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A new approach to estimating a profit frontier using the censored stochastic frontier model

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ABSTRACT

The translog profit functional form is widely used to study technical efficiency for banks. Although this functional form is known as being flexible, it is not applicable to those banks incurring economic losses. The recently developed approach, i.e., the censored stochastic frontier model (CSFM), by Tsay et al. (2013) appears to be superior to existing approaches, since CSFM does not need to transform negative profit into positive profit before taking the natural logarithm. The transformation with respect to the profit variable tends to bias the parameter estimates of the profit frontier and the subsequent profit efficiency measure. We show that the parameter estimates of CSFM have the desirable statistical properties. Moreover, empirical results reveal that the mean profit efficiency of CSFM is more robust than those models using transformed profits across the sub-periods 1991–1998 and 1999–2009.

1. Introduction

There have been many studies on the profit efficiencies (PE) of financial institutions over the past several decades that differ largely from the cost efficiency scores, implying the importance of the revenue side on the measurement of bank performance.[1] The study of profit frontiers for banks can provide more information for managers' references than cost frontiers that use information up to the expenditures of inputs hired and completely exclude the role of total revenue. This paper estimates the profit frontiers of Taiwan's banking industry, together with technical efficiency, under the assumption of perfect competition. Taiwan's banking sector consists of many small-sized banks and each of them takes a small market share. Therefore, perfect competition seems to be a valid assumption to characterize this condition of "over-banking".

Because banks in the sample may unfortunately incur losses during the period covered, the popular translog functional form is not specifically applicable for negative profits. One cannot take the natural logarithm with respect to a negative value. To date, three methods are suggested to deal with this problem. First, one simply dismisses those observations with negative profit values (henceforth, the dismissal approach). See, for example, Huang (2000). This approach obviously does

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[1] Existing research studies on the estimation of a profit function can be classified into the standard profit function and the alternative profit function. The former assumes that the input and output markets are perfect competition. Given the input and output price vectors, firms maximize their profits by adjusting the quantities of inputs and outputs. The latter allows for the possibility of imperfect competition in output markets so that output prices are replaced by output quantities. See, for example, Maudos, Pastor, Perez, and Quesada (2002), Maudos and Pastor (2003), Kasman and Yildirim (2006), Bos and Schmiedel (2007), Ariff and Can (2008), Berger, Hasan, and Zhou (2009), and Ray and Das (2010).

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not fully utilize the sample information, since these omitted observations contain useful information on a firm's input–output and exit decisions.

Second, one can inflate the profits (henceforth, the inflating approach) of all sample firms by adding the sum of the absolute value of the minimum (negative) profit in the sample and unity to each firm's observed profit, e.g., Maudos et al. (2002), Berger & di Patti (2006), and Fitzpatrick and McQuinn (2008). This second approach is the mainstream in the literature, but it distorts the original information embedded in the dependent (profit) variable due to an ad hoc amount being added. This causes the parameter estimates to be inconsistent. Even worse, the subsequent measures of technical efficiency and scale and scope economies, calculated by using these parameter estimates, are also misleading.[2]

Third and finally, an indicator approach, which Hasan, Koetter, and Wedow (2009) and Bos and Koetter (2011) propose, recommends that only profits of loss-incurring firms are altered into unity and an extra explanatory variable, say z, is created and takes the absolute value of the negative profit for the loss-incurring firm. The profits of profit-making firms remain intact, and the variable z takes a value of unity. Similar to the inflating approach, estimators from the indicator approach are devoid of desirable statistical properties.

This paper proposes a different approach from the above three methods, under the framework of a censored regression model with composed errors, i.e., the censored stochastic frontier model (CSFM). A bank incurring economic losses is treated as a censored sample whose (log)profit is set to equal an arbitrarily small value of $c > 0$, which is a threshold level of profit. In this manner, the dependent variable of profit is not continuous for loss-incurring banks and is set to equal c, but is continuous and recorded as the actual level if the bank makes a positive profit. The advantages of such a treatment for a loss-incurring bank are threefold. First, the censored sample remains in the data and can be used to estimate the parameter in the profit frontier – that is, CSFM allows one to fully utilize the entire sample. Second, no adjustment for profits is required, thus avoiding any potential distortion of the dependent variable. Amemiya (1973) proves that the maximum likelihood estimators of the Tobit model (Tobin, 1958) are consistent and asymptotically normal. Third, as can be seen in Section 3, CSFM is easily implemented and the resulting parameter estimates have the desirable properties.

