INSURERER'S RISKS AND UNDERWRITING CYCLES IN PROPERTY-LIABILITY INSURANCE INDUSTRY

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摘 要

本文主要目的是對於產險利潤循環之原因提出一新的解釋。

產險之核保利潤長久以來皆呈現週期性波動，此種波動於保險價格之穩定與保險公司之清償能力有重要影響。過去的研究大多認為利潤循環是由於保險人非理性的定價決策所造成，本研究則證明即使保險人以精算公平原則理性制定價格，仍可能出現利潤週期性波動。因此，本文結論認為利潤循環可能是產險業本身之產業特質所引起，而非保險人不理性的定價行為所造成。

本研究之結論雖與 Cummins & Outreville (1987) 之研究類似，認為產業特質是造成利潤循環的主要原因，然而他們並未能指出是何種特質引起，也未能說明為何不同保險產品其循環方式不相同。本文則明確指出損失理賠展期是造成循環的主因，且展期期間之長短會影響利潤波動的型式。

Abstract

This paper provides a potential explanation for the unanswered questions of underwriting cycles in the property-liability insurance industry. Underwriting profits of property-liability insurers have been characterized by significant cyclical fluctuations. Such fluctuations have important impact to insurance price and solvency.

Some of previous research attributed the reason for the cycles to insurer's irrational pricing decision. This paper shows that underwriting cycles may be generated even if price is set based on fair rate principle and the insurer's expectations are rational. Hence, this paper suggests that underwriting cycles may be caused by the inherent characteristics of the property-liability insurance industry.

The finding of this paper is consistent with the institutional intervention hypothesis proposed by Cummins and Outreville (1987). But this paper points out directly that the loss settlement delay is the major factor to cause the cycles. Furthermore, this paper shows that length of autocorrelation lag is related to length of loss settlement delays. These results have not been indicated in the previous literature yet.

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I. Introduction

Since at least the 1950's, underwriting returns of property-liability insurers in the U.S. have been characterized by significant cyclical fluctuations. Such fluctuations have important impact to insurance price and solvency. Research on underwriting cycles in property-liability insurance attributed the reasons for cycles to insurer's decision behaviour or market mechanism or institutional interventions.

Wilson (1981) and Stewart (1984) attribute cycles to insurers who decide to constrict supply and change prices to achieve financial objectives, e.g. investment income and operating ratios. The arguments in these articles imply that insurance prices are set subjectively to fit financial purposes of the insurers rather than to compensate the expected losses of the insured. Additionally, Venizian (1985) suggests that the rate-making procedures through extrapolation used by insurers caused the cycles.

Some other studies attribute cycles to market mechanism or institutional intervention factors, which are not under the control of individual insurers. For example, Cummins and Outreville (1987) suggest regulatory lag and accounting reporting procedures are possible causes of cycles. Doherty and Kang (1988) consider fluctuation of interest rates is the important reason for the cycles. Gron (1990) explains that the cycles are caused by insurer's capacity constraint.

The primary objective of this paper is to provide an explanation for underwriting cycles. We consider the cycles are a natural phenomenon in this industry due to several risks faced by the insurers. Basically, it belongs to the second category of cycles research; however, this paper adopts a different approach. Most previous research in the second category concerns the impact of external factors like financial and economic variables and analyzes all-lines combined ratios or profits. Although this paper concerns external impacts on underwriting returns, it also takes into account the differences between short-tail and long-tail insurance products because each line of insurance may have different underwriting characteristics, which is consistent with the study by Venezian (1985) and Warren-Boulton (1987).

This paper assumes that insurers set price equal to the fair rate based on rational expectations about losses. Then it is shown that underwriting cycles can be generated under these two assumptions due to loss settlement delays and forecasting risks which are special features of this industry.

The implication of the this analysis is to highlight the impact of the inevitable forecasting risk on insurance pricing and profits. Most of the previous academic insurance literature based on financial theory emphasized on the speculative risk of insurance products, while forecasting risk did not received much attention. Forecasting in fact is the key to insurance operation because an insurance contract may involve several time periods. Consequently, forecasting risk should not be ignored any more.

The remainder of this paper consists of the following sections. Section II describes the risks faced by the insurers. Section III discusses the loss settlement procedures. Assumptions and notations for pricing decision are presented in section IV. Section V shows the mathematical proof of autocorrelation functions for underwriting returns. Empirical evidence is provided in section VI to support the theoretical arguments. Conclusions and suggestions are in section VII.

II. Risks Faced by the Insurers

As the economics progress from traditional static analysis to the studies of uncertainty, "risk" becomes a day-to-day vocabulary in the academic papers. Each field creates its own terminology, e.g., systematic/unsystematic risk in finance, pure/speculative risk in insurance, ..., etc. Finally, it becomes confusing in some studies when they mix up the RISK's defined in different fields.

