

考試科目	微積分(一) = 以微積分為主	系別	應用數學系 = 甲級	考試時間	7月8日(星期五) 第二節
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1. Evaluate the limits.

(a) (10%) $\lim_{(x,y) \rightarrow (0,0)} \frac{xy \sin y}{3x^2 + y^2}$.

(b) (10%) $\lim_{x \rightarrow 0} \left(\frac{1}{x^3} - \frac{1}{2x^2} + \frac{1}{3x} - \frac{\ln(1+x)}{x^4} \right)$.

(c) (10%) $\lim_{x \rightarrow 0} \frac{\sin(2x) \tan(4x)}{1 - \cos(3x)}$.

2. Find the derivatives of the following functions

(a) (10%) $f(x) = x \sqrt{2x + \sqrt{3x + 1}}$.

(b) (10%) $f(x) = \int_{x^3}^{3^x} e^{t^2} dt$.

3. (10%) Let $f(x) = 3 + x^2 + \tan\left(\frac{\pi x}{2}\right)$, $-1 < x < 1$. Find $(f^{-1})'(3) = ?$

4. (10%) Let $a_1 = 1$, and $a_{n+1} = 3 - \frac{1}{a_n}$ for $n \geq 1$. Determine whether the sequence $\{a_n\}_{n=1}^{\infty}$ is convergent or divergent.

5. (10%) Find the Maclaurin series for $f(x) = \sqrt[3]{8+x}$.

6. (10%) Find the maximum rate of change of $f(x, y) = \sin(xy)$ at $(1, 0)$.

7. (10%) Find the extreme values of $f(x, y, z) = xyz$ subject to $x^2 + 2y^2 + 3z^2 = 6$.

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註

一、作答於試題上者，不予計分。

二、試題請隨卷繳交。

考試科目	微積分(二) : 以積分為主	系別	應用數學系 二年級	考試時間	7月8日(星期五) 第四節
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Problem 1 Evaluate the integrals.

(1) (10%) $\int \frac{1}{\sqrt{x^2 - 4x}} dx.$

(2) (10%) $\int_0^1 \int_0^1 \frac{1}{1 - xy} dx dy.$

(3) (10%) $\int_0^\infty \frac{e^{-x}}{\sqrt{x}} dx.$

(4) (10%) $\int_0^1 \int_{\sqrt{y}}^1 \frac{ye^{x^2}}{x^3} dx dy.$

(5) (10%) $\iint_R \cos\left(\frac{y-x}{y+x}\right) dA$ where R is the trapezoidal region with vertices $(1, 0)$, $(2, 0)$, $(0, 2)$ and $(0, 1)$.

Problem 2 Determine whether the series is convergent or divergent

(a) (10%) $\sum_{n=1}^{\infty} \ln\left(\frac{n^2}{n^2 + 1}\right).$

(b) (10%) $\sum_{n=1}^{\infty} (-1)^{n-1} a_n$, where $a_n > 0$ for $n \in \mathbb{N}$ and $\lim_{n \rightarrow \infty} a_n = 0$.

Problem 3 (10%) Find the interval of convergence of the series

$$\sum_{n=1}^{\infty} \ln\left(\frac{(2x-1)^n}{5^n \sqrt{n}}\right).$$

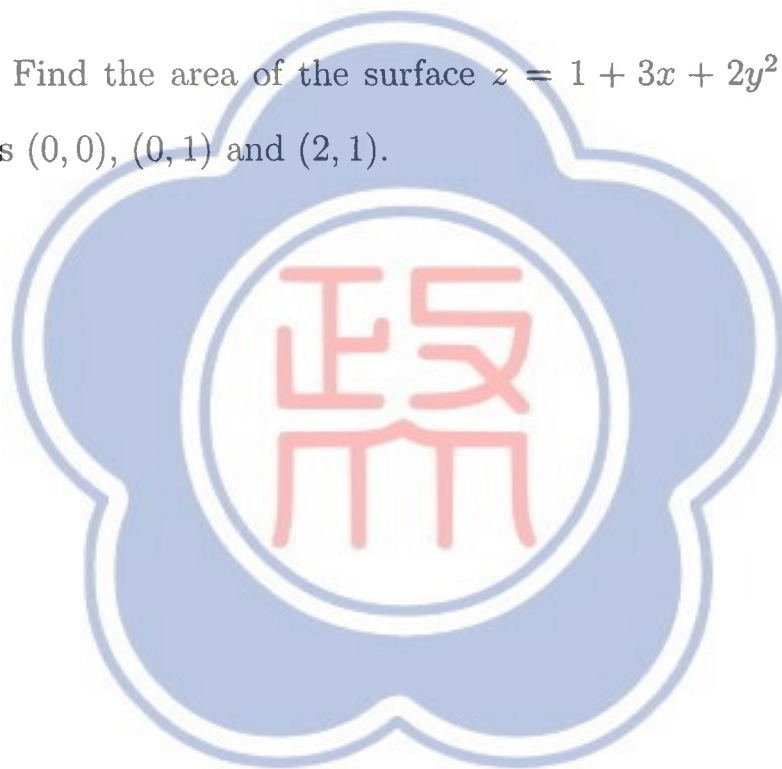
考試科目	微積分(二) ：以積分為主	系別	應用數學系 二年級	考試時間	7月8日(星期五) 第四節
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Problem 4 (10%)

$$\int_C (y + e^{\sqrt{x}}) dx + (2x + \cos y^2) dy$$

where C is the positive oriented boundary curve of the region enclosed by $y = x^2$ and $x = y^2$.

Problem 5 (10%) Find the area of the surface $z = 1 + 3x + 2y^2$ that lies above the triangle with vertices $(0, 0)$, $(0, 1)$ and $(2, 1)$.



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註

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- 二、試題請隨卷繳交。

考試科目	微積分	系別	應用數學系 三年級	考試時間	7月8日(五)第二節
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無演算過程者不予計分。

(1) (8% for each sub-problem) Find the limit. (Please clearly explain how you obtain your answers.)

(a) $\lim_{x \rightarrow 2} \tan^{-1}\left(\frac{x^2 - 4}{3x^2 - 6x}\right)$

(b) $\lim_{x \rightarrow \infty} \left(\tan \frac{\pi x}{2x+1}\right)^{1/x}$

(2) (8% for each sub-problem) Find $\frac{dy}{dx}$ evaluated at the specified point.

(a) $\cos(x-y) = (2x+1)^3 y$ at $(x, y) = (1, 1)$.

(b) $y = x \tan^{-1}(\sqrt{x})$ at $x = 4$.

(3) (10% for each sub-problem) Evaluate the following integrals.

(a) $\int_0^1 \frac{1}{(4-x^2)^{3/2}} dx$

(b) $\int_0^{\pi/2} (\sin^5 x \cdot \cos^2 x) dx$

(c) $\iint_D 1 dx dy$ where $D = \{(x, y) \mid y \geq x^2, x+y \leq 2\}$

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註

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