This article introduces a new methodology to formally address the above issue by extending the conventional censored regression model (Tobit model) to the stochastic frontier context, characterized by composed errors. To our knowledge, it is the first work that applies CSFM to investigate profit efficiencies. The emergence of the composed errors in the profit frontier poses difficulty in deriving a closed-form cumulative distribution function (cdf) for the censored sample, as the probability density function (pdf) of the composed error cannot be directly integrated due to the fact that it has no closed form. This impedes the Tobit model with error components from being estimated by the maximum likelihood. Differing from Greene (2003,2010), who proposes the simulated maximum likelihood to approximate the integration, this paper derives a closed-form formula for the cdf of the profit frontier with censored observations, as first proposed by Tsay, Huang, Fu, and Ho (2013). Given the foregoing statements, the current paper contributes to the research on profit efficiency in the banking industry.

The remainder of the paper is organized as follows. Section 2 briefly reviews the literature mainly on the studies of bank efficiency scores. Section 3 introduces the methodology. Section 4 describes the data and variables employed. Section 5 delineates the empirical results, while Section 6 concludes the paper.

2. Literature review on bank efficiency

There are two definitions on Farrell's (1957) radial measure of technical efficiency: output-oriented and input-oriented measures. An output technically efficient firm can produce maximal output from a given input mix, and an input technically efficient firm is able to use a minimal input mix to produce a given set of outputs. Given an input mix, the output-oriented technical efficiency score of a firm measures how close the firm's actual output (cost or profit) level is to that of the best-practice firm on the efficient frontier. Given a set of outputs, the input-oriented technical efficiency score of a firm assesses deviations in its input mix from the predicted amounts of the best firms in the industry.

There are two popular approaches to measuring technical efficiency in the literature: the stochastic frontier approach (SFA) and data envelopment analysis (DEA). Aigner, Lovell, and Schmidt (1977) and Meeusen and van den Broeck (1977) first introduce the former approach. Later, most existing works in the literature estimate either production or cost frontiers to measure technical efficiency. Profit frontiers are relatively less applied by empirical researchers. Huang (2000), Altunbas, Gardener, Moyneux, and Moore (2001), Kasman and Yildirim (2006), Fitzpatrick and McQuinn (2008), Berger et al. (2009), Koutsomanoli-Filippaki, Mamatzakis, and Staikouras (2009), and Akhigbe and Stevenson (2010), to name a few, employ SFA to estimate profit frontiers, as SFA allows for the presence of composite errors. One of them is a two-sided random disturbance, accounting for the random shocks uncontrollable by the bank under consideration, and the other is a one-sided non-negative random variable that represents production inefficiency. Bhaumik, Das, and Kumbhakar (2012) use SFA to model the financial constraints of firms. Kumbhakara, Parmeter, and Tsionas (2013) introduce the zero inefficiency stochastic frontier model to accommodate the presence of both efficient and inefficient firms in the sample.

[2] It is easy to show that the inflation approach leads to inconsistent parameter estimates in the context of a simple regression model.

Charnes, Cooper, and Rhodes (1978) introduce the latter approach (DEA), which involves the use of linear programming techniques. DEA attributes all deviations of the observed output (cost or revenue) of a bank from the piece-wise linear production frontier to the managerial inability of that bank. See, for example, Maudos and Pastor (2003), Ariff and Can (2008), and Ray and Das (2010). Juo (2014) uses the non-parametric approach to decompose the change in profit of Taiwanese banks into various drivers, taking risk (represented by non-performing loans) into account. Juo, Fu, and Yu (2012) propose a non-oriented slacks-based measure model to divide the change in operating profit into various components, i.e., quantity and price effects. Mulwa and Emrouznejad (2013) demonstrate the measurement of productive efficiency using the Nerlovian indicator and the metafrontier with DEA techniques. Fu, Juo, Chiang, Yu, and Huang (2016) estimate and compare the profit efficiencies of Taiwanese and Chinese banks, under the framework of the metafrontier and by including equity capital as a quasi-fixed input.

The second stage of the three-stage DEA analysis, dating back to Fried, Lovell, Schmidt, and Yaisawamg (2002), recommends the standard SFA, which ignores the fact that the dependent variable is in fact censored at zero from below.[3] In other words, some observations have the dependent variable equaling zero, which corresponds to the best-practice firms in the sample. Therefore, the censored SFA model should be a better choice than the standard SFA.

Tsay et al. (2013) generalize the standard Tobit model to the CSFM that allows for the presence of both composed errors and a censored dependent variable. They propose an approximation approach to derive the cdf of the composed errors for censored data and show that the finite sample performance of their proposed MLE of the CSFM is acceptable. Huang and Liu (2014) examine the degree of market competition in the banking industries of 17 Central and Eastern European countries, using the H statistic proposed by Panzar and Rosse (1987). They apply CSFM to test whether these markets have achieved long-run equilibrium.