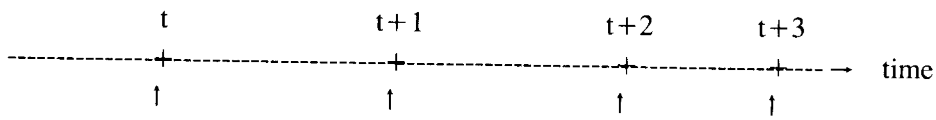
Cummins (1992) summarizes the risks which should be considered in insurance pricing decision. Six risks are pointed out in his paper. It is hard to have a neat formula for insurance price if it contains six risk variables simultaneously. In face these RISK's are not completely independent. Their relationships could be outlined by figure 1.

At the pricing moment, insurer (actuary) faces estimation risk of frequency and severity. Appropriate classification and pooling of the policyholders may diversify this risk.

Forecasting risk due to state of nature is not diversifiable even if insurer utilizes all the information available at pricing time, i.e., rational expectation (Lucas and Sargent, 1981). This is because human being cannot predict the future perfectly. Thus forecasting risk always exists since an insurance contract involves at least two timing points (t and t+1). Forecasting errors may result in default risk, and payout pattern risk is a special case of forecasting errors when long-tail insurance product is considered.

If we assume no insurer is naive or stupid, the forecasting error due to individual insurer's behaviour can be ignored. Consequently, the major reasons for forecasting error come from external factors of economy and society such as

Figure 1.
Insurer's Risks



	pricing	losses incurred & paid	losses developed and paid (for long-tail lines)
risk involved	frequency & severity risks ↓ estimation risk	default risk ↘ forecasting risk	payout pattern risks ↙ forecasting risk
reasons (causality)	law of large number	macroeconomic factors e.g., inflation (risk), interest rate fluctuation (risk) stock market fluctuations (risk)	
nature	diversifiable risk	undiversifiable risk	

economic and social inflations (Cummins and Nye, 1984), interest rate changes, and stock market fluctuations.

Although the insurer may reduce its portfolio risk by increasing the number of policyholders, forecasting risk due to environmental changes of different time periods is inevitable and cannot be reduced even through pooling. That is, the insurer faces a type of intertemporal risk (Machina, 1984).

III. The Loss Settlement Process

Property-liability insurance policies cover losses due to accidents occuring within a fixed period such as one year. However, the settlement of loss claims typically is not resolved until several years later. Particularly in the case of serious personal injuries, claimants are reluctant to settle until they have a clear picture of the prognosis on their injuries (Webb, et al, 1984).

Liability insurance claims may remain in an "open" status for several years. Compared with the liability insurance, the loss settlement of property insurance is faster. The length of the payout tail is important in the insurance rate-making decision because it results in forecasting errors. Table 1 exhibits payment patterns in practice for some major liability lines for the period 1979-83. We can see there are significant differences between these insurance products. For example, auto insurance almost finish the claims within five years, but other liability only pays up to 62% of losses in five years.

Table 1.

Emergence Pattern of Major Liability Insurance[1]

type of insurance	cumulative % of final settlements in dev. year				
	0	1	2	3	4
Multi. Peril	55.8%	79.9%	86.2%	89.4%	93.6%
Auto Laibility	36.4%	65.7%	80.0%	88.9%	93.7%
Worker's Comp.	26.7%	51.9%	65.5%	74.1%	79.9%
Other Liab.	9.0%	20.6%	34.3%	48.8%	62.5%

Data Source: Best's Aggregates and Averages, Schedule P, 1985-87

Since losses may be settled over a long time period, the loss inflation rate becomes an important factor in the insurance rate-making decision. The accurate forecasting of the loss inflation rate is crucial. For example, an error of 0.01 in the value of loss severity trend factor would represent 43 percent of the average underwriting profit rate in automobile liability insurance for the period 1971-1982 (Cummins and Griepentrog, 1985).

1 The payout ratios in the table are the average of five years (1979-83), but payout ratios of each year for each line are in fact very stable and almost the same as the average.

The loss settlement process and its reporting procedures (see Troxel and Bouchie, 1990) are specific features of property-liability insurance industry. Therefore, it is interesting to analyze the impact of loss settlement delay on underwriting cycles.

IV. Assumptions for Pricing Decision

The fair rate principle of an insurance pricing is that the price (pure premium) must equal the present value of total loss payments. Since expenses and profit loading charged are a percentage of pure premiums and expense ratios are quite stable in the past decades, they are supposed not the reasons for underwriting cycles and thus omitted in the following discussions to simplified the notations.

As the concept shown in figure 1, insurance price for policy year t+1 is set at the end of time t (the beginning of time t+1). At the pricing moment, the insurer needs to forecast the losses incurred in t+1 that may be settled in t+1 and the following development years. This paper assumes the insurer makes pricing decision based on rational expectations. That is, they will take into account all the information available at the moment of forecasting and the ex ante forecasting error is zero.

When losses are to be settled in more than one period, the pricing decision involves the forecast of loss payments in each development period. We assume it is a constant proportion of the total claim costs paid in that calendar year. This assumption will make the notations and discussion much easier than the traditional approach.[2] Because the total claim costs paid in that year is a realized value while estimated total losses involve further estimation problems, it is a convenient way to consider the forecast of losses based on total claim costs instead of incurred losses. Besides, this approach is consistent with the Taylor's separation method (see Lemaire, 1985) and can promote future insurance research to connect with the actuarial models.

