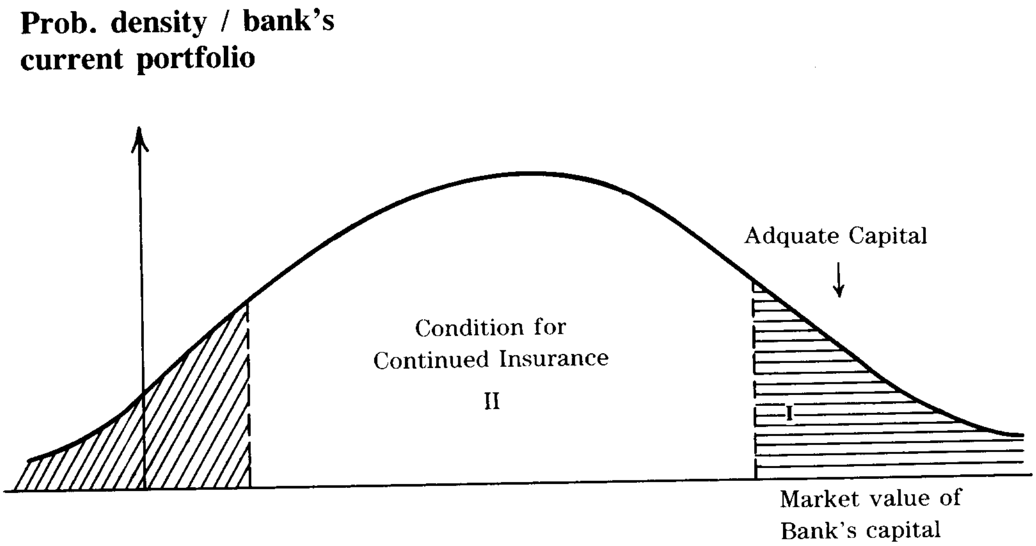


Note: D* is the optimal level of insured deposits
π is the flat-rate premium per $1 deposit.
V_{FIR} is the market value of insured bank with flat-rate premium system.

(Source: Buser, Chen, and kane, [1981], JF)

Figure 3-3

Capital Adequacy and Deposit Insurance



Note: 1. In state II, bank manager should strengthen bank's asset and liability portfolio or rasing new capital or restricting dividend payment in order to get continued insurance.

2. In state III, Only when the market value of bank's portfolio and physical assets falls below that of its deposit liability, then FDIC forces a bank to "technical insolvency".

3. Minimum Level for Capital Adequacy (MLCA): New York Federal Reserve Bank assigns a specific weight (ranging from 0 to 1) to each asset category and calculate the sum of these weighted value as the MLCA.

If it falls into state II, then the bank managers will be forced to strengthen bank's asset and liability portfolio or raising new capital, or restricting dividend payment in order to get continued insurance.

Once the market value of bank's portfolio and assets is less than that of its deposit liability, FDIC will force a bank to ''technical insolvency''.

(to be continued)