Journal
Educational Studies
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An ecological approach to adolescent mathematics ability development: differences in demographics, parenting, mathematics teaching, and student behaviors and emotions

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Received 22 Oct 2016, Accepted 22 Sep 2019, Published online: 01 Oct 2019

Download citation https://doi.org/10.1080/03055698.2019.1672522 Check for updates

ABSTRACT

This study used an ecological approach to studying adolescent mathematics ability development by classifying their mathematics ability growth trajectories and examining contextual measures differentiating the identified classes. Longitudinal student and parent data were collected for Taiwanese students in Grades 7, 9, 11, and 12 (n = 4,163). Growth mixture modelling identified 4 growth classes: low-increase, middle-flat, middle-increase, and high-increase. Multivariate analysis of variance revealed that girls' programmes, persistent parental monitoring related to desirable ability development, and student-perceived teaching quality related to student ability. High-increase students reduced their engagement in leisure activities when preparing for examinations, but they felt little mathematics frustration, whereas the opposite was true for low-increase students.

KEYWORDS: Demographics, growth mixture modelling, mathematics ability, mathematics teaching, parenting, TEPS

Introduction

Variation, diversity, and inequity remain persistent educational challenges even after the six decades of educators' endeavours to increase educational outcomes and effectiveness (Lee 2016). To address the complex challenge, a starting point is to propose, identify, and clarify the ecological context of students' development by taking account of diverse sociocultural, teaching, and learning factors over time. Using an ecology-based theoretical background with appropriate methodologies to study student ability development will contribute to the knowledge and literature about ecological approaches to education and the policy and practice of a comprehensive educational design for promoting student short- and long-term ability development.

An ecological approach to student mathematics ability development

The theory and purpose of this study

The ecology-based theories for students' development aim to examine the dynam relationships between multiple contextual factors informed by diverse disciplines such as behavioural, cognitive, and sociocultural theories (Bronfenbrenner 1986, 1994; Bronfenbrenner and Ceci 1994; Lee 2010). The factors emphasised by the ecological theories are temporal, biological or physiological, environmental or cultural, and psychosocial or dispositional factors.

Based on the ecological theories, this study proposed a contextual approach with a related methodology to study mathematics ability development. The ecological contexts psychosocial (e.g. student behaviours and emotions) contexts (Figure 1). The purpose of this study, therefore, was to identify the patterns of mathematics ability development over time and explore the patterns by using diverse contextual factors.

Figure 1. An ecological approach to mathematics ability development.

The methodology

Student abilities (represented by achievement scores) are context- or ecology-dependent and should be addressed by variations rather than just means in terms of educational measurement. The mean of and variation in achievement scores reveal different information regarding the distribution of academic performance among students. For example, Taiwanese adolescents perform well and achieve favourable mathematics scores in international cognitive tests such as the eighth-grade mathematics test of the 2011 Trends in International Mathematics and Science Study (TIMSS), in which Taiwan was ranked third (Mullis et al. 2012, 42). An underlying challenge, however, is the wide spectrum of the scores. For example, compared with all the participating countries in the 2012 Programme for International Student Assessment (PISA), Taiwan had the highest variation in its student mathematics scores, with a mean of 560 and 13% and 37% of students being classified as low and high achievers, respectively (Organization for Economic Cooperation and Development [OECD], 2014, pp. 19, 72). This shows that, despite the high mean score, Taiwan has a significant number of low achievers in mathematics.

Traditional evaluation studies investigating linear relationships between mathematics scores and contextual measures may be ineffective in identifying the key to resolving the challenge of high variation in achievement scores. Furthermore, a major educational stressor for adolescents is the sharp increase in the difficulty of mathematics learning content during secondary education and the high-stakes examinations for progressing to the next stages of education (e.g. high school or college entrance examinations). The roles of parents, mathematics teachers, and students in facing educational stressors are

Growth mixture modelling (GMM) is both a variable- and person-centred and linear and nonlinear statistical technique for identifying different patterns of longitudinal growth trajectories (Vanhalst et al. 2013). GMM is usually used by psychologists or health professionals to identify longitudinal development in psychological constructs such as cognitive ability, self-esteem, and loneliness (Birkeland et al. 2012; Hong and You 2012; Qualter et al. 2013) or behavioural and health indicators such as school dropout, delinquency, and smoking (Bowers and Sprott 2012; Morin, Rodriguez, Fallu, Maïano, & Janosz, 2012; Wiesner and Windle 2004). For GMM in this study, longitudinal data from the Taiwan Education Panel Survey (TEPS) were used (Chang, 2001-2007). The TEPS was mainly designed on the basis of sociology, which provides fruitful contextual measures that offer opportunities for educators or policy makers to address issues of macro educational design and policy making. The sociology database also offers this study an opportunity to creatively use GMM with the data.