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[2] In traditional literature, we usually assumed losses paid in each development year is a constant proportion of total losses for that policy. That is, $LP_{t,i} = \Theta_i(L_t)$ where $LP_{t,i}$ = loss payment for policy issued at year t and paid at year t+i, Θ_i is payout proportion at year t+i, L_t is total losses for policy issued at year t which is unknown at pricing moment.

In this paper, we assume $LP_{t,i} = \alpha_i(C_{t+i})$ where C_{t+i} = total claim payments paid in year t+i, α_i is the proportion of C_{t+i} used to pay claims of policy of year t but paid at year t+i.

In either case, the proportion Θ_i or α_i is assumed constant and will not affect the discussions of autocorrelations of underwriting returns.

The fluctuation of interest rates might be an important reason for the cycles as indicated by Doherty and Kang (1988). In this paper, the risk from discount factor or interest rae is incorporated into forecasting risk since the forecast errors of discount factor also come from economic changes. However, the term for discount factor is not directly shown in the price equation to keep notations neat.

According to the above assumptions, the price equation can be described as equation (1):

$$P_{t+1} = E_t L_{t+1}$$

$$= \sum_{i=0}^{n-1} \alpha_i E_t C_{t+1+i} \tag{1}$$

where, P_{t+1} = price for insurance policy of year t+1

E_t = rational expectation based on all information available at the end of year t

L_{t+1} = total losses of insurance policy of year t+1

C_{t+1+i} = total claim costs paid in year t+1+i

α_i = proportion of total claim cost used to pay claims in its i-th development year

n = total number of loss development years

V. Autocorrelation of Underwriting Returns

Rational expectations imply the insurer sets price based on the loss distribution being the same as the theoretical loss distribution, conditioned on all the information available at the time of pricing. Therefore, a rational price implies that the ex ante expected value of forecasting errors is zero. Underwriting profits are the difference between price and losses which are just equal to the realized forecast errors.

Based on the assumptions of loss process and price equation developed in the previous section, the patterns of underwriting returns under rational price are discussed for short-tail lines and long-tail lines respectively because of their differences in loss settlement.

Additionally, we must distinguish the actual (ultimate) underwriting returns and reported underwriting returns due to the accounting procedures. The observed six-year underwriting cycles are based on underwriting returns reported for each calendar year. They are not actual returns for each insurance policy. We believe that insurers will concern actual returns for each policy as well as financial reports because actual returns reflect underwriting performance. Besides, it is interesting to compare the reported returns with actual returns so that we can know the impact of accounting methods. If the actual returns randomly dispersed but the reported returns cyclically fluctuated, then we can find the way to remove cycles simply by changing accounting procedures. Otherwise, we must search for other solutions.

A. Actual Underwriting Returns

The actual underwriting return of an insurance policy is defined as the premiums minus total loss payments for that policy, i.e., ultimate returns. They are not revealed until the claims completely settled. They are eqùal to the forecasting errors of loss payments due to random shocks of each loss development year. That is (see appendix 1 for derivation),

$$AR_{t+1} = P_{t+1} - L_{t+1}$$

$$= \sum_{i=0}^{n-1} a_{t+1+i} \qquad (2)$$

$$E_t AR_{t+1} = E_t\left(\sum_{i=0}^{n-1} a_{t+1+i} \right) = 0$$

Since the ex ante expectation of returns is zero, it is a rational price. Based on equation (2), it is easy to show that the length of autocorrelation lag of underwriting is related to loss settlement process. The longer delay in loss settlement, the longer the autocorrelation lag. The proof is shown as follows:

Case 1: losses paid immediately, i.e., $n = 1$,

$$\begin{aligned} \text{cov}\ (AR_{t+i}, AR_{t+j}) &= \text{cov}\ (a_{t+i}, a_{t+j}) \\ &= 0 \quad \text{for } i \neq j \end{aligned}$$

Case 2: losses settled through several years, i.e., $n > 1$,

$$\text{cov}\ (AR_{t+i}, AR_{t+j}) = \text{cov}\ \left(\sum_{k=0}^{n-1} a_{t+i+k}, \sum_{k=0}^{n-1} a_{t+j+k} \right)$$

$$\begin{cases} \neq 0 & \text{if } j < i+n \\ = 0 & \text{if } j \geq i+n \end{cases}$$

These mathematical results explain that there is no autocorrelation for underwriting returns if no loss settlement delays. On the other hand, if the losses must be settled for several years, the returns will be correlated due to the overlapping of loss payment years. The length of autocorrelation lag depends on the number of overlapping years.