Many previous works have adopted SFA to perform empirical studies. Altunbas et al. (2001) examine the relationship between ownership type and technical efficiency for Germany's banking sector, using both cost and profit frontiers spanning 1989–1996, and find that private banks are more efficient than state-owned banks. Kasman and Yildirim (2006) explore cost and profit efficiencies of the eight new members of the European Union (EU) from Central and Eastern European countries across the period 1995–2002 and note that the cost inefficiency, on average, is equal to 0.207, whereas the profit inefficiency, on average, is equal to 0.367. In addition, foreign banks are found to be more efficient than domestic banks. Fitzpatrick and McQuinn (2008) evaluate profit efficiency in Canada, the UK, Ireland, and Australia over 1996–2002 and conclude that UK banks are, on average, the least efficient versus those of other countries.

Berger et al. (2009) analyze the efficiency of Chinese banks covering 1994–2003 and find that the Big Four banks are by far the least profit efficient, that foreign banks are the most efficient, and that minority foreign ownership is related to considerably improved efficiency of Chinese banks. Koutsomanoli-Filippaki et al. (2009) examine the effect of structural reforms on profit efficiency in four new EU member states from 1999 to 2003 and offer evidence that reforms support a positive impact on the profit efficiency of the banking sector, whereas reforms in the non-bank financial sector appear to hamper the enhancement of profit efficiency. Akhigbe and Stevenson (2010) investigate profit efficiency and its relationship with non-interest income for bank holding companies over 2003–2006, finding that higher levels of non-interest income, especially underwriting/brokerage income, are associated with lower profit efficiency. Furthermore, a bank with a larger amount of assets tends to have higher profit efficiency.

3. Methodology

Consider a stochastic profit frontier model:

$$y_i = \beta' x_i + \varepsilon_i, i = 1, \cdots, N, \tag{1}$$

where y_i and ε_i are the (log)total profit and composite error of the i^{th} observation, respectively, x_i is a $k \times 1$ vector of intercept, (log)output, and (log)input prices, and β is the corresponding vector of unknown parameters to be estimated. Composite error ε_i is specified as:

$$\varepsilon_i = v_i - u_i, \tag{2}$$

where $v_i \sim N(0, \sigma_v^2)$ is an independently and identically distributed (iid) normal random variable with a mean of zero and a constant variance, and u_i is assumed to be a half-normal random variable with a constant variance σ_u^2, i.e., $u_i \sim |N(0, \sigma_u^2)|$. Terms v_i and u_i are independent of each other.

It is well known that the pdf of ε_i, $f(\varepsilon_i)$, can be derived as:

$$f(\varepsilon_i) = \frac{2}{\sigma} \phi\left(\frac{\varepsilon_i}{\sigma}\right) \Phi\left(\frac{-\lambda \varepsilon_i}{\sigma}\right), \tag{3}$$

where $\lambda = \sigma_u / \sigma_v$, $\sigma^2 = \sigma_v^2 + \sigma_u^2$, and $\phi(.)$ and $\Phi(.)$ respectively signify the pdf and cdf of the standard normal distribution.[4]

[3] The dependent variable is obtained by the sum of the radial and non-radial inefficiency measures, derived from the first-stage DEA.

[4] Readers can refer to, e.g., Kumbhakar and Lovell (2000), for the detailed derivation of $f(\varepsilon_i)$.

If the dependent variable y_i is censored at a constant c from below, then Eq. (1) becomes CSFM, which can be expressed as follows:[5]

$$\begin{cases} y_i^* = \beta' x_i + \varepsilon_i, & i = 1, \cdots, N; \\ y_i = y_i^* & \text{if } y_i^* > c \\ y_i = c & \text{if } y_i^* \leqslant c \end{cases} \tag{4}$$

where y_i^* is a latent variable that is only observed when $y_i^* > c$ and when y_i^* is equal to the actual profit of y_i. It is noticeable from the first section that c can be viewed as the threshold level of profit. Notation y_i is set to be equal to c (a small number), when the actual profit is less than or equal to zero, since the logarithm of a non-positive value is undefined. A censored regression model should be estimated by the maximum likelihood, whose log likelihood function is written as:

$$L = \sum_{y_i > c} \ln[f(\varepsilon_i)] + \sum_{y_i = c} \ln[F(c - \beta' x_i)], \tag{5}$$

where $F(.)$ denotes the cdf of ε_i.

A difficulty immediately emerges for CSFM, since $f(.)$ of (3) contains $\Phi(.)$, which is not a closed form. Consequently, the cdf function of $F(.)$ cannot be directly integrated, because it is computed as:

$$F(Q_i) = \int_{-\infty}^{Q_i} f(\varepsilon_i) d\varepsilon_i, \tag{6}$$

where $Q_i = c - \beta' x_i$. Tsay et al. (2013) resolves this problem using the approximation approach, under the assumption that u_i is a half-normal random variable.[6] In other words, only the approximated $F(Q_i)$, $F_{app}(Q_i)$, can be derived, which has a closed form. Appendix shows the detailed derivation for $F_{app}(Q_i)$.

