

考試科目	微積分	系別	應用數學系	考試時間	7月6日(五) 第二節
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1. Use the definition of a limit to show that

$$(10\%) (a) \lim_{x \rightarrow 1} \frac{x^2 - 1}{x - 1} = 2.$$

$$(10\%) (b) \lim_{x \rightarrow \infty} \frac{1}{x-1} = 0.$$

2. Prove or disprove that

(10\%) (a) if $f(x)$ is continuous at $x=a$, then $f(x)$ is differentiable at $x=a$.

(10\%) (b) if $f(x)$ is differentiable at $x=a$, then $f(x)$ is continuous at $x=a$.

3. Show that

(10\%) (a) if $f'(x) > 0$, then $f(x)$ is increasing.

(10\%) (b) if $f'(x) < 0$, then $f(x)$ is decreasing.

4. Suppose money is being transferred continuously into an account over a time period $0 \leq x \leq T$ at a rate given by the function $f(x)$ and the account earns interest at an annual rate r compounded continuously.

(10\%) (a) Show that the future value over the term T is $\int_0^T f(x) e^{r(T-x)} dx$.

(10\%) (b) Show that the present value over the term T is $\int_0^T f(x) e^{-rx} dx$.

5. Let $z = f(x, y)$ be an objective function and $g(x, y) = 0$ be a constrained equation and λ be a Lagrange multiplier.

(10\%) (a) Show that the method of Lagrange multipliers works.

(10\%) (b) Interpret the meaning of λ in the real world and justify your answer.

考試科目	線性代數	系別	應用數學系	考試時間	7月6日(五)第四節
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1. Let W_1 and W_2 be subspaces of a vector space V . Prove that $W_1 \cup W_2$ is a subspace of V if and only if $W_1 \subseteq W_2$ or $W_2 \subseteq W_1$. (15 pts)
2. Let T be a linear transformation defined by $T(x, y) = (3x+2y, 2x-5y)$. Let $\beta = \{(1, 0), (0, 1)\}$ and $\gamma = \{(1, 1), (1, 0)\}$ be two ordered bases for \mathbb{R}^2 . Find some nonsingular matrix Q such that $[T]_\gamma = Q^{-1}[T]_\beta Q$. (20 pts)
3. It can be shown that the vectors $\vec{u}_1 = (2, -3, 1)$, $\vec{u}_2 = (1, 4, -2)$, $\vec{u}_3 = (-8, 12, 4)$, $\vec{u}_4 = (1, 37, -17)$, and $\vec{u}_5 = (-3, -5, 8)$ generate \mathbb{R}^3 . Find a subset of $\{\vec{u}_1, \vec{u}_2, \dots, \vec{u}_5\}$ that is a basis for \mathbb{R}^3 . (15 pts)
4. Prove that if $M \in M_{n \times n}(\mathbb{F})$ can be written in the form $M = \begin{pmatrix} A & B \\ 0 & C \end{pmatrix}$, where A and C are square matrices, then $\det(M) = \det(A) \cdot \det(C)$. (15 pts)
5. Let $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -2 & 1 \end{bmatrix}$. Evaluate A^{100} . (15 pts)
6. Let $A = \begin{bmatrix} 2 & -2 & 1 \\ -2 & -1 & 2 \\ 1 & 2 & 2 \end{bmatrix}$. Evaluate $A^9 + 3A^6 - 15A^5 + 200A^4 - 150A^2 + 10I$. (20 pts)

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