考試科目	微積分	所別	應用數學系	考試時間	3月14日 星期六	第1節

1. (15%) Let $f\colon [0,5] \to \mathbb{R}$ be a function defined by

$$f(x) = \begin{cases} 1 & \text{if } x \neq 1 \\ 0 & \text{if } x = 1. \end{cases}$$

Prove that $f(x)$ is Riemann integrable on $[0,5]$ and evaluate its integral on $[0,5]$.

2. (15%) Let $\sum_{n=1}^{\infty} a_n$ be a series with nonnegative terms and s_n be its n-th partial sum. Show that $\sum_{n=1}^{\infty} a_n$ converges if and only if $\{s_n\}$ is a bounded sequence.

3. (15%) Let $f\colon [a,b] \to \mathbb{R}$ be a continuous function. Show that there exists $c \in [a,b]$ such that

$$\int_a^b f(t)\,dt = f(c)(b-a).$$

4. (15%) Suppose that $f(x)$ is a continuous real-valued function on the real line $\mathbb{R}$. Show that if $f(x+y) = f(x) + f(y)$ for all $x, y \in \mathbb{R}$, then there exists a constant a such that $f(x) = ax$ for all $x \in \mathbb{R}$.

5. (20%) Prove the following identities.

(a) $\displaystyle\int_x^1 \frac{dt}{1+t^2} = \int_1^{1/x} \frac{dt}{1+t^2}$ for $x > 0$.

(b) $\displaystyle\int_0^1 x^m(1-x)^n\,dx = \int_0^1 x^n(1-x)^m\,dx.$

6. (20%) State the Green's Theorem and show that the area of the region $R = \{(x,y) \in \mathbb{R}^2 \mid x^2+y^2 \leq 1\}$ can be expressed as a line integral over the circle $x^2+y^2 = 1$ in the counterclockwise direction.

備考	試題隨卷繳交
命題委員：	（簽章）

命題紙使用說明：1.試題將用原件印製，敬請使用黑色墨水正楷書寫或打字（紅色不能製版請勿使用）。
2.書寫時請勿超出格外，以免印製不清。
3.試題由郵寄遞者請以掛號寄出，以免遺失而示慎重。

考試科目	線性代數	所別	應用數學系	考試時間	3月14日 星期六 第2節

1. (20%) Let A, B be two $n \times n$ complex matrices such that $AB = BA$. Suppose A has n distinct eigenvalues. Show that B is diagonalizable.

2. (20%) Let A be a 5×5 real matrix. Suppose that $A^3 = 0$, but $A^2 \neq 0$. Find all possible Jordan canonical form for A.

3. (20%) Let W_1 and W_2 be subspaces of a finite-dimensional inner product space V over a field $\mathbb{F}$. Suppose $\dim W_1 = \dim W_2$. Show that there is an orthogonal linear operator T on V such that $T(W_1) = T(W_2)$.

4. (20%) Let V and W be finite-dimensional vector spaces over a field $\mathbb{F}$ with ordered bases $\beta = \{\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_n\}$ and $\gamma = \{\mathbf{w}_1, \mathbf{w}_2, \ldots, \mathbf{w}_m\}$, respectively. Let $\mathcal{L}$ be the vector space of all linear transformation from V to W. Suppose that $T_{ij} : V \to W$ is the linear transformation such that

$$T_{ij}(\mathbf{v}_k) = \begin{cases} \mathbf{w}_i & \text{if } k = j \\ 0 & \text{if } k \neq j. \end{cases}$$

Show that $S = \{T_{ij} \mid 1 \leq i \leq m, 1 \leq j \leq n\}$ is a basis for $\mathcal{L}$.

5. (20%) Let V and W be two subspaces of a vector space over a field $\mathbb{F}$. Show that $V/(V \cap W)$ is isomorphic to $(V + W)/W$. Please prove directly by the definition of two vector spaces being isomorphic.

備考	試題隨卷繳交
命題委員：	（簽章）

命題紙使用說明：1.試題將用原件印製，敬請使用黑色墨水正楷書寫或打字（紅色不能製版請勿使用）。
2.書寫時請勿超出格外，以免印製不清。
3.試題由郵寄遞者請以掛號寄出，以免遺失而示慎重。