After replacing $F(Q_i)$ with $F_{app}(Q_i)$ in Eq. (5), the log-likelihood function of CSFM can be written as:

$$L = \sum_{y_i > c} \ln[f(\varepsilon_i)] + \sum_{y_i = c} \ln[F_{app}(Q_i)]. \tag{7}$$

Eq. (7) is suitable for conducting empirical studies, whenever the dependent variable is subject to censoring with composed errors. Amemiya (1973) proves that the MLEs of the standard Tobit model are consistent and asymptotically normal. Tsay et al. (2013) claim that the MLEs of CSFM exhibit excellent finite sample performance under various model configurations. We therefore conclude that our CSFM leads to consistent estimates of β in (1), even though the dependent variable of censored data is set to be $y_i = c$.

Following Jondrow, Lovell, Materov, and Schmidt (1982), the conditional mean of u_i given ε_i is calculated as the estimate of technical inefficiency for each observation. Note that we use $y_i = c$ to compute the residual of ε_i for censored data, while the actual values of y_i are used for profit-making banks. Estimates of the technical efficiency of each bank are derived from taking the natural exponent of the negative technical inefficiency measure.

4. Data

To highlight the merits of CSFM in estimating a profit frontier, we compile data from Taiwan's domestic commercial banks, extracted mainly from the databank of Taiwan Economic Journal.[7] It is important to note that Taiwan's banking industry can be characterized by over-banking, i.e., too many banks compete with one another in the small island country, such that the net interest margin is below 1.5%. Viewed from this angle, this financial market is nearly perfectly competitive, which validates the use of the profit frontier to describe the production technology of Taiwan's banks.

Following the intermediation approach, which views commercial banks as financial intermediaries employing inputs to provide various financial outputs, we identify two variable outputs, three variable inputs, and a netput. The output categories include loans (Y_1), consisting of short- and long-term loans, and investments (Y_2), including government and corporate securities. The ratios of their respective revenues to the corresponding output quantities are defined as their prices, denoted by P_1 and P_2, respectively. Non-interest income (Z) is treated as the netput, which is equal to the sum of fee income, credit card income, and other related incomes.

Labor (X_1), physical capital (X_2), and all deposits and borrowed money (X_3) are regarded as inputs. It is noteworthy that one of the salient features of the intermediation approach is treating borrowed money as one of the inputs. This highlights the important role of commercial banks in the allocation of scarce loanable funds to alternative projects. Conversely, the production approach defines the number of accounts issued by banks as one of the inputs. Unfortunately, this variable is usually

[5] In conducting the empirical study, we arbitrarily set the censoring point for loss-incurring banks at the logarithm of 10% of the minimum profit level among profit-making banks. In effect, the parameter estimates of the profit frontier are robust, because a change of the censoring point into 1%, for example, of the same minimum profit results in almost the same parameter estimates.

[6] Note that u_i can be assumed to have other forms of distribution, such as a truncated normal or an exponential distribution.

[7] Before 2001, the entity of personnel expenses is taken from the publication of the Ministry of Finance, Taiwan, the R.O.C, whereas for the remaining years the same item is provided by the Central Bank's publications.

Table 1
Descriptive statistics.

Profit variables	Mean	Std Dev
Profits*	3790.0490	5292.4010
Loans*	297685.4245	346895.3159
Investments*	65801.0594	97503.3040
Labor (number of employees)	2514.0723	2134.8958
Physical capital*	9367.9936	12473.2389
Borrowed funds*	377607.9123	456040.2203
Price of loans	0.0916	0.3537
Price of investments	0.0814	0.1111
Price of labor	1.0321	0.3221
Price of physical capital	0.3919	0.3126
Price of borrowed funds	0.0405	0.0224

Note: Variables with "*" mean that they are measured by millions of real New Taiwan Dollars (NT$) with base year 2006.

not available from most publicly accessible data banks. In addition, the number of accounts may not be positively associated with the amounts deposited in the individual accounts.

The price of labor (w_1) is calculated as the ratio of personnel expenses to the number of employees. The price of physical capital (w_2) is identified as the ratio of non-interest expenses minus personnel expenses to physical capital. The price of borrowed funds (w_3) is defined as the ratio of interest expenses to all deposits and borrowed money. Economic profit (π) is equal to total revenues, i.e., the sum of loan income and investment income, minus total expenditures on the employment of the three inputs. We arbitrarily choose labor as the numeraire and use its price to normalize the profit as well as all output and input prices.[8] All nominal variables are further deflated by the consumer price index with base year 2006.

