

考試科目	微積分(一)	系別	應用數學系	考試時間	7月6日 上午 第二節 星期六
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- Let f and g be functions from \mathbb{R} to \mathbb{R} and $f \circ g(x) = x, \forall x \in \mathbb{R}$.
 - (10%) Prove or disprove that f is one-to-one.
 - (10%) Prove or disprove that f is onto.
 - (10%) Prove or disprove that g is one-to-one.
 - (10%) Prove or disprove that g is onto.
- (10%) Use definition to show that $\lim_{x \rightarrow 2} \frac{x^2 - 4}{x - 2} = 4$.
- (10%) Let $f(x) = x^5 - x + 1$. Show that there is a number b such that $f(b) = 10$.
- (10%) Given any real number r , show that $\frac{d}{dx} x^r = r x^{r-1}$.
- (10%) Show that if $f'(x) = 0$ on an interval, then f is constant on that interval.
- (10%) Show that if $f''(x) > 0$ on an interval, then f is concave upward on that interval.
- (10%) Let f be continuous on $[a, b]$ and $g(x) = \int_a^x f(t) dt$, $a \leq x \leq b$. Prove or disprove that g is differentiable on (a, b) .

考試科目	微積分(=)	系別	應用數學系	考試時間	7月6日 星期六 下午第4節
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一. 設 n 表非負整數, 令 $a_n = \int_0^{\frac{\pi}{2}} \sin^n x dx$,

① 試求 $n a_n a_{n-1} = ?$ (10%)

② 試證 $\{a_n\}$ 為遞減數列。 (10%)

二. 設函數 $f: [0, 1] \rightarrow \mathbb{R}$ 滿足 $1 - f(x) = f(1-x)$.

試計算 $\int_0^1 f(x) dx = ?$ (20%)

三. 令函數 $F(x) = \int_x^{x+1} \frac{t-1}{t^2+1} dt$,

① 試求 $F'(x) = ?$ (10%)

② 試求 $F(x)$ 之值為最小時之 x 值。 (10%)

四. 設函數 $y = f(x)$ 為微分方程式 $\frac{dy}{dx} = 2y$ 的解, 且

$f(0) = 1$. 設 a 表正實數時, 若 $2 \int_0^a f(x) dx = \int_a^{2a} f(x) dx$ 成立
試求 $a = ?$ (20%)

五. 設 R 表由曲線 $y = \sqrt{x} \sin x$ ($0 \leq x \leq \pi$) 與 x 軸所圍成

區域。試求區域 R 繞著 x 軸旋轉一周所成立體之體積。 (20%)