國立政治大學104學年度轉學生招生考試試題

考試科目	微積分一（以微分為主）	系別	應用數學系（二年級）	考試時間	7月10日(五) 第二節

無演算過程者不予計分

1. **(10%)** Use the $\varepsilon-\delta$ definition to prove that $\lim_{x\to -4} x^2+4x=0$.

2. **(10%)** Prove that if f is differentiable on $(-\infty,\infty)$ and $f'(x)<1$ for all real numbers, then f has at most one fixed point. (A fixed point of function f is a real number c such that $f(c)=c$.)

3. **(8% +8%+8%)** Find the limit (if it exists). If the limit does not exist, explain why.

   (a) $\lim_{x\to 0^+}(\sin x)^x$

   (b) $\lim_{(x,y)\to(0,0)} \dfrac{\sqrt{x^2+y^2}-\tan\sqrt{x^2+y^2}}{(x^2+y^2)^{\frac{3}{2}}}$

   (c) $\lim_{(x,y)\to(0,0)} \dfrac{x^2y}{x^4+y^2}$

4. **(8% +8%+8%)** Evaluate $\dfrac{dy}{dx}$ at the specific point.

   (a) $y=\left(\dfrac{3x^2-2}{2x+3}\right)^3$ at $x=0$

   (b) $y=\ln\left(\dfrac{1+e^x}{1-e^x}\right)$ at $x=1$

   (c) $y=\cos^{-1}(x^2)$ at $x=\dfrac{1}{2}$

5. **(8%)** Discuss the continuity of the function $f(x,y)=\begin{cases}\dfrac{\sin(x^2-y^2)}{x^2-y^2}, & x^2\neq y^2\\ 1, & x^2=y^2\end{cases}$.

6. **(10%)** Find the points on the surface $z^2=xy+4$ closest to the origin.

7. **(6%+8 %)** Let $f(x,y)=\begin{cases}\dfrac{xy(x^2-y^2)}{x^2+y^2}, & (x,y)\neq(0,0)\\ 0, & (x,y)=(0,0)\end{cases}$. Find the following derivatives.

   (a) $f_x(0,0)$

   (b) $f_{xy}(0,0)$

備註	一、作答於試題上者，不予計分。 二、試題請隨卷繳交。

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考試科目	微積分二（以積分為主）	系別	應用數學系（二年級）	考試時間	7月10日(五) 第四節

無演算過程者不予計分

1. **(5% for each sub-problem)** Let $G(x)=\int_0^x [s\int_0^s f(t)dt]ds$, where f is continuous for all real t. Find

   (a) $G'(x)$,

   (b) $G''(x)$.

2. **(8% for each sub-problem)** Evaluate the integral.

   (a) $\int_1^9 \frac{1}{\sqrt{x}(1+\sqrt{x})^2}dx$

   (b) $\int_0^1 x^3 e^{-2x}dx$

   (c) $\int_1^5 \frac{x-1}{x^2(x+1)}dx$

   (d) $\int_0^\infty \int_0^\infty xye^{-(x^2+y^2)}dxdy$

   (e) $\int_0^{\sqrt{3}/2} \frac{t^2}{(1-t^2)^{3/2}}dt$

3. **(10%)** Evaluate the integral $\int_{-\infty}^{\infty} e^{-\frac{1}{2}x^2}dx$.

4. **(10%)** Find the interval of convergence of the power series $\sum_{n=0}^{\infty}\frac{(-1)^n(x-2)^n}{(n+1)^2}$.

5. **(10%)** Find the positive value of p for which the series $\sum_{n=2}^{\infty}\frac{1}{n(\ln n)^p}$ converges.

6. **(10%)** Evaluate the integral $\int_C xydx+(x+y)dy$ along the curve $y=x^2$ from $(-1,1)$ to $(2,4)$.

7. **(10%)** Evaluate the integral $\iint_R \frac{y}{x^2+y^2}dA$, where R is the trapezoid bounded by $y=x$, $y=2x$, $x=1$, and $x=2$.

備註	一、作答於試題上者，不予計分。 二、試題請隨卷繳交。

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考試科目	微積分	系別	應用數學系三年級	考試時間	7月10日(五) 第二節

Show all your work

1. (16pts) Find the limit L for $\lim_{x \to 2}(1 - x^2)$. Then use $\epsilon - \delta$ definition to prove that the limit is L.

2. (20pts) State and prove the Mean Value Theorem.

3. (16pts) Use a power series to approximate

$$\int_0^1 e^{-x^2}\,dx$$

with an error of less than 0.01.

4. (16pts) Find the length of the arc from $\theta = 0$ to $\theta = 2\pi$ for the cardioid

$$r = f(\theta) = 2 - 2\cos\theta.$$

5. (16pts) Find the highest point on the curve of intersection of the surfaces.

$$\text{Cone} : x^2 + y^2 - z^2 = 0$$

$$\text{Plane} : x + 2z = 4.$$

6. (16pts) Let R be the region bounded by the square with $(0,1), (1,2), (2,1)$ and $(1,0)$. Evaluate the integral

$$\int_R\int (x+y)^2 \sin^2(x-y)\,dA.$$

備註	一、作答於試題上者，不予計分。 二、試題請隨卷繳交。

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考試科目	線性代數	系別	應用數學系（三年級）	考試時間	7月10日(五)第四節

Show all your work.

1. (16pts) Let $W = \mathrm{sp}(e^{2x}, e^{4x}, e^{8x})$ be the subspace of the vector space of all real-valued functions with domain $\mathbb{R}$, and let $B = (e^{2x}, e^{4x}, e^{8x})$. Find the matrix representation A relative to B, B of the linear transformation $T : W \to W$ defined by $T(f) = D^2(f) + 2D(f) + f$.

2. (16pts) Prove that a square matrix is invertible if and only if its adjoint is an invertible matrix.

3. (20pts) Find the eigenvalues and the corresponding eigenvectors of the matrix

$$A = \begin{bmatrix} -2 & 0 & -1 \\ 0 & 2 & 0 \\ 3 & 0 & 2 \end{bmatrix}$$

and also find an invertible matrix C and a diagonal matrix D such that $D = C^{-1}AC$.

4. (16pts) Find an orthonormal basis for $\mathrm{sp}(1, e^x)$ for $0 \le x \le 1$ if the inner product is defined by $\langle f, g\rangle = \int_0^1 f(x)g(x)dx$.

5. (16pts) Let $\{\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_n\}$ be a basis for a vector space V, and let $\mathbf{w} = t_1\mathbf{v}_1 + t_2\mathbf{v}_2 + \cdots + t_k\mathbf{v}_k$, with $t_k \neq 0$. Prove that

$$\{\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_{k-1}, \mathbf{w}, \mathbf{v}_{k+1}, \ldots, \mathbf{v}_n\}$$

is a basis for V.

6. (16pts) Let $T : \mathrm{P}_2 \to \mathrm{P}_2$, $T(a + bx + cx^2) = (a + b - 2c) + (a - 2b + c)x + (b - 2a)x^2$. Compute the characteristic polynomial $c_T(x)$.

備註	一、作答於試題上者，不予計分。 二、試題請隨卷繳交。