B. Reported Underwriting Returns

The reported underwriting profits observed on the financial statement every year are defined as the premiums earned minus incurred losses of the reporting year. It involves the forecast errors of loss payment and loss reserves because the insurer will revise estimation of loss reserves as the information updated. Smith (1980) and Weiss (1985) have shown that loss reserve errors are important to financial performance of the insurers. That is (see appendix 1 for derivation),

$$RR_{t+1} = EP_{t+1} - IL_{t+1}$$

$$= a_{t+1} + \sum_{i=1}^{n-1} \epsilon_{t+1,i} + \sum_{i=1}^{n-1} \epsilon_{t,i} + \xi_{t+1} \tag{3}$$

According to equation (3), we can show that reported returns are autocorrelated. The reported returns exhibit first order autocorrelation when the losses cannot be settled immediately. The autocorrelation is due to loss reserve forecast errors and accounting procedures for premium earned and reported losses (see appendix 2 for proof).

Case 1: losses paid immediately, i.e., $n = 1$, so no loss reserves

$$\begin{aligned} \text{cov}\ (RR_{t+i}, RR_{t+j}) &= \text{cov}\ (a_{t+i} + \xi_{t+i}, a_{t+j} + \xi_{t+j}) \\ &= 0 \quad \text{for } i \neq j \end{aligned}$$

Case 2: losses settled through several years, i.e., $n > 1$,

$$\text{cov}\ (RR_{t+i}, RR_{t+j})$$

$$= \text{cov}\ \left(\sum_{k=1}^{n-1} \epsilon_{t+i,k} + \sum_{k=1}^{n-1} \epsilon_{t+i-1,k},\ \sum_{k=1}^{n-1} \epsilon_{t+j,k} + \sum_{k=1}^{n-1} \epsilon_{t+j-1,k} \right)$$

$$\begin{cases} \neq 0 & \text{if } j = i+1 \\ = 0 & \text{if } j > i+1 \end{cases}$$

C. Extension: Autocorrelation of Returns with Information Lag

The above discussion provides mathematical proof that, because of loss settlement delays, both actual and reported underwriting returns are autocorrelated even under the "perfect" conditions (i.e., fair rate, rational expectations, and no information lag) although their autocorrelation patterns are different. Therefore, we consider the autocorrelation of returns could be a natural phenomenon in property-liability insurance industry.

The above discussion just let us familiar with theoretical analysis for underwriting cycles, but it is not practical enough because of information lag. In practice price charged for insurance policy of year $t+1$ usually is not based on the immediate information at the end of year t. Because of the administration delays inside the insurance company or prior approval regulation for insurance rates, the price charged for policy of t+1 in fact is based on the information at the of year t−1 instead of year t. Therefore, the price equation may be modified as equation (4).

$$P_{t+1} = E_{t-1}L_{t+1}$$

$$= \sum_{i=0}^{n-1} \alpha_i E_{t-1} C_{t+1+i} \qquad (4)$$

$$AR_{t+1} = P_{t+1} - L_{t+1}$$

$$= \sum_{i=0}^{n} a_{t+i} \qquad (5)$$

$$E_t AR_{t+1} = E_{t-1}\left(\sum_{i=0}^{n} a_{t+i} \right) = 0$$

It is still a rational price since the ex ante expectation of forecasting errors is zero based on available information at pricing moment. The length of autocorrelation lag of actual underwriting is increased with number of information lags. For example, the case of no loss settlement delay become first order autocorrelated due to one information lag.

Case 1: losses paid immediately, i.e., n = 1,

$$\text{cov}\ (AR_{t+i}, AR_{t+j}) = \text{cov}\ (a_{t+i-1} + a_{t+i}, a_{t+j-1} + a_{t+j})$$

$$\begin{cases} \neq 0 & \text{if } j = i+1 \\ = 0 & \text{if } j > i+1 \end{cases}$$

Case 2: losses settled through several years, i.e., n > 1,

$$\text{cov}\ (AR_{t+i}, AR_{t+j}) = \text{cov}\ \left(\sum_{k=0}^{n} a_{t+i-1+k}, \sum_{k=0}^{n} a_{t+j-1+k} \right)$$

$$\begin{cases} \neq 0 & \text{if } j \leq i+n \\ = 0 & \text{if } j > i+n \end{cases}$$

The autocorrelation lag of reported returns is also increased because of information lag. For the case of immediate loss settlement, the result is the same as actual returns which exhibits first order auotcorrelation due to one information lag. It is somewhat complicated for the case with loss settlement delays. If claims can be finished in two period, it only generates first order autocorrelation. But if the claims must be paid in three periods or more, then the reported returns will exhibit second order autocorrelation. Therefore, we may suggest that the length of settlement delays has an impact on underwriting cycles. The results are shown as follows (see appendix 3 for proof).