The growth context related to mathematics ability development

The ecological approach to student mathematics ability development (Figure 1) emphasises the diversity of student development. This means that students may possess different patterns of growth trajectories, which deserve identification and tailored educational provisions. GMM can serve as a methodology to address the diverse patterns or different growth trajectories of student mathematics ability development.

Most studies using GMM to investigate mathematics ability growth trajectories have focused on the stage from kindergarten to early primary education and have identified ability differences at the start, followed by an increasing or flat trend. For example and You (2012) studied Latino children in the United States from kindergarten to G and identified four trajectories of mathematics ability development starting with four levels of mathematics ability, all followed by an increasing trend. Jordan et al. (2007) studied the number sense of U.S. children from kindergarten to Grade 1 and found three classes: low/flat, middle/steep, and high/flat. In the study by Chen, Hughes, and Kwok (2014), the mathematics achievements of U.S. children in Texas were followed from Grades 1 to 6 and two classes were identified: high start/slow growth and low start/fast growth. Aunola et al. (2004) investigated the mathematics ability of preschool

No studies to date have adequately identified the mathematics ability growth trajectories of adolescents. A partial finding is that Bowers and Sprot (2012) examined U.S. students' general grades (including mathematics grades) from the first three semesters in high school and found four growth trajectories: high stable, middle stable, middle decrease, and low increase.

The generally increasing growth rates in GMM research are consistent with findings from regression or path analyses showing that early mathematics ability predicts later mathematics ability (Grimm 2008). The increase in the rate of early mathematics ability (e.g. from prekindergarten to Grade 1) has even been shown to strongly predict later (e.g. age 15 years) mathematics ability (Watts et al. 2014).

Other ecological contexts related to mathematics ability development

According to the ecological approach proposed by this study (Figure 1), in addition to the growth context, three contexts need further consideration. The biosocial context includes family backgrounds (e.g. demographics) and parental involvement in children's learning (e.g. parenting). The sociocultural context includes diverse cultural designs and artefacts in a given society, of which the most salient example is mathematics teaching. The psychosocial contexts relate to students' characteristics such as behaviours and emotions.

Demographics

Three aspects of demographics in the biosocial context are relevant to this investi
gender, socioeconomic status (SES), and educational programme choice.

(a) Gender: Research comparing boys' and girls' mean scores of mathematics achievement finds no definite universal gender differences in mathematics ability (Grimm 2008). Gender differences in mathematics achievement are small and vary across countries (OECD, 2014, 72). Mathematics slightly favour girls before middle school and favour boys from high school (Reilly, Neumann, and Andrews 2015). Taking into account different levels of achievements, boys are found to be top achievers and good at advanced mathematics (Mullis et al. 2016). Research using GMM has led to a similar

(b) SES: SES relates to cultural capital (as indicated by parental education, family income, official language use at home, and residency) and generally have positive relationships with student academic achievement (Tucker-Drob and Harden 2012) with few exceptions (Murray and Harrison 2011). Students with high mathematics achievement have more opportunities to speak their official language at home (Chen 2014), and have a higher SES (e.g. parent education and family income), compared with students with low mathematics achievement (Hong and You 2012; Jordan et al. 2007; Yoshino 2012). In Germany, adolescents' mathematics ability is unrelated to their parents' jobs but related to residence in East Germany (Trautwein et al. 2002). Some other research, however, has shown that school grades are unrelated to either urban or rural residence (Bowers and Sprott 2012). In terms of the growth context, the effects of SES on mathematics ability occur only in early kindergarten, with no effect on later growth rates (Jordan et al. 2006).

