國立政治大學 103 學年度碩士班招生考試試題

考試科目	微積分	所別	應用數學系 8116, 8111	考試時間	2 月 23 日(日) 第一節

★ **Problems:**

1. Evaluate the following limits if they exists.

   (a) **(5 points)** $\lim_{x\to 0} \dfrac{(e^{2x^2}-1-2x^2)(\cos x-1)}{(\sin 3x-\ln(1+3x))x^4}$.

   (b) **(5 points)** $\lim_{x\to 0} \dfrac{x\cos x - xe^{-x^2}}{\sin^3 x}$.

   (c) **(5 points)** $\lim_{x\to 0} \dfrac{e^x - e^{\sin x}}{x-\sin x}$.

2. **(10 points)** Let $0 \le \theta \le 2\pi$ and consider the series

$$\sum_{n=1}^{\infty}(-1)^{n+1}\tan^{2n}(\theta).$$

   Determine the values of θ for which the series converges and compute the sum.

3. **(15 points)** Suppose that f is a continuous real-valued function defined on the closed interval $[0,1]$. Which of the following must be true?

   I) There is a constant $C>0$ such that $|f(x)-f(y)|\le 1$ for all x and y in $[0,1]$ that satisfy $|x-y|\le C$.

   II) There is a constant $D>0$ such that $|f(x)-f(y)|\le D$ for all x and y in $[0,1]$.

   III) There is a constant $E>0$ such that $|f(x)-f(y)|\le E|x-y|$ for all x and y in $[0,1]$.

4. The intersection of the two surfaces $x^2+\frac{y^2}{2}=1$ and $z^2+\frac{y^2}{2}=1$ consists of two curves.

   (a) **(4 points)** Parameterize each curve in the form $r(t)=(x(t),y(t),z(t))$.

   (b) **(3 points)** Set up the integral for the arc length of one of the curves.

   (c) **(3 points)** What is the arc length of this curve?

5. **(10 points)** Find all real solutions to the differential equation $\cos^2 x\frac{dy}{dx}+y=e^{\tan x}$.

6. **(10 points)** Discuss the convergence and divergence of

$$\int_0^{\infty} x^p e^{-x}\,dx$$

   for $p<\infty$.

備註	試題隨卷繳交

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7. Let S be the part of the spherical surface $x^2 + y^2 + z^2 = 2$ lying in $z > 1$. Orient S upwards and give its bounding circle, C, lying in $z = 1$ the compatible orientation.

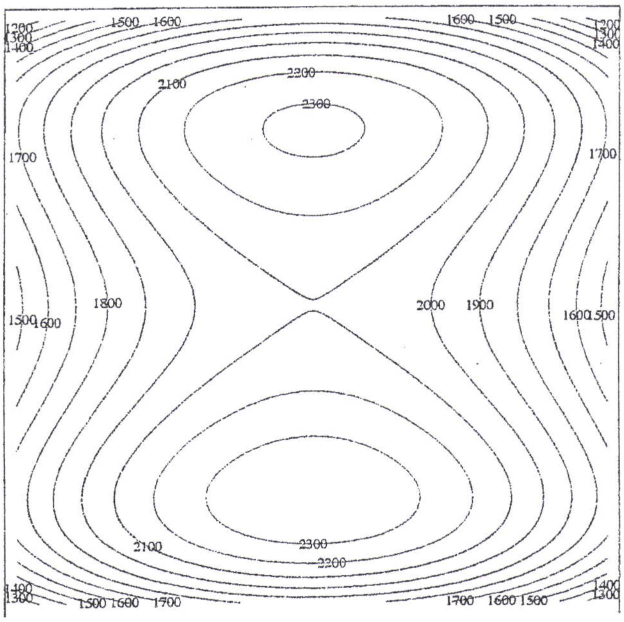
   (a) **(6 points)** Parametrize C and use the parametrization to evaluate the line integral

   $$I = \oint_C xz\,dx + y\,dy + y\,dz.$$

   (b) **(6 points)** Compute the curl of the vector field $\mathbf{F} = xz\vec{i} + y\vec{j} + y\vec{k}$.

   (c) **(8 points)** Write down a flux integral through S which can be computed using the value of I.

8. **(10 points)** On the contour plot below, mark the portion of the level curve $f = 2000$ on which $\frac{\partial f}{\partial y} \geq 0$.



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