Case 1: losses paid immediately, i.e., $n = 1$,

$$\text{cov}\,(RR_{t+i}, RR_{t+j}) = \text{cov}\,(a_{t+i-1} + a_{t+i} + \zeta_{t+i}, a_{t+j-1} + a_{t+j} + \zeta_{t+j})$$

$$\begin{cases} \neq 0 & \text{if } j = i+1 \\ = 0 & \text{if } j > i+1 \end{cases}$$

Case 2: losses settled through several years, i.e., $n > 1$,

$$\text{cov}\,(RR_{t+i}, RR_{t+j})$$

$$= \text{cov}\,\Big(a_{t+i-1}+a_{t+i}+\sum_{k=1}^{n-1} \epsilon_{t+i,k}+\sum_{k=1}^{n-1} \epsilon_{t+i-1,1+k}+\sum_{k=1}^{n-1} \epsilon_{t+i-2,1+k}+\zeta_{t+i}\ ,$$

$$a_{t+j-1}+a_{t+j}+\sum_{k=1}^{n-1} \epsilon_{t+j,k}+\sum_{k=1}^{n-1} \epsilon_{t+j-1,1+k}+\sum_{k=1}^{n-1} \epsilon_{t+j-2,1+k}+\zeta_{t+j}\Big)$$

$$\begin{cases} \neq 0 & \text{if } j = i+1, \text{ or if } j = i+2 \text{ and } n \geq 3 \\ = 0 & \text{otherwise} \end{cases}$$

VI. Empirical Evidence

The theoretical proof in the previous section is consistent with the observed real world phenomenon. The losses of short-tail lines, e.g., fire insurance and auto physical damage insurance, usually can be finished within two periods and their returns present only up to first order autocorrealtion (see table 2). On the other hand, the loss settlement for long-tail lines, i.e., liability insurance, requires more than two periods (see table 1). Consequently, most of their reported returns are autocorrelated for two lags as shown in table 2.

Most of the liability insurance lines present significant autocorrelation functions up to the second lag. The only exception is commercial peril insurance which presents only first order autocorrelation. The possible reason is that about 80% of its claims can be settled in two periods (see table 1) and thus loss settlement delay is not substantial.

Besides, table 2 also shows that the lines with slower loss settlement process, e.g., other liability, exhibit stronger autocorrelation of returns. The autocorrelation values will affect the length of cycles (Box and Jenkins, 1976). This is consistent with the findings of Venezian (1985) that different lines present different cyclical fluctuations.

The random shocks which caused the forecast errors (i.e., underwriting returns) and consequent cycles during the past 40 years might come from several sources as pointed out in the paper by Harrington (1987) rather than a certain specific factor like interest rate or capacity constraint.

Another remark is concerning about the difference between actual returns and reported returns. The observed six-year underwriting cycles are fluctuations of returns reported on the financial statements instead of the actual returns for each insurance policy. It is important to know the patterns of actual returns because they reflect the underwriting performance of insurers and have an impact on insurance price. Although in this paper we cannot provide complete empirical evidence of autocorrelation functions for actual returns due to unavailability of data, a brief comparison between reported returns and actual returns for recent ten years is provided by figure 2. The loss ratios calculated based on the fully developed losses from schedule P, which are close to actual underwriting returns in this paper,[3] also exhibit cyclic patterns even if they are somewhat different from those of reported returns.

[3] The empirical data usually are in form of loss ratio. Return can be obtained by one minus the loss ratio. However, the autocorrelation patterns are unchanged.

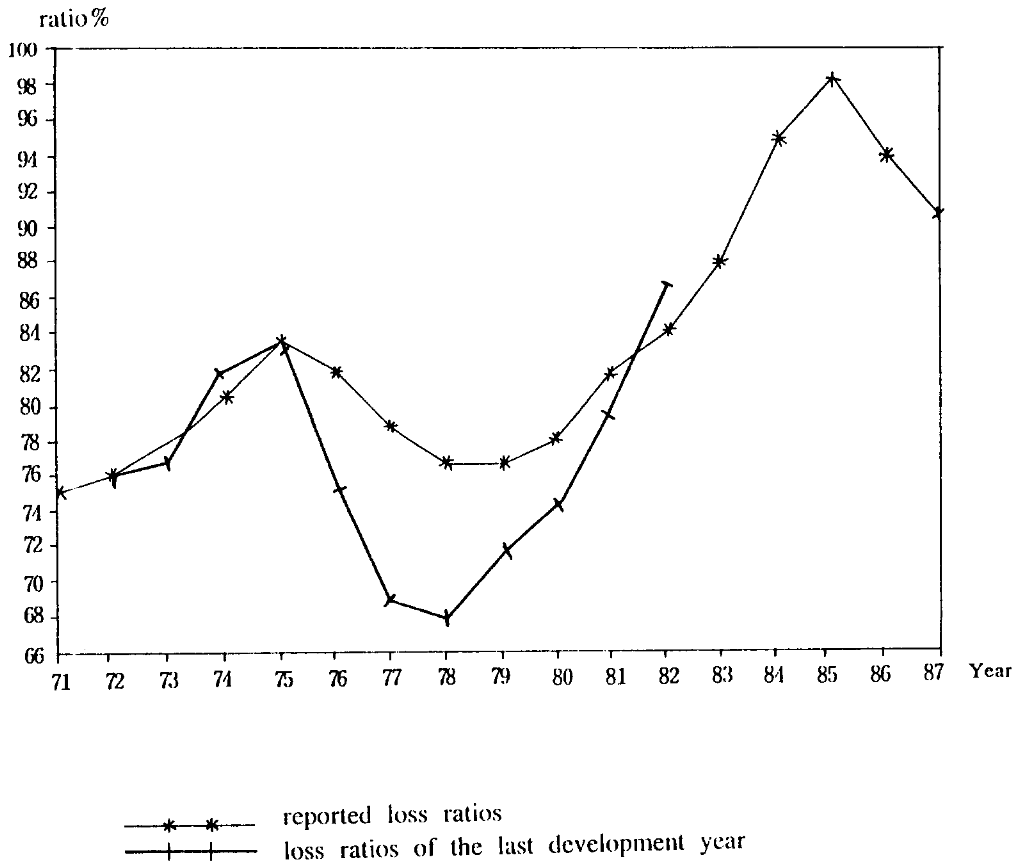
Table 2.
Autocorrelation Functions of Underwriting Returns
of Major Property and Liability Insurance Products