(c) Educational programme choice: This is included in demographics because educational programme choice is a negotiation among educational design, student ability, family SES, and parenting in Taiwan (cf. the Study Context section in this paper). For example, the tuition fees of private schools are normally higher than those of public schools (Hoare and Johnston 2011). Low-SES students may not be able to afford the tuition fees of private schools. Similarly, German adolescents studying in the academic track were shown to have higher mathematics ability than those in the vocational track (Trautwein et al. 2002). Gifted educational programmes only accept the highest achievers but their effects on achievement are uncertain (Bui, Craig, and Imberman 2014). Choosing or tracking into academic and gifted education may relate to achievement, but these in turn relate to SES.

Parenting

Positive parenting practices relate to their children's positive learning outcomes (Wang and Sheikh-Khalil 2014). In addition, both paternal and maternal positive parenting practices (e.g. talking with and monitoring) relate to their adolescents' academic achievement (Hsu et al. 2011). The positive relationships between parenting and achievement may come from detailed active interaction activities such as shared reading, checking homework, and parent-teacher communication (Jeynes 2012). Long-term effects of parenting on children' achievement are also expected. Maternal numeracy activities (e.g. practicing sums and talking about time) and informal ones (e.g. playing card games and shopping) in childhood can predict children's later mathematics ability (Chiu 2018a; Dunst et al. 2017; LeFevre et al. 2009; Skwarchuk, Sowinski, and LeFevre 2014).

In contrast, negative parenting practices relate to negative learning outcomes for their children. "Negative parenting practices", however, may be an issue of detailed degrees in the manner of control. For example, parental support, democratic control, and authoritative style relate to positive adolescents' academic achievement (Morin et al. 2012); parental control over children's academic performance as well as permissive and authoritarian styles relate to children's lower achievement (Chen 2015; Su et al. 2015). At the negative extreme, parental use of physical discipline in kindergarten was shown to relate to low mathematics achievements in Grade 5 (Bodovski and Youn 2010).

Recent research indicates that the pattern of the positive relationship between parenting (e.g. autonomy-supportive, democratic and authoritative vs. controlling, authoritarian, and permissive) and achievement may be a cross-cultural phenomenon (Cheung et al. 2016; Masud, Thurasamy, and Ahmad 2015). Parenting by comparing their children or adolescents with other people may be a unique parenting behaviour in Taiwan because there appears to be no research on this to date. It is worth investigating the effect of parental social comparing behaviour, which may be inferred as an undesirable parenting behaviour because social comparison may undermine children's academic self-concept, which in turn decrease their academic achievement (Chiu 2012; Marsh et al. 2014). The negative effect of parental social comparison behaviour may be especially true when children are inferior to others.

Mathematics teaching

Positive learning outcomes (e.g. mathematics achievement) generally relate to high-quality mathematics teaching using both teacher- and student-centred teaching. High-quality teacher-centred or traditional teaching include direct teaching, frequent formative assessment, and reasoning (Thorvaldsen, Vavik, and Salomon 2012). In contrast, focusing on rules is an ineffective teaching method in terms of learning outcomes (Hinostroza, Labbé, Brun, and Matamala 2011). High-quality student-centred teaching includes formative feedback (Espasa and Meneses 2010), learning models that

2009). A balanced use of traditional mathematics teaching and scaffolding tools benefits the mathematics achievement of students with different levels of mathematics ability (Tan and Tan 2015).

Relatively few studies find uncertain relationships between high-quality teaching and achievement. For example, teacher responsiveness is unrelated to students' mathematics achievement in Grade 10 but is related to that in Grade 12 (Byrnes and Miller 2007). There are also few studies on linking inside- and outside-school/class teaching, the effect of which is uncertain. For example, teacher monitoring of homework completion does not relate to adolescents' mathematics ability (Trautwein et al. 2002). The boundary between inside- and outside-class teachings may gradually diminish because of the development of information and communication technology such as online social networks (Junco and Cotten 2012). This offers more opportunities for teachers to pay attention to students' after-class learning, the effect of which deserves investigation.

