考試科目	微積分	所別	應用數學系	考試時期	3 月 19 日 星期六 8:20~10:00

1. Let $f(x)=x^3-9x^2+15x$

   (a) Prove that $f(x)\geq 0$ for all $x\in[0,2]$ and find the absolute maximum value of $f(x)$ on $[0,2]$. (10%)

   (b) Evaluate $\lim_{n\to\infty}\left(\int_0^2[f(x)]^n\,dx\right)^{\frac{1}{n}}$ if exists. (Justify your answer.) (10%)

2. Show that $\int_0^{\pi} xf(\sin x)dx=\frac{\pi}{2}\int_0^{\pi} f(\sin x)dx$ and deduce the formula

$$\int_0^{\pi}\frac{x\sin x}{1+\cos^2 x}dx=\pi\int_0^1\frac{1}{1+x^2}dx$$ (20%)

3. Find the area of the region enclosed by the hypocycloid $x^{\frac{2}{3}}+y^{\frac{2}{3}}=a^{\frac{2}{3}}$, where $a>0$. (20%)

4. (a) State and prove the fundamental theorem of calculus. (10%)

   (b) Using (a) to compute $F'(0)$, where $F(x)=\int_{\sin x}^{x^2+1}e^{-t^2}dx$. (10%)

5. Find the volume of the largest rectangular parallelepiped that can be inscribed in the ellipsoid

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$$

   where a, b, and c are positive constants. (10%)

6. Let $\sum_{n=1}^{\infty}a_n$ be a convergent series, where $a_n\geq 0$ for all $n=1,2,\cdots$. Discuss the convergence of $\sum_{n=1}^{\infty}\frac{\sqrt{a_n}}{n^p}$ for $p\geq\frac{1}{2}$. (Justify your answer.) (10%)

備考	試題隨卷繳交

命題老師： 040 （簽章） 94 年 2 月 28 日

考試科目	線性代數	所別	應用數學系	考試時期	3月19日 星期六	10:20~12:00

1. Let $\mathbf{y}_1 = \begin{pmatrix}1\\1\\1\end{pmatrix}$, $\mathbf{y}_2 = \begin{pmatrix}1\\1\\0\end{pmatrix}$, $\mathbf{y}_3 = \begin{pmatrix}1\\0\\0\end{pmatrix}$, and let L be the linear transformation from R^3 to R^3 defined by $L(c_1\mathbf{y}_1 + c_2\mathbf{y}_2 + c_3\mathbf{y}_3) = (c_1 + c_2 + c_3)\mathbf{y}_1 + (2c_1 + c_3)\mathbf{y}_2 - (2c_2 + c_3)\mathbf{y}_3$.
   (a) Find a matrix representing L with respect to the ordered basis $[\mathbf{y}_1, \mathbf{y}_2, \mathbf{y}_3]$. (10%)
   (b) For each of the following, write the vector $\mathbf{x}$ as a linear combination of $\mathbf{y}_1, \mathbf{y}_2, \mathbf{y}_3$ and use the matrix from part (a) to determine $L(\mathbf{x})$.
   (i) $\mathbf{x} = (7, 5, 2)$ (ii) $\mathbf{x} = (3, 2, 1)$ (10%)

2. The town of Midvale maintains a constant population of 300,000 people from year to year. A political science study estimated that there were 150,000 independents, 90,000 Democrats, and 60,000 Republicans in the town. It was also estimates that each year 20 percent of the independents become Democrats and 10 percent become Republicans. Similarly, 20 percent of the Democrats become independents and 10 percent become Republicans, while 10 percent of Republicans defect to the Democrats and 10 percent become independents each year. Let

$$\mathbf{x} = \begin{pmatrix}150{,}000\\ 90{,}000\\ 60{,}000\end{pmatrix}$$

   and let $\mathbf{x}^{(1)}$ be a vector representing the number of people in each group after 1 year.
   (a) Find a matrix A such that $A\mathbf{x} = \mathbf{x}^{(1)}$. (5%)
   (b) Show that $\lambda_1 = 1.0$, $\lambda_2 = 0.5$, and $\lambda_3 = 0.7$ are eigenvalues of A. Find the eigenvectors corresponding to λ_1, λ_2, and λ_3. (10%)
   (c) Factor A into product SDS^{-1}, where D is diagonal. (10%)
   (d) Which group will dominate in the long run? Justify your answer by computing $\lim_{n\to\infty} A^n\mathbf{x}$. (5%)

備考	試題隨卷繳交

命題老師： 041 （簽章） 年 月 日

考試科目	線性代數	所別	應用數學系	考試時期	3 月 19 日 星期六	10:20~12:00

3. Given

$$A=\begin{pmatrix}5 & 4 & 7\\ 2 & -4 & 3\\ 2 & 8 & 6\end{pmatrix},\quad \mathbf{b}=\begin{pmatrix}2\\ -5\\ 4\end{pmatrix},\quad \mathbf{c}=\begin{pmatrix}5\\ -4\\ 2\end{pmatrix}$$

(a) Solve the system $A\mathbf{x}=\mathbf{b}$ using complete pivoting. In complete pivoting the pivot element is chosen to be the element of maximum modulus among all the elements in the remaining rows and columns. (10%)

(b) Let P be the permutation matrix determined by the pivot rows and let Q be the permutation matrix determined by the pivot columns. Factor PAQ into a product LU. (10%)

(c) Use the LU factorization from part (b) to solve the system $A\mathbf{x}=\mathbf{c}$. (10%)

4. Consider a system of the form

$$\begin{pmatrix}A & \mathbf{a}\\ \mathbf{c}^T & \beta\end{pmatrix}\begin{pmatrix}\mathbf{x}\\ x_{n+1}\end{pmatrix}=\begin{pmatrix}\mathbf{b}\\ b_{n+1}\end{pmatrix}$$

where A is a nonsingular $n\times n$ matrix and $\mathbf{a}$, $\mathbf{b}$, and $\mathbf{c}$ are vectors in R^n.

(a) Multiply both side of the system by a matrix to obtain an equivalent triangular system. (10%)

(b) Set $\mathbf{y}=A^{-1}\mathbf{a}$ and $\mathbf{z}=A^{-1}\mathbf{b}$. Show that if $\beta-\mathbf{c}^T\mathbf{y}\neq 0$, then the solution of the system is given by

$$x_{n+1}=\frac{b_{n+1}-\mathbf{c}^T\mathbf{z}}{\beta-\mathbf{c}^T\mathbf{y}}$$

$$x=\mathbf{z}-x_{n+1}\mathbf{y}$$ (10%)

備考	試題隨卷繳交

命題老師： 042 （簽章） 年 月 日