考試科目	線性代數	所別	應用數學系	考試時間	5月26日 星期六 第一節

1. (10%) Prove or give a counter example: A row echelon form of an $n \times n$ matrix A is a diagonal matrix, then A is diagonalizable.

2. (10%) Prove or give a counter example: Let A, B be two 77×77 real matrices such that $AB = -BA$, then A or B is not invertible.

3. (20 %) Let V be a finite-dimensional inner product space over a field $\mathbb{F}$, and let $g\colon V \to \mathbb{F}$ be a linear functional. Then there exists a unique vector $\mathbf{u} \in V$ such that $g(\mathbf{x}) = \langle \mathbf{x}, \mathbf{u} \rangle$ for all $\mathbf{x} \in V$.

4. (20 %) Let $P_2(\mathbb{R})$ be the vector space of real polynomials with degrees at most 2. Define an inner product on $P_2(\mathbb{R})$ by $\langle f(x), g(x) \rangle = \int_{-1}^{1} f(t)g(t)\,dt$. Let $\beta = \{1, x, x^2\}$ be the standard basis of $P_2(\mathbb{R})$.
   (a) For any $h(x) \in P_2(\mathbb{R})$, denote the coordinate vector (as a column vector) of $h(t)$ with respect to β by $[h(x)]_\beta$. Find a matrix A such that $\langle f(x), g(x) \rangle = [g(x)]_\beta^T A [f(x)]_\beta$.
   (b) Use the Gram-Schmid process to replace β by an orthogonal basis for $P_2(\mathbb{R})$.

5. (20 %) Let A be an $n \times n$ matrix whose characteristic polynomials splits. Show that A and A^t are similar.

6. (20 %) Find a real matrix B such that $B^{43} = \begin{pmatrix} \frac{5}{2} & -\frac{3}{2} \\ -\frac{3}{2} & \frac{5}{2} \end{pmatrix}$.

備考	試題隨卷繳交

命題委員： 194 （簽章） 年 月 日

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國立政治大學九十六學年度研究所博碩士班入學考試

第1頁，共1頁

考試科目	分析概論	所別	應用數學系	考試時間	5月26日 星期六 第二節

1. (20%) Let $A \subseteq \mathbb{R}^n$ be an open set, $f: A \to \mathbb{R}$ be a C^1 function and $S \subseteq A$ be a C^1 hypersurface defined by $g(x) = 0$. If the gradient $\nabla g(x_0)$ of g at $x_0 \in S$ is nonzero and $f(x)$ has a maximum or minimum value at x_0, then there exists a real number λ such that $\nabla f(x_0) = \lambda \nabla g(x_0)$.

2. (20%) Let $(X, \mathcal{S}, \mu)$ be a probability measure space, $\varphi: (a, b) \to \mathbb{R}$ be a convex function, where $-\infty < a < b < \infty$, and $f: X \to (a, b)$ be a measurable function on X. Show that

$$\varphi\Big(\int_X f\, d\mu\Big) \le \int_X \varphi \circ f\, d\mu.$$

3. (20%) Let $(X, \mathcal{S}, \mu)$ be a measure space and $p > 0$. Show that if $f_n \to f$ in $L^p(X, \mathcal{S}, \mu)$, then $f_n \to f$ in measure.

4. (20%) Let X be a normed linear space. Show that the set X^* of all bounded linear functionals on X is a Banach space.

5. (20%) Let

$$f(x) = \begin{cases} \dfrac{\sin x}{x} & \text{if } x \ne 0, \\ 1 & \text{if } x = 0. \end{cases}$$

Evaluate the improper Riemann integral $\displaystyle\int_{-\infty}^{\infty} f(x)\, dx$. Is $f(x)$ Lebesgue integrable over $(-\infty, \infty)$?

備考	試題隨卷繳交

命題委員：　　　　195（簽章）　96年5月4日

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