15%)

Show that if a function is one-to-one and onto, then its inverse is a one-to-one and onto function.

(15%)

Show that if the limit exists, then it is unique.

(10%) (a) Prove or disprove: "If a function is continuous then it is differentiable."

(10%) (b) Prove or disprove: "If a function is differentiable then it is continuous."

(10%)

Show that $\frac{d}{dx}(e^x) = e^x$.

(10%)

Show that if $f'(x) > 0$ on an interval, then f is increasing on that interval.

(15%)

Let $f: R \to R$ be a continuous function with the absolute maximum value M and the absolute minimum value m. Show that for every $x \in R$, there is $y \in R$ such that $f(x) + f(y) = M + m$.

(15%)

Let $f(x,y) = \frac{x^2}{x^2+y^2}$. Determine whether $\lim_{(x,y)\to(0,0)} f(x,$ exists or not. If the limit exists, please find it. If the limit doesn't exist, please explain.

科目	微積分 II	系別	應用數學系	考試時間	7月7日(星期六)上午/下午 第2

試計算 $\int_0^1 \ln(x+\sqrt{1+x^2})\,dx = ?$ (10%)

設 $z = f(y+x) + g(y-x)$，且 f 與 g 都是二次可微分函數時，
試求 (i) $\frac{\partial z}{\partial x} = ?$ (ii) $\frac{\partial z}{\partial y} = ?$ (iii) $\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial y^2} = ?$

(i). 設 n 表自然數，試證

$$1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+\frac{1}{5}-\frac{1}{6}+\cdots-\frac{1}{2n} = \frac{1}{n+1}+\frac{1}{n+2}+\frac{1}{n+3}+\cdots+\frac{1}{2n}$$ (8%)

(ii). 試利用(i)題之結果，求 $\sum_{n=0}^{\infty}\frac{(-1)^n}{n+1}$ 之和 $= ?$ (7%)

設 $f(x) = x + e^{x-1}$，令 R 表由曲線 $y = x + e^{-x}$，直線 $x=0$，$y=0$ 及 $x=1$ 所圍成封閉區域，試求區域 R 之面積 $= ?$ (10%)

設 $f(x) = \begin{cases} \sin t, & \text{當 } 0 \le t \le \frac{\pi}{2} \text{ 時.} \\ 2\cos\frac{2t}{3}, & \text{當 } \frac{\pi}{2} < t \le \pi \text{ 時.} \end{cases}$

$\forall x \in [0,\pi]$，定義 $F(x) = \int_0^x f(t)\,dt$.

試求函數 F 在 $[0,\pi]$ 中之極值(極大值或極小值)。 (10%)

試求由圓形拋物面 $z = x^2+y^2$，圓柱 $x^2+y^2=4$ 及坐標平面在第一對限($x\ge0$, $y\ge0$, $z\ge0$)內所圍成立體之體積 $= ?$ (15%)

試求原點至平面 $3x+2y-z=14$ 間的最短距離 $= ?$ (15%)

若 $\int_0^{\pi}(x-a-b\cos x)^2\,dx$ 之值為最小時，則 $a = ?$ $b = ?$ (10%)