考試科目	微積分	所別	數學教學碩士在職專班	考試時間	2月26日(六)第3節

1. Assume that f is continuous on $[0,2]$ and $f(0) = f(2) = 0$. Show that there exists $c \in [0,1)$ such that $f(c + 1) = f(c)$. (15%)

2. Suppose that f and g are differentiable functions in $[0,1]$ and $f(0) = 5$, $f(1) = 0, g(0) = 2, g(1) = -3$. Show that there exists a point $x_0 \in (0,1)$ such that $f'(x_0) = g'(x_0)$. (15%)

3. Find the maximum and minimum values of the function $f(x, y, z) = x + 2y + z$ on the sphere $x^2 + y^2 + z^2 = 24$. (15%)

4. Show that $(1 + x)^\alpha < 1 + \alpha x$ for all $x > 0$ and $0 < \alpha < 1$. (15%)

5. Compute the volume of the solid enclosed by the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$, where a, b, c are positive constants. (15%)

6. Evaluate the following integrals:

   (a) $\int \ln x \, dx$ (b) $\int \frac{1}{x^2+1} dx$ (c) $\int \frac{1}{x^2-1} dx$ (15%)

7. Does the series $\sum_{n=1}^{\infty} \left(\frac{2n}{2n+1} - \frac{2n-1}{2n} \right)$ converge? Explain. (10%)

備註	試題隨卷繳交

國立政治大學 100 學年度研究所碩士班招生考試試題

考試科目	線性代數	所別	數學教學碩士在職專班	考試時間	2月26日六第四節

1. (15 points) Let A be an $n \times n$ invertible real matrix. Is A diagonalizable? Justify your answer.

2. (15 points) Let $\mathcal{F} = \{f : \mathbb{R} \to \mathbb{R}\}$ be the vector space of all real-valued functions defined on $\mathbb{R}$. Let $f(x) = \cos(x)$ and $g(x) = \sin(x)$. Clearly, f and g belong to $\mathcal{F}$. Are f, g linearly independent? Justify your answer.

3. (15 points) Show that any linear transformation $T : \mathbb{R}^3 \to \mathbb{R}$ is of the form $T(x, y, z) = ax + by + cz$ for some real number $a, b, c \in \mathbb{R}$.

4. (15 points) Let W be the subspace of $\mathbb{R}^3$ spanned by two vectors $(1, 1, 0)$ and $(1, 0, 1)$. Find a basis for $W^\perp$. ($W^\perp = \{\mathbf{v} \in \mathbb{R}^3 | \mathbf{v} \cdot \mathbf{u} = 0 \text{ for all } \mathbf{u} \in \mathbb{R}^3\}$)

5. (20 points) Let $\beta_1 = \{(1, 1), (1, 0)\}$ and $\beta_2 = \{(-1, 1), (0, 1)\}$ be two bases of $\mathbb{R}^2$.
   (a) For any vector $\mathbf{v}$ in $\mathbb{R}^2$, the coordinate of $\mathbf{v}$ corresponding to the basis β is denoted by $[\mathbf{v}]_\beta$. Let $\mathbf{v}$ be a vector in $\mathbb{R}^2$ such that $[\mathbf{v}]_{\beta_1} = (2, 3)$. Find $[\mathbf{v}]_{\beta_2}$.
   (b) Find the matrix B such that $B[\mathbf{u}]_{\beta_1} = [\mathbf{u}]_{\beta_2}$, for all $\mathbf{u}$ in $\mathbb{R}^2$.

6. (20 points) Let V be a finite-dimensional vector space over $\mathbb{R}$. Let T be a linear operator on V. Suppose that $T^2 = T$. Show that $V = \ker \mathrm{T} + \mathrm{im}\, \mathrm{T}$.

備註	試題隨卷繳交