line	τ_1 (t value)	τ_2 (t value)	τ_3 (t value)	No. of sig. τ_k
Fire	0.64 (3.89)*	0.23 (1.03)	−0.008 (0.03)	1
Home owner	0.48 (2.57)*	−0.09 (0.40)	−0.21 (0.97)	1
Inland marine	0.67 (4.06)*	0.16 (0.79)	−0.12 (0.54)	1
Auto p. damage	0.57 (3.48)*	0.003 (0.01)	−0.15 (0.74)	1
Comm. peril	0.76 (4.12)*	0.37 (1.33)	0.08 (0.27)	1
Auto. liab.	0.82 (4.96)*	0.51 (2.03)*	0.24 (0.84)	2
Worker Comp.	0.82 (4.98)*	0.56 (2.24)*	0.31 (1.10)	2
Other liab.	0.88 (5.35)*	0.65 (2.48)*	0.40 (1.33)	2

Data Source: Best's Aggregates and Averages, 1951-87
Note: * = significant at 0.05 level
τ_i = i-th order autocorrelation function
Higher order autocorrelations are very small and need not to be quoted here.

Figure 2.

Comparison of Incurred Loss Ratios and Loss Ratios of Last Development Year of Schedule P for Combined Liability Lines.[4]



Data Source: Best's Aggregates and Averages, 1971-87

VII. Concluding Remarks

The above discussion provides mathematical proof that both actual and reported underwriting returns are autocorrelated when there exist loss settlement delays even though price is set rationally and equal to fair rate. The length of autocorrelation

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4 The data of loss ratios for the last development year are available only from 1972 to 1982.

lag for actual underwriting return of each policy is depending on the length of loss settlement delays. The longer the delays, the longer the autocorrelation lag.

The autocorrelations of observed reported returns result from loss settlement delays as well as accounting procedures. The reported returns usually exhibit first order autocorrelation, but they may present second order autocorrelation when losses must be settled for more than two periods.

The findings of this paper is consistent with the study of Cummins and Outreville (1987). However, it is not clearly explained in their study about what kind of institutional intervention causes the cycles and how the length of autocorrelation lag is affected by the intervention factor. This paper directly points out that loss settlement delay is the major factor to cause the cycles and proves that the length of autocorrelation lag is related to the length of loss settlement delays.

Therefore, this paper suggests that underwriting cycles may be an unavoidable phenomenon in property-liability insurance industry because loss settlement delay is an inherent nature of property-liability insurance products. Therefore cycles cannot be attributed to the irrationality of the insurers since the random shocks occur unexpectedly ex post to the price decision rather than decided by the insurers.

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Appendix 1.

A. Actual Underwriting Returns (AR)

The actual underwriting return of an insurance policy is defined as the premiums written minus total loss payments for that policy.

$$AR_{t+1} = P_{t+1} - L_{t+1}$$

$$= P_{t+1} - \sum_{i=0}^{n-1} \alpha_i C_{t+1+i}$$

$$= \sum_{i=0}^{n-1} \alpha_i \{ E_t C_{t+1+i} - C_{t+1+i} \}$$

$$= \sum_{i=0}^{n-1} a_{t+1+i}$$

$$E_t AR_{t+1} = E_t(\sum_{i=0}^{n-1} a_{t+1+i}) = 0,$$ because a_{t+1+i} is random shock.

where, AR_{t+1} = actual returns for insurance policy of t+1

a_{t+1+i} = forecasting errors of loss payments of t+1+i

other notations are the same as those defined in equation (1) in the text.

B. Reported Underwriting Returns (RR)

The reported underwriting returns are equal to premiums earned minus incurred losses. The premiums earned for year t+1 involved insurance policies (rates) made for years t and t+1. Thus, $EP_{t+1} = \phi P_t + (1 - \phi)P_{t+1}$.

The incurred losses (IL) of reporting year t+1 is defined as the loss payments (C) plus loss reserves (LR) of t+1 minus loss reserves of year t (see Webb, et al, 1984). That is, $IL_{t+1} = C_{t+1} + LR_{t+1} - LR_t$.

