國立政治大學九十八學年度轉學生入學考試命題紙

考試科目	微積分(一) 微分為主	系別	應用數學系	考試時間	7月10日 星期五	第二節

1.

(10%) (a) Show that $\lim_{x\to 1} \frac{x^2-1}{x-1} = 2$.

(10%) (b) Show that $\lim_{x\to 0} x\cos\frac{1}{x} = 0$.

2. Let the sequence $a_n = (1+\frac{1}{n})^n$.

(10%) (a) Show that a_n is increasing.

(10%) (b) Show that a_n is bounded.

3. From 2, we know that a_n is convergent, let $\lim_{n\to\infty} a_n = e$.

(10%) (a) Show that $\frac{d}{dx}\log_e x = \frac{1}{x}$.

(10%) (b) Show that $\frac{d}{dx} e^x = e^x$.

4. Let $f(x) = \int_x^{x^2} e^{(t^3)}\,dt$.

(10%) (a) Find $\frac{d}{dx} f(x)$.

(10%) (b) Find an equation of the tangent line of $f(x)$ at $x=1$.

5. Determine whether the following limits exist. Justify your answers.

(10%) (a) $\lim_{(x,y)\to(0,0)} \frac{(x+y)^2}{x^2+y^2}$.

(10%) (b) $\lim_{(x,y)\to(0,0)} \frac{x^4-y^4}{x^2+y^2}$.

備考	試題隨卷繳交
命題委員：	（簽章） 98年 6月 29日

命題紙使用說明：1.試題將用原件印製，敬請使用黑色墨水正楷書寫或打字（紅色不能製版請勿使用）。
2.書寫時請勿超出格外，以免印製不清。
3.試題由郵寄遞者請以掛號寄出，以免遺失而示慎重。

考試科目	微積分(二)	系別	應用數學系	考試時間	7月10日 星期五	第四節

積分為主

1. (20%) Let $\Gamma(x) = \int_0^\infty t^{x-1}e^{-t}\,dt$.
   (a) Show that $\Gamma(x)$ is well-defined for $x > 0$.
   (b) Evaluate the value $\Gamma(\frac{1}{2})$.
   (c) Show that $\Gamma(n+1) = n!$ for all positive integers.

2. (20%) Prove or disprove the following statements.
   (a) If both $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ converge absolutely, then $\sum_{n=1}^{\infty} a_n b_n$ also converges absolutely.
   (b) If $p > 1$, then the series $\sum_{k=2}^{\infty} \frac{1}{k(\ln k)^p}$ converges.

3. (20%) Let $S = \{(x,y,z) \in \mathbb{R}^3 \mid x^2 + y^2 + z^2 \leq 1\}$.
   (a) Prove that the volume of S is $\frac{4}{3}\pi$.
   (b) Evaluate $\iiint_S (x^2 + y^2 + z^2)^{5/2}\,dxdydz$.

4. (20%) Let C be the curve in $\mathbb{R}^3$ parametrized by $\gamma: [0, 2\pi] \to \mathbb{R}^3$, $\gamma(t) = (\cos t, \sin t, t)$, $t \in [0, 2\pi]$.
   (a) Find the arc-length of C.
   (b) Evaluate the integral $\int_C (x + y + z)\,ds$.

5. (20%) Let $D = [a,b] \times [c,d]$ be a rectangle in $\mathbb{R}^2$ and $f: D \to \mathbb{R}$ be a continuous function whose partial derivatives are continuous. Show that the surface area of $z = f(x,y)$ is given by

$$\iint_D \left[1 + \left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial y}\right)^2\right]^{1/2} dxdy.$$

備考	試題隨卷繳交
命題委員：	（簽章） 98 年 6 月 25 日