To enhance the banking industry's competitive viability and to restructure the financial system, Taiwan's authorities initiated a series of financial reforms, i.e., the First Financial Restructuring from 2002 to 2003 and the Second Financial Restructuring from 2004 to 2008. Moreover, the United States subprime mortgage crisis ended around June 2009. The crisis did incur substantial losses for some commercial banks in Taiwan. Starting from 2011, some banks in Taiwan began to provide their accounting reports according to International Financial Reporting Standards (IFRS). In order to extract variables that are consistently defined over the entire sample period and after taking financial reforms into account, we set the sample period spanning from 1991 to 2009. All sample banks are domestic commercial banks and more than one half of them are listed in Taiwan's stock exchange market. Cooperative banks, industrial banks, and branches of all foreign banks are excluded. Our unbalanced panel data consist of 775 observations from 67 Taiwanese domestic commercial banks. Around 5% of the sample observations have negative profit values. Table 1 reports the descriptive statistics for all these variables mentioned.

5. Empirical results

This section first estimates the profit frontiers for four models. The dismissal approach (Model I) only considers data with positive economic profits. The inflating approach (Model II) inflates each bank's profit by a fixed amount (as suggested by, e.g., Berger & di Patti, 2006). The indicator approach (Model III) creates an explanatory variable, called the negative profit indicator (NPI), that is equal to unity for profit-making banks and is equal to the absolute value of the negative profit for loss-incurring banks (as suggested by Bos & Koetter, 2011) whose profits are set to be equal to unity. Lastly, Model IV is our preferred CSFM. The parameter estimates are next exploited to evaluate and compare profit efficiency scores among the four models.

Since the Asian financial crisis, which erupted in the second half of 1997, impacted the operations of Taiwan's banking sector to some extent, it is crucial to investigate the effect of the crisis on profit efficiency. Therefore, the entire sample period is further split into two sub-periods, i.e., 1991–1998 and 1999–2009, in order to see the changes in efficiency scores across the two sub-periods. Specifically, each of the four models is estimated three times, using data covering 1991–1998, 1999–2009, and the entire sample period, respectively.

5.1. Parameter estimates

We specify the profit function as the conventional translog form with the composed error ε of (2). The translog form is known as being flexible in describing the relationship between a firm's profit and its input and output prices. Note that the time trend is also included and treated as an additional netput in order to capture any possible technological change. All four models are estimated by the maximum likelihood.

Table 2 presents the parameter estimates of the four models, showing that most of the estimated parameters attain at least the 10% level of significance in all models. In particular, Model IV has the largest number of parameter estimates reach-

[8] This is one of the two ways for imposing the homogeneity restriction on a profit function.

Table 2
Parameter estimates of the four models.