Student behaviours and emotions

During secondary education, students experience high-stakes testing for entering into subsequent stages of schooling (i.e. from middle school to high school, and to university). Because of this, students need to exercise test-preparation and goal-setting behaviours. Student intentions to take college entrance examinations, as well as inhibitory control, ego resiliency, attention, and prosocial behaviours have been reported to predict high mathematics scores (Byrnes and Miller 2007; Chen, Hughes, and Kwok 2014; Dulaney, Vasilyeva, and O'Dwyer 2015). The mechanism may be that goal setting for tests and entry into preferred schools directs students' behaviour (Pintr 2003). The goal transforms into grit and self-regulating learning behaviours (e.g. inhibitory control) and thus relates to higher achievement (Wolters and Hussain 2015).

Students' emotions (or affects) are an indispensable aspect of mathematics learning (Chiu and Whitebread 2011; Gomez-Chacon 2000; Mason, Burton, and Stacey 1996; McLeod 1994). As such, students' emotions (e.g. joy or frustration) towards learning mathematics generally relate to their mathematics achievement (Peixoto et al. 2017) even along a period of time (Ahmed et al. 2013). Adolescent mathematics ability perceptions or confidence can further predict mathematics ability and career selfteachers about opportunities for effective intervention.

Research questions

Two research questions were derived from the literature review, GMM application, and TEPS measures. Research Question 1 focuses on identifying different growth trajectories. Research Question 2 focuses on class differences in contextual measures.

1. Are there distinct growth trajectory classes (patterns) in adolescent mathematics ability during secondary education? (The possible patterns are depicted in the upper part of Figure 2, including the latent trajectory classes, intercept, slope, and Wave 1–4 mathematics ability.)

   Figure 2. The hypothesised ecological model of adolescent mathematics ability development. The latent trajectory classes are the growth context in Figure 1; the demographics and parenting are the biosocial context; the mathematics teaching is the sociocultural context; and the student behaviours and emotions are the psychosocial context.

2. What are the differences among the identified classes in terms of demographics, parenting, mathematics teaching, and student behaviours and emotions?

Method

Data sources

Longitudinal student and parent data were taken from the TEPS, which was compiled by the Survey Research Data Archive in Taiwan (Chang, 2001-2007). This study used the data collected from the same 4,163 students (51% girls; all without mental disability) in Grades 7, 9, 11, and 12 (Waves 1–4, respectively) throughout Taiwan from 2001 to 2007.

During 2001–2007 (and even until 2018), Taiwan's educational system has used the same middle school (Grades 7–9) curriculum for all students. Most students study at local public middle schools, but in order to gain admission to public high-achieving academic high schools, some parents might enrol their children in private middle schools, the tuition fees of which are generally higher than those of public high schools. Students are tracked into different educational programmes (academic, mixed, or vocational) in high school (Grades 10–12), mostly from Grade 10, according to their performance in the high school entrance examination, their preferences, and partially on the basis of their middle school grades. The highest achieving students in the examination tend to elect to go to top-achieving public academic high schools in urban areas.

Students studying in the academic educational programme of high schools typically apply to leading universities, whereas most students studying in vocational high schools plan to enter the workforce or study at technical colleges or universities. Similar to the previous middle school stage, high school students are tracked into different higher education institutions after Grade 12, mostly on the basis of their performance on college entrance examinations, their preferences, and partially according to their high school grades.

Measures

The ecological approaches suggested that four sets of measures should be investigated. Firstly, the growth context was based on students' mathematics abilities across waves. Next, the biosocial context was represented by demographics and parenting. The sociocultural context was indicated by mathematics teaching. The psychosocial context was addressed by students' behaviours and emotions. Table 1 details the contents score ranges of each measure.

Table 1. Measure descriptions, descriptive statistics, and post hoc test results for the contexts between the four mathematics ability classes.

Student mathematics ability was repeatedly examined through cognitive testing across four waves (i.e. Grades 7, 9, 11 and 12). The test scores in the four waves were scaled using item response theory, so student ability scores of the four waves could be compared.

Demographics

Demographics contained 11 measures surveyed. In Grade 7, five basic demographics were surveyed. The demographics measures included gender, parental education, family income, the official language use at home (i.e. Mandarin), and living in remote, rural areas (i.e. remote residence).

Two measures were surveyed in both Grade 7 and 11: urban residence and studying in a private school. The repeated measures were used to reflect students' changes from middle to high school. These measures were added because public high schools are generally only located in areas with large enough populations, and many students experience changes from public to private schools or vice versa after the examination selection procedure.