At the end of each year the insurer will revise the loss reserves based on the updated available information if under the assumption of rational expectations.

$$RR_{t+1} = EP_{t+1} - IL_{t+1}$$

$$= \{ \phi P_t + (1 - \phi)P_{t+1} \} - IL_{t+1}$$

$$= P_{t+1} - IL_{t+1} + \phi \{ P_t - P_{t+1} \}$$

Then,

$$P_{t+1} - IL_{t+1} = P_{t+1} - \{C_{t+1} + \sum_{i=1}^{n-1} (\sum_{j=0}^{n-1-i} \alpha_{i+j}) (E_{t+1}C_{t+1+i} - E_tC_{t+i})\}$$

$$= E_tC_{t+1} - C_{t+1} - \sum_{i=1}^{n-1} (\sum_{j=0}^{n-1-i} \alpha_{i+j}) (E_{t+1}C_{t+1+i} - E_tC_{t+1+i})$$

$$= E_tC_{t+1} - C_{t+1} + \sum_{i=1}^{n-1} \delta_i(E_{t+1} - E_t)C_{t+1+i}$$

$$= a_{t+1} + \sum_{i=1}^{n-1} \epsilon_{t+1,i}$$

$$P_t - P_{t+1} = E_{t-1}L_t - E_tL_{t+1}$$

$$= \sum_{i=0}^{n-1} \alpha_i E_{t-1} C_{t+i} - \sum_{i=0}^{n-1} \alpha_i E_t C_{t+1+i}$$

$$= \alpha_0 E_{t-1} C_t - \sum_{i=0}^{n-2} (\alpha_i E_t - \alpha_{i+1} E_{t-1}) C_{t+1+i} - \alpha_{n-1} E_t C_{t+n}$$

$$= \sum_{i=1}^{n-1} \epsilon_{t,i} + \alpha_0 E_{t-1} C_t - \alpha_{n-1} E_t C_{t+n}$$

$$= \sum_{i=1}^{n-1} \epsilon_{t,i} + \xi_{t+1}$$

Therefore,

$$RR_{t+1} = a_{t+1} + \sum_{i=1}^{n-1} \epsilon_{t+1,i} + \sum_{i=1}^{n-1} \epsilon_{t,i} + \xi_{t+1}$$

where, RR_{t+1} = accounting returns reported in year t+1

EP_{t+1} = premiums earned reported in year t+1

IL_{t+1} = incurred losses reported in year t+1

a_{t+1} = forecast error for loss payment of year t+1

$\epsilon_{t+1,i}$ = forecast error for loss reserves of year t+1+i but revealed at the end of t+1

ξ_{t+1} = other remaining term of year t+1

δ_i, ϕ = coefficients

other notations are the same as those in the text

Appendix 2.

The proof of autocorrelation of reported returns when losses settled through several years, i.e., n > 1, is shown as follows:
The assumptions about the covariance and variance of random shocks and reserve errors are as follows:

$$\begin{aligned}
\mathrm{cov}\,(a_{t+i}, a_{t+j}) &= 0, \quad \text{for } i \neq j \\
\mathrm{cov}\,(\epsilon_{t+i,k}, \epsilon_{t+i,l}) &= 0, \quad \text{for } k \neq l \\
\mathrm{cov}\,(\epsilon_{t+i,k}, \epsilon_{t+j,k}) &= 0, \quad \text{for } i \neq j \\
\mathrm{cov}\,(\xi_{t+i}, \xi_{t+j}) &= 0, \quad \text{for } i \neq j \\
\mathrm{cov}\,(a_{t+i}, \epsilon_{t+j,k}) &= 0, \quad \text{for all } i,j,k \\
\mathrm{cov}\,(\xi_{t+i}, \epsilon_{t+j,k}) &= 0, \quad \text{for all } i,j,k \\
\mathrm{cov}\,(a_{t+i}, \xi_{t+j}) &= 0, \quad \text{for all } i,j
\end{aligned}$$

Based on these assumptions, we have

$\mathrm{cov}\,(RR_{t+i}, RR_{t+j})$

$$= \mathrm{cov}\,\Big(a_{t+i}+\sum_{k=1}^{n-1}\epsilon_{t+i,k}+\sum_{k=1}^{n-1}\epsilon_{t+i-1,k}+\xi_{t+i},\ a_{t+j}+\sum_{k=1}^{n-1}\epsilon_{t+j,k}+\sum_{k=1}^{n-1}\epsilon_{t+j-1,k}+\xi_{t+j}\Big)$$

$$= \mathrm{cov}\,\Big(\sum_{k=1}^{n-1}\epsilon_{t+i,k}+\sum_{k=1}^{n-1}\epsilon_{t+i-1,k},\ \sum_{k=1}^{n-1}\epsilon_{t+j,k}+\sum_{k=1}^{n-1}\epsilon_{t+j-1,k}\Big)$$

Since a_{t+i}, a_{t+j}, ξ_{t+i}, and ξ_{t+j} are uncorrelated when $i \neq j$, the resources for autocorrelation come from ϵ's. When $j=i+1$,

$\mathrm{cov}\,(RR_{t+i}, RR_{t+j})$

$$= \mathrm{cov}\,\Big(\sum_{k=1}^{n-1}\epsilon_{t+i,k}+\sum_{k=1}^{n-1}\epsilon_{t+i-1,k},\ \sum_{k=1}^{n-1}\epsilon_{t+i+1,k}+\sum_{k=1}^{n-1}\epsilon_{t+i,k}\Big)$$

$$= \mathrm{cov}\,\Big(\sum_{k=1}^{n-1}\epsilon_{t+i,k},\ \sum_{k=1}^{n-1}\epsilon_{t+i,k}\Big)$$

$$= \mathrm{var}\,\Big(\sum_{k=1}^{n-1}\epsilon_{t+i,k}\Big)$$

Appendix 3.