Variable	Model I	Model II	Model III	Model IV
Constant	6.0069***	9.2063***	−0.1890	1.9254**
	(0.7815)	(0.4149)	(1.7537)	(0.9599)
$\ln w_2$	0.8640***	0.2738*	5.5176***	3.3068***
	(0.3581)	(0.1781)	(0.7590)	(0.5242)
$\ln w_3$	−1.7728***	−1.4449***	−8.6146***	−5.0338***
	(0.4741)	(0.2865)	(1.0118)	(0.5440)
$\ln w_2 * \ln w_2$	−0.2820***	−0.0258	−0.8747***	−0.5519***
	(0.0587)	(0.0363)	(0.1349)	(0.0587)
$\ln w_3 * \ln w_3$	−0.8956***	−0.6216***	−2.5713***	−2.0103***
	(0.1540)	(0.0761)	(0.2760)	(0.2394)
$\ln p_1$	−0.5778	0.3830**	−0.1041	−2.0655***
	(0.5097)	(0.2701)	(0.9433)	(0.6392)
$\ln p_2$	−0.2311*	0.0941	0.0576	0.8535**
	(0.1566)	(0.0972)	(0.3599)	(0.3693)
$\ln p_1 * \ln p_1$	−0.2239*	−0.2971***	−0.1453***	−0.6012*
	(0.1443)	(0.0670)	(0.2605)	(0.3419)
$\ln p_2 * \ln p_2$	0.0393**	0.0021	0.0204	0.0939*
	(0.0199)	(0.0125)	(0.0468)	(0.0563)
$\ln w_2 * \ln w_3$	0.0982	0.0070	0.4956***	−0.2332
	(0.1048)	(0.0437)	(0.1606)	(0.2426)
$\ln p_1 * \ln p_2$	−0.1352**	−0.0336	−0.1710	−0.4565***
	(0.0783)	(0.0347)	(0.1222)	(0.1143)
$\ln p_1 * \ln w_2$	0.3491***	0.0605	1.5251***	1.3553***
	(0.1111)	(0.0533)	(0.2090)	(0.2102)
$\ln p_1 * \ln w_3$	0.0476	0.2064***	−0.3207**	−0.2266*
	(0.1308)	(0.0461)	(0.1765)	(0.1281)
$\ln p_2 * \ln w_2$	−0.0578*	−0.0223	0.0338	0.1268
	(0.0422)	(0.0257)	(0.0952)	(0.0919)
$\ln p_2 * \ln w_3$	0.0124	0.1132***	0.0935	0.5460***
	(0.0709)	(0.0385)	(0.1360)	(0.1093)
$\ln z$	−0.2191*	−0.1358**	−0.5366*	−0.8355***
	(0.1545)	(0.0756)	(0.3453)	(0.2684)
$\ln z * \ln z$	0.0907***	0.0271**	0.1485***	0.1762***
	(0.0263)	(0.0131)	(0.0585)	(0.0328)
$\ln p_1 * \ln z$	0.1810***	0.0777**	0.2199***	0.0929**
	(0.0512)	(0.0282)	(0.1064)	(0.0477)
$\ln p_2 * \ln z$	0.0172	0.0286	0.0128	0.0250
	(0.0186)	(0.0121)	(0.0444)	(0.0299)
$\ln w_2 * \ln z$	−0.0217	−0.0256***	−0.3471***	−0.1974***
	(0.0353)	(0.0205)	(0.0796)	(0.0674)
$\ln w_3 * \ln z$	−0.1190***	−0.1100	−0.0022	−0.1197***
	(0.0357)	(0.0228)	(0.0818)	(0.0491)
t	0.0269	0.0138	−0.6082***	−0.0675
	(0.0565)	(0.0308)	(0.1163)	(0.0762)
$t * t$	−0.0311***	0.0022	−0.0773***	−0.0335***
	(0.0035)	(0.0019)	(0.0071)	(0.0053)
$t * \ln w_2$	0.0017	−0.0016***	0.1734***	0.0519*
	(0.0147)	(0.0076)	(0.0295)	(0.0307)
$t * \ln w_3$	−0.0910***	0.0451***	−0.1630***	−0.0611***
	(0.0218)	(0.0112)	(0.0422)	(0.0258)
$t * \ln p_1$	0.0032	−0.0618	−0.2875***	−0.1248**
	(0.0231)	(0.0118)	(0.0470)	(0.0541)
$t * \ln p_2$	0.0091	0.0042***	0.0052	0.0288**
	(0.0093)	(0.0048)	(0.0177)	(0.0126)
$t * \ln z$	0.0040	−0.0097*	0.0242**	−0.0104
	(0.0056)	(0.0029)	(0.0122)	(0.0089)
NPI	–	–	0.3158***	–
			(0.1067)	
Observations	737	775	775	775

Note:
1. Numbers in parentheses are standard errors.
2. ***, **, and * denote significance at the 1%, 5%, and 10% levels, respectively.
3. Each input and output prices, as well as the dependent variable, have been normalized by the price of labor (w_1).

ing statistical significance at least at the 10% level. This can be attributed to its capability of fully utilizing the sample information, while the other three models suffer from either discarding data (Model I) or transforming the profit variable (Models II and III), which is apt to distort the sample information. To ensure that the parameter estimates are acceptable, we check

Table 3
Average profit efficiency scores across models.

Model	1991–2009	1991–1998	1999–2009
Model I	0.2795	0.4619	0.3897
	(0.1977)	(0.1897)	(0.2088)
Model II	0.3694	0.2326	0.5302
	(0.3159)	(0.2925)	(0.3846)
Model III	0.3337	0.4604	0.3041
	(0.1206)	(0.1400)	(0.1206)
Model IV	0.3121	0.3605	0.3707
	(0.2219)	(0.1803)	(0.2369)

Note: Numbers in parentheses are standard deviations.

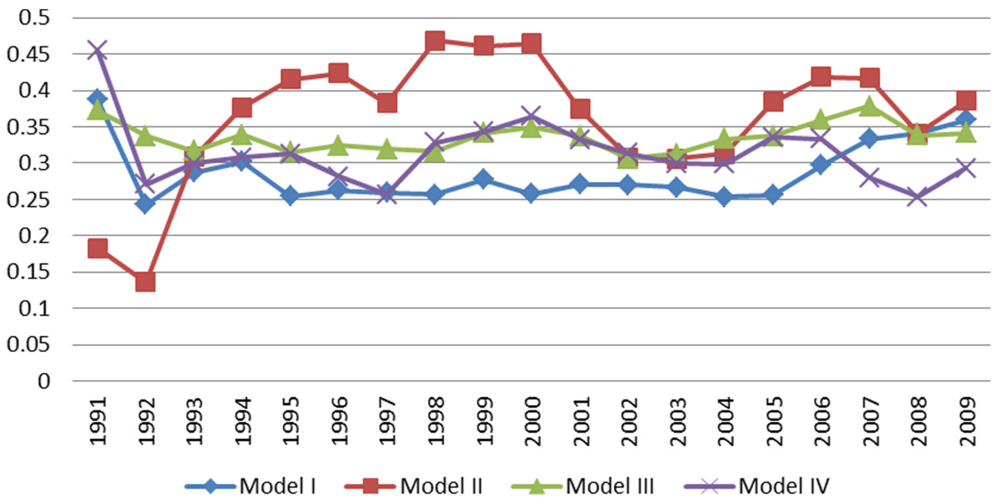

Fig. 1. Average profit efficiency scores of different models over time.