Two measures related to educational programmes were only considered during Grade 11 (or high school): (A) Educational programmes were rescaled on a 3-point scale (1 = vocational, 2 = mixed, 3 = academic), with higher scores indicating a stricter academic programme in high school. For Grade 7, educational programme was not included because all students attended middle school and had the same educational programme. (B) Students were asked whether they attended mathematics or science education programmes for the gifted.

Parenting

Students rated both paternal and maternal parenting practices pertaining to career discussions, listening, and monitoring in both Grades 7 and 11. These items were measured using a 4-point scale (1 = *never* to 4 = *often*).

The last parenting measure was parents' social comparison behaviour surveyed in Grade 11. Students indicated whether their parents compared them with other people (e.g.

Mathematics teaching

Students rated whether their mathematics teachers provided clear and thorough lectures, provided adequate class interactions, and cared about their students' learning outside of class in both Grades 9 and 12. These measures were rated on a 2-point scale (0 = *no* or 1 = *yes*).

Student behaviours and emotions

Grades 9 and 12 are the last year of middle school and high school, respectively, in Taiwan, so student behaviours for preparing for high-stakes high school and university entrance examinations were surveyed. Students indicated the extent to which they reduced the number of hobbies they engaged in to prepare for tests in Grade 9, which was measured on a 4-point scale (1 = *never* to 4 = *always*), and whether they reduced their engagement in leisure activities to prepare for tests in Grade 12, which was measured on a 2-point scale (0 = *no* or 1 = *yes*).

In Grade 12, students also indicated their future academic goals, indicating whether or not they intended to study social sciences and humanities, physical sciences, or life sciences in higher education. Each of the three items was rated on a 2-point scale (0 = *no* or 1 = *yes*).

Students' emotions were surveyed in Grade 11. Students reflected on when they felt frustration in learning mathematics: before Grade 4, in Grades 5 and 6, in middle school, in the first year of high school, and in the second year of high school separately. Finally, a derived variable was used to indicate students' overall frustration with scores of 0 *least once* and 1 = *never feeling frustration in any of the above periods of time*.

Data analysis

Research question 1

The measures of student mathematics ability in the four waves (Table 1) were used in the GMM procedure to identify distinct classes of growth trajectories (Models 1–4; Table 2) by using Mplus Version 7.11 (Muthén and Muthén 2013). By default, Mplus addresses in class solutions, six model fit criteria were used to compare the models. A model more closely fitting to empirical data than its competing models has a lower Akaike information criterion (AIC), Bayesian information criterion (BIC), and sample-size adjusted BIC (aBIC; Lewis, Butler, and Gilbert 2011). An entropy value (ranging from 0 to 1) close to 1 indicates "clear classification" (Muthén et al. 2002, 465). A significant P value (i.e. $p < .05$) provided by the Lo–Mendell–Rubin (LMR) likelihood ratio test and parametric bootstrapped likelihood ratio test (BLRT) indicates that the present examined k class model fits the data more closely than the $k - 1$ class model does (Lo, Mendell, and Rubin 2001; Nylund, Asparouhov, and Muthén 2007).

Table 2. Model Fit Indices for the growth mixture models.

CSV Display Table

Research question 2

Mplus provides multinomial logistic regression (log odds) and odds ratio results when performing GMM with covariates (Muthén and Muthén 2012). Multinomial logistic regression has functions similar to multivariate analysis of variance (MANOVA), both of which compare class differences in covariates. This study compared the MANOVA and multinomial logistic regression results and determined that the MANOVA results appeared to be more reasonable and easier to be understood. The reason might be that longitudinal covariates repeated in several waves increased the problem of multicollinearity or suppression effects in the regression analysis (Beckstead 2012 odds ratios were not used because they are relatively rare in education research a general education audience might be unfamiliar with them. Accordingly, MANOVA was used to examine the class differences in the measures of demographics, parenting, mathematics teaching, and student behaviours and emotions, which enabled examining the numerous measures from the different waves simultaneously without causing the problem of multicollinearity. MANOVA-related methods for examining between-class differences identified through GMM or latent class analysis in covariates have been

Results

Classes of growth trajectories

Determining the class number

GMM was used to identify the most distinct classes of mathematics ability growth trajectories that fit the empirical data. The GMM procedure started from setting two classes in the Mplus syntax and ended in setting five classes. It ended there because one class in the five-class solution had only one case, so that case could be seen as an outlier and provided little interpretive plausibility. The five-class model, therefore, was excluded from further consideration. On the other hand, all of the two- to four-class models generated distinct classes or patterns of growth trajectories and were thus considered further by looking at their values of model fit criteria.

