考試科目	微積分	所別	數學碩士專班 A811	考試時間	3月19日上午第3節 星期六

每題十分

1. Show that $f(x)$ is differentiable at $x=a$, then $f(x)$ is continuous at $x=a$.
2. Show that if $f'(x)>0$ on open interval (a,b), then $f(x)$ is increasing on (a,b).
3. Let $f(x)=\begin{cases} 2x-1, & \text{if } x>1 \\ x^2, & \text{if } x\le 1 \end{cases}$. Determine if $f(x)$ is differentiable at $x=1$?
4. Find $\frac{d}{dx}[f(x)\cdot g(x)]$ at $x=2$, when $f(2)=f'(2)=3$ and $g(2)=g'(2)=4$.
5. Let $f(x^2-2)=g(x)$, find $f'(2)$ if $g'(2)=1$.
6. Use the limit of Riemann sum to find $\int_0^1 x^2\,dx$.
7. Let $f(x)=\int_1^x \frac{1}{\sqrt{t}+t^2}\,dt$, find $f'(x)$ and $f'(4)$.
8. Find the average of $f(x)=\sqrt{x}$ from $x=1$ to $x=9$.
9. Find $\int \ln x\,dx$.
10. Find $\int \frac{x}{\sqrt{x+1}}\,dx$.

備考	試題隨卷繳交

命題委員：　　057　　（簽章）94年3月10日

考試科目	線性代數	所別	數學教學碩士在職專班	考試時間	3月19日 星期六 下午第4節

1. Find the appropriate matrix inverse to solve the system

$$\begin{cases} x_1 - x_2 - 2x_3 = 1 \\ 2x_1 - 3x_2 - 5x_3 = 3 \\ -x_1 + 3x_2 + 5x_3 = -2. \end{cases} \quad (15 \text{ pts})$$

2. Let $L: \mathbb{R}^3 \to \mathbb{R}^3$ be defined by $L\left(\begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix}\right) = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 2 \\ 2 & 1 & 3 \end{bmatrix}\begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix}$.

   (a) Is L onto? (8 pts)
   (b) Find a basis for range L. (8 pts)
   (c) Find Ker L. (8 pts)
   (d) Is L one-to-one? (5 pts)

3. Consider the basis $S = \{\vec{u}_1, \vec{u}_2, \vec{u}_3\}$ for $\mathbb{R}^3$, where $\vec{u}_1 = (1, 1, 1)$, $\vec{u}_2 = (-1, 0, -1)$, and $\vec{u}_3 = (-1, 2, 3)$.
   Use the Gram-Schmidt process to transform S to an orthonormal basis for $\mathbb{R}^3$. (17 pts)

4. (a) Show that the following matrix A is diagonalizable.
   (b) Find a diagonal matrix D that is similar to A.
   (c) Determine the similarity transformation that diagonalizes A.

$$A = \begin{bmatrix} -4 & -6 \\ 3 & 5 \end{bmatrix} \quad (8 \text{ pts each})$$

5. To evaluate the determinant $\begin{vmatrix} 1 & 2 & -3 & 4 \\ -4 & 2 & 1 & 3 \\ 3 & 0 & 0 & -3 \\ 2 & 0 & -2 & 3 \end{vmatrix}$. (15 pts)

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