the monotonicity condition, required by the microeconomic theory, for each input and output prices and find that most of the conditions are satisfied.[9]

5.2. Profit efficiency scores

For Models I, III, and IV, we calculate the PE scores for each bank, using the conditional expectation of u_{it} given ε_{it} (Jondrow et al., 1982). The efficiency formula of Model II is the same as in Berger & di Patti (2006), who define the measure as the ratio of actual profits to predicted profits of the best-practice bank after adjusting for the inflation value. A problem immediately arises, i.e., loss-incurring banks will have a negative value for the efficiency score. We set these banks' efficiency scores to be equal to zero.

Table 3 shows that, over the entire sample period, Model I, dismissing loss-incurring banks from the sample, has the lowest average efficiency measure (column 1). This indicates that the omission of loss-incurring banks tends to underestimate the efficiency score on average, compared with the preferred Model IV. Conversely, the two inflation models (II and III) are inclined to overestimate the efficiency scores on average relatively to Model IV. The overestimation is likely to stem from the fact of the distortion of the dependent variable by their respective adjustment procedures. In other words, the dependent variable of profit is inflated either for all observations (Model II) or a part of observations (Model III).

We re-estimate the four models over sub-periods 1991–1998 and 1999–2009. The coefficient estimates are not shown to save space, and Table 3 summarizes the average efficiency scores in the last two columns. The average efficiency measures of Model IV are quite stable over the two sub-periods, while the remaining three models fluctuate substantially relative to Model IV. We further utilize the standard log-likelihood ratio testing approach to examine the presence of structural change between the two sub-periods. The null hypothesis of no structural change is not rejected on the basis of Model IV, while the remaining three models reject the hypothesis, i.e., they support the occurrence of a structural break. This may be due to the fact that these models involve either removing banks with negative profit (Model I) or transforming the

[9] Monotonicity conditions require that, at least on average, the derivatives of the profit with respect to each output price and each input price are positive and negative, respectively.

Table 4
Bootstrapped SEs and confidence intervals of the mean profit efficiency.

Model	PE[a]	$\overline{PE}$[b]	SE[c]	Lower bound	Upper bound
Model I	0.2795	0.2792	0.0071	0.2660	0.2936
Model II	0.3694	0.3690	0.0114	0.3484	0.3918
Model III	0.3338	0.3340	0.0044	0.3253	0.3417
Model IV	0.3121	0.3121	0.0083	0.2975	0.3277

Note:
[a] Mean estimated PE with original full sample.
[b] Average mean PE after bootstrapping.
[c] Standard errors.

dependent variable on the basis of non-positive profit levels (Models II and III), whereas our CSFM is immune from converting the dependent variable, leading its profit frontiers and efficiency scores to be relatively robust.

Fig. 1 depicts the average efficiency measures over time for the four models. For Model IV, the mean PE ranges between 0.2533 (2008) and 0.4556 (1991) and fluctuates up and down during the entire sample period. It is crucial to note that the two troughs occurring at 1997 and 2008 coincide with the Asian financial crisis and the sub-prime mortgage crisis in the U.S., respectively. These crises hit Taiwan's banking sector to some extent and reduced profit levels. However, the adverse effect is relatively small and persists for a short period. Model II also captures the two troughs, and Model III finds only a trough in 2008, while Model I fails to do so.

Since there is no way to know which of the four models is the correct specification, we follow Atkinson & Wilson, 1985 and Bos and Koetter (2011) to conduct bootstrapping for 1000 replications. Table 4 reports the mean PE, $\overline{PE}$, standard errors (SE), and the lower and upper bounds of the 95% confidence intervals. The bootstrapped $\overline{PE}$ of Model IV is exactly equal to its estimate, PE. Model II has the largest bootstrapped SEs, followed by Model IV, Model I, and Model III. Bos and Koetter (2011) also find that the rescaled model (Model II) has the largest SE. Although the SE of our preferred Model IV is not the lowest, it gains discriminatory power without a loss of precision and appears to be unaffected by possible outliers that bias the PE scores.