Among the two- to four-class models, the four-class model had the lowest AIC, BIC, and aBIC, indicating that the four-class model attained the closest fit to the data (Table 2). The entropy values of the two-, three-, and four-class models were .69, .64, and .65 respectively, indicating that the classification was clearest for the two-class model, followed by the four- and three-class models. The LMR and BLRT P values for the two- to four-class models were all significant ($p < .05$), showing that the two-class model was more appropriate than the one-class model, the three-class model was more appropriate than the two-class model, and the four-class model was more appropriate than the three-class model. Reviewing the plausibility and fit indices of the two–five-class models (Table 2) suggested that the four-class model tended to have the closest fit to the data and provided theoretical interest and interpretive plausibility in its growth trajectory patterns and case numbers (Figure 3). The four-class model was therefore selected and further investigated in this study.

Figure 3. Observed mean trends in mathematics ability in the four trajectory classes.

The results of the intercept and slope tests conducted through GMM revealed that the mathematics ability of Class A students was initially below average but exhibited a generally increasing trend (Table 1; Figure 3). Class B students maintained an average, similar mathematics ability along the first three waves, with a slight decrease in Wave 4 (a similar decreasing trend to Class A and C). Moreover, Class C students exhibited average mathematics ability in Grade 7, similar to that observed for the Class B students, but their general trend was increasing. Class D students had initially high mathematics ability with an increasing trend over the four waves.

MANOVA test results revealed significant class differences among the four waves of mathematics ability, with a large effect size (Wilks' λ = .14; $F_{(12, 10,422)} = 984.67$, $p < .05$, $\eta^2 = .49$; Table 1). Class D students maintained higher mathematics ability compared with the other three groups of students. Initially, the ability of Class C students was similar to that of Class B students in Grade 7, but the Class C students outperformed Class B students in Grade 9. Class B students outperformed Class A students during Grades 7, 9, and 11, but their performance levels became similar in Grade 12. Grade A students kept lower ability than did the other three groups during Grades 7, 9, and 11, but in Grade 12, their ability became similar to that of Grade B students. From the combined results, Classes A–D were named low-increase (13%), middle-flat (4%), middle-increase (41%), and high-increase (43%), respectively.

A unique phenomenon shown in Figure 3 and Table 1 is that the mathematics ability of Classes A, B, and C decreased from Grades 11 to 12, whereas that of Class D increased. Repeated measures tests investigating the ability differences between Grades 11 to 12 for each class revealed significant differences in mathematics ability between Grad[illegible] and 12 for each group (Class A: $F_{(1, 499)} = 66.99$, $p < .05$, $\eta^2 = .12$; Class B: $F_{(1, 150)} = 30.01$, $p < .05$, $\eta^2 = .19$; Class C: $F_{(1, 1613)} = 125.69$, $p < .05$, $\eta^2 = .07$; Class D: $F_{(1, 1710)} = 34.92$, $p < .05$, $\eta^2 = .02$).

Class differences in contextual measures

MANOVA was performed to separately investigate the class differences in the four sets for contextual measures. The results (Table 1) revealed class differences in demographics (Wilks' λ = .60; $F_{(33, 11,423)} = 66.58$, $p < .05$, $\eta^2 = .16$; large effect size), student behaviours and emotions (Wilks' λ = .64; $F_{(33, 11,417)} = 55.78$, $p < .05$, $\eta^2 = .14$; large effect size). Detailed differences in each type of contextual measures are presented as follows.

Demographics

The measures with large effect sizes were Grade 11 private school and educational programmes, with Class D students attending the public academic programme. The results were expected because students are tracked into academic, mixed, or vocational educational programmes in Grade 10. The measure with a medium effect size was parent education, with Class D (high-increase) students having the highest parent education levels and Class C students having higher parent education levels compared with Class A students.

The other demographic measures had small effect sizes, with SES-related measures including family income, official language use at home, urban residence, and private schooling (Grