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| 考試科目 | 微積分(-) | 系別 | 應用數學系 | 考試時間 | 7月12日(五)第二節 |
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Show all your work.

- Find the limit L of $\lim_{x \rightarrow 4} \sqrt{x}$. Then use the ϵ - δ definition to prove that the limit is L .
- Find the derivative of $f(x) = \frac{1}{x^2}$ by the limit process.
- The edges of a cube are expanding at a rate of 8 centimeters per second. How fast is the surface area changing when each edge is 6.5 centimeters?
- Use differentials to approximate the value of $\sqrt{99.4}$.
- Analyze and sketch a graph of the function of $f(x) = \frac{x^2+1}{x}$. Label any intercepts, relative extrema, points of inflection, and asymptotes.
- Find the minimum value of $f(x, y, z) = 2x^2 + y^2 + 3z^2$ subject to the constraint $2x - 3y - 4z = 49$.
- Find all relative extrema and saddle points on the graph of $f(x, y) = 2xy - \frac{1}{2}(x^4 + y^4) + 1$.

- (15 pts)
- (10 pts)
- (15 pts)
- (10 pts)
- (20 pts)
- (15 pts)
- (15 pts)

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| 備註 | 試題隨卷繳交 |
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| 考試科目 | 微積分(二) | 系 別 | 應用數學 31 8111 | 考試時間 | 七月 12 日(五) 第 四 節 |
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Instruction: Full credit will be given only if the necessary work is shown justifying your answer.

- Determine whether the improper integral $\int_1^\infty \frac{1}{\sqrt{x^3+1}} dx$ converges and justify your answer. (20%)
- Evaluate the following integrals:
 - $\int_0^1 \frac{x}{\sqrt{x+1}} dx$
 - $\int \csc x dx$
- Evaluate the integral $\int_0^1 \int_{3y}^3 e^{x^2} dx dy$. (20%)
- (a) State Green's theorem in the plane.
 (b) Use the Green's theorem to evaluate the line integral: $\oint_C (x^2 + 4xy)dx + (2x^2 + 3y)dy$, where C is the ellipse $9x^2 + 16y^2 = 144$. (20%)
- Evaluate the double integral: $\iint_R \frac{1}{4+x^2+y^2} dxdy$, where R is the region in quadrant one bounded by $x^2 + y^2 = 4$ and the lines $y = 0$ and $y = x$. (20%)

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| 考試科目 | 微積分 | 系別 | 應用數學系 8116 | 考試時間 | 7月2日(五) 第二節 |
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Instruction: Full credit will be given only if the necessary work is shown justifying your answer.

1. (a) State the fundamental theorem of calculus.

(b) Let $F(x) = \int_{\cos x}^{x^2+x} e^{t^2} dt$. Use (a) to evaluate $F'(0)$. (20%)

2. Determine whether the series $\sum_{n=1}^{\infty} (\sqrt{n^3 + 1} - \sqrt{n^3})$ converges and justify your answer. (20%)

3. Evaluate the integral $\iiint_B x^2 + y^2 + z^2 dx dy dz$, where $B = \{(x, y, z) | x^2 + y^2 + z^2 < 1\}$. (20%)

4. Let $f(x, y) = 2x^2 + 3y^2 - 4x - 5$. Find the absolute maximum and minimum of $f(x, y)$ on the region $x^2 + y^2 \leq 16$. (20%)

5. Let

$$f(x, y) = \begin{cases} \frac{xy}{x^2 + y^2} & \text{if } (x, y) \neq (0, 0), \\ 0 & \text{if } (x, y) = (0, 0). \end{cases}$$

- (a) Find $\frac{\partial f}{\partial x}(0, 0)$ and $\frac{\partial f}{\partial y}(0, 0)$ if exists;
(b) Is $f(x, y)$ continuous at $(0, 0)$? (20%)

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| 考試科目 | 線性代數 | 系別 | 應用數學系 ⁸¹¹⁶ | 考試時間 | 7月12日(五)第 四 節 |
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Show all your work.

1. Prove that the set of all $n \times n$ symmetric matrices forms a subspace of $M_n(\mathbb{R})$. Find a basis for this subspace. (15 pts)
2. Show that the product of two upper triangular matrices is upper triangular. (15 pts)
3. Let T be a linear transformation from V to W . Show that the null space of T is a subspace of V and the range of T is a subspace of W . Let $T(2, -1, 3) = (-1, 1)$, $T(0, 4, -2) = (3, 2)$ and $T(1, 2, 3) = (-2, 0)$. Is T a linear transformation from \mathbb{R}^3 to \mathbb{R}^2 ? If yes, find the null space and range of T . (20 pts)
4. Let W_1 and W_2 are subspaces of V . Show that $\dim W_1 + \dim W_2 = \dim(W_1 + W_2) + \dim(W_1 \cap W_2)$. (15 pts)
5. Find the eigenvalues, eigenvectors, the algebraic multiplicity and geometric multiplicity of each eigenvalue of the matrix $\begin{bmatrix} 1 & -1 & -1 \\ 1 & 3 & 2 \\ 1 & 1 & 0 \end{bmatrix}$. (20 pts)
6. For the matrix $\begin{bmatrix} 1 & 2 & 1 \\ -1 & 1 & 2 \\ 0 & 1 & 1 \end{bmatrix}$, compute the rank and the inverse if it exists. (15 pts)