6. Conclusion

When assessing the performance of a firm, profit efficiency plays an important role. However, some firms' profit level has negative values, making the popular translog functional form unsuitable when estimating profit efficiency scores. Previous researchers have overcome the above problem by discarding the negative value (Model I) or transforming the dependent variable of profit (Models II and III).

In this paper we introduce CSFM, which allows us to derive a closed-form pdf for the censored regression model with composed error terms. The implied log-likelihood function can then be used to estimate a firm's profit frontier, and the resulting parameter estimates should have the desirable asymptotic properties like the usual MLEs. The suggested model is used to estimate the translog profit function for a sample of Taiwan's domestic banks. For the purpose of comparison, we bootstrap the standard error of mean profit efficiency and show that our preferred model is more precise than the other three models. The empirical results of our CSFM differ considerably from those of the other three models, in terms of the parameter estimates and profit efficiency measures.

This paper presents a feasible way of estimating profit frontiers and evaluating profit efficiency for banks when some banks earn negative profits, hindering the standard translog profit function from being directly utilized. Empirical results reveal that the mean profit efficiency scores of CSFM are relatively stable across the two sub-periods: the pre- and post-financial crisis periods. Conversely, the mean efficiency scores from the remaining three models vary substantially, due mainly to the use of distorted profits. Bank performance in the two sub-periods is, in fact, found to be quite stable. This implies that those commonly proposed policies for regulators or bank managers, such as tax cuts, capital regulations, deregulation of deposit rates, removal of geographic restrictions on branching, and mergers and acquisitions among financial intermediaries, in order to weather a financial crisis may not be necessary. Resources misallocation can then be avoided.

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Appendix A. Derivation of $F_{app}(Q_i)$

Eq. (6) can be equivalently expressed as:

$$F(Q_i) = \int_{-\infty}^{Q_i} f(\varepsilon_i)d\varepsilon_i = \frac{2}{\sigma}I(Q_i), \tag{A.1}$$

where

$$I(Q_i) = \int_{-\infty}^{Q_i} \left[\int_{-\infty}^{a\varepsilon_i} \phi(\xi) dxi \right] \phi(b\varepsilon_i) d\varepsilon_i, \tag{A.2}$$

and the new parameters of $a = -\lambda/\sigma < 0$ and $b = 1/\sigma$.

We can at best derive an approximation formula $I_{app}(Q_i)$ to $I(Q_i)$ in Eq. (A.2), in which $I_{app}(Q_i)$ has a closed form and can be used to estimate the parameters of interest. The derivation of $I_{app}(Q_i)$ is more involved and tedious and hence omitted here. Readers are suggested to refer to Tsay et al. (2013) for a detailed derivation. The resulting function of $I_{app}(Q_i)$ is expressed as:

$$I_{app}(Q_i) = \frac{1}{2b}\left[1 + erf\left(\frac{bQ_i}{\sqrt{2}}\right)\left(\frac{1 - sign(Q_i)}{2}\right)\right] - \frac{1}{4\sqrt{b^2 - a^2c_2}} \exp\left(\frac{a^2c_1^2}{4(b^2 - a^2c_2)}\right)\left\{1 + erf\left[\frac{-ac_1 - \sqrt{2}Q_i(b^2 - a^2c_2)sign(Q_i)}{2\sqrt{b^2 - a^2c_2}}\right]\right\}$$

where the error function $erf(z)$ is defined as:

$$erf(z) = \frac{2}{\sqrt{\pi}} \int_0^z e^{-t^2} dt = 2\int_0^{\sqrt{2}z} \phi(t) dt,$$

and the sign function is defined as $sign(Q_i) = 1, 0,$ or -1, if Q_i >, =, or <0. The two constants $c_1 = -1.09500814703333$ and $c_2 = -0.75651138383854$ are particularly selected to ensure that the error function $erf(z)$ can be well approximated by another function, $g(z) = 1 - e^{c_1 z + c_2 z^2}$, for $z \geqslant 0$.

We substitute $I_{app}(Q_i)$ for $I(Q_i)$ in Eq. (A.2) to yield:

$$F_{app}(Q_i) = \frac{2}{\sigma} I_{app}(Q_i) \tag{A.3}$$

Tsay et al. (2013) demonstrate that $F_{app}(Q_i)$ carries a very accurate approximation to $F(Q_i)$. For example, letting $\sigma_u^2 = \sigma_v^2 = 0.25$, we compute the true values of $I(Q)$ for $Q = -2,\ -1,\ 0,\ 1,$ and 2. The results are equal to 0.0016519, 0.0534267, 0.265165, 0.3513664, and 0.3535515, respectively, while the corresponding values of $I_{app}(Q)$ are equal to 0.0016515, 0.0534066, 0.265107, 0.3513463, and 0.3535511, respectively. These two groups of values are quite close to one another.

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