When there is one information lag, price for year t+1 (P_{t+1}) is made based on information set of t−1. That is $P_{t+1} = E_{t-1}L_{t+1}$. Therefore,

$$P_{t+1} - IL_{t+1} = P_{t+1} - \{C_{t+1} + \sum_{i=1}^{n-1} (\sum_{j=0}^{n-1-i} \alpha_{i+j}) (E_{t+1}C_{t+1+i} - E_tC_{t+i})\}$$

$$= \sum_{i=0}^{n-1} \alpha_i E_{t-1}C_{t+1+i} - \{C_{t+1} + \sum_{i=1}^{n-1} (\sum_{j=0}^{n-1-i} \alpha_{i+j}) (E_{t+1}C_{t+1+i} - E_tC_{t+i}) \}$$

$$= \alpha_0(E_{t-1}C_{t+1} - C_{t+1}) + \sum_{i=1}^{n-1} \alpha_i(E_tC_{t+1} - C_{t+1}) - \sum_{i=1}^{n-1} \alpha_i(E_{t+1} - E_{t-1})C_{t+1+i} - \sum_{i=1}^{n-2} (\sum_{j=1}^{n-2-i} \alpha_{i+j}) (E_{t+1} - E_t)C_{t+1+i}$$

$$= \alpha_0(E_{t-1}C_{t+1} - C_{t+1}) + \sum_{i=1}^{n-1} \alpha_i(E_tC_{t+1} - C_{t+1}) - \sum_{i=1}^{n-1} \alpha_i\{(E_{t+1} - E_t) + (E_t - E_{t-1})\}C_{t+1+i} - \sum_{i=1}^{n-2} (\sum_{j=1}^{n-2-i} \alpha_{i+j}) (E_{t+1} - E_t)C_{t+1+i}$$

$$= a_t + a_{t+1} + \sum_{i=1}^{n-1} \epsilon_{t+1,i} + \sum_{i=1}^{n-1} \epsilon_{t,1+i}$$

$$P_t - P_{t+1} = E_{t-2}L_t - E_{t-1}L_{t+1}$$

$$= \sum_{i=0}^{n-1} \alpha_i E_{t-2}C_{t+i} - \sum_{i=0}^{n-1} \alpha_i E_{t-1}C_{t+1+i}$$

$$= \alpha_0 E_{t-2}C_t - \sum_{i=0}^{n-2} (\alpha_i E_{t-1} - \alpha_{i+1}E_{t-2})C_{t+1+i} - \alpha_{n-1}E_{t-1}C_{t+n}$$

$$= \sum_{i=1}^{n-1} \epsilon_{t-1,1+i} + \alpha_0 E_{t-2}C_t - \alpha_{n-1}E_{t-1}C_{t+n}$$

$$= \sum_{i=1}^{n-1} \epsilon_{t-1,1+i} + \zeta_{t+1}$$

Therefore,

$$RR_{t+1} = a_t + a_{t+1} + \sum_{i=1}^{n-1} \epsilon_{t+1,i} + \sum_{i=1}^{n-1} \epsilon_{t,1+i} + \sum_{i=1}^{n-1} \epsilon_{t-1,1+i} + \zeta_{t+1}$$

$$RR_{t+2} = a_{t+1} + a_{t+2} + \sum_{i=1}^{n-1} \epsilon_{t+2,i} + \sum_{i=1}^{n-1} \epsilon_{t+1,1+i} + \sum_{i=1}^{n-1} \epsilon_{t,1+i} + \zeta_{t+2}$$

$$RR_{t+3} = a_{t+2} + a_{t+3} + \sum_{i=1}^{n-1} \epsilon_{t+3,i} + \sum_{i=1}^{n-1} \epsilon_{t+2,1+i} + \sum_{i=1}^{n-1} \epsilon_{t+1,1+i} + \zeta_{t+3}$$

. . . . . . ,

$$RR_{t+i} = a_{t+i-1} + a_{t+i} + \sum_{k=1}^{n-1} \epsilon_{t+i,k} + \sum_{k=1}^{n-1} \epsilon_{t+i-1,1+k} + \sum_{k=1}^{n-1} \epsilon_{t+i-2,1+k} + \zeta_{t+i}$$

The general form of covariance for RR_{t+i} and RR_{t+j} is quite lengthy. However, we know that the covariance will not equal zero when they have common items according to the assumptions of covariance in appendix 2. Therefore, it is easy to show that reported returns may have first-order or second-order autocorrelation. For example, RR_{t+1} is first order autocorrelated with RR_{t+2} by a_{t+1}, ϵ_{t+1}'s. Additionally, it is second order autocorrelated with RR_{t+3} by ϵ_{t+1}'s if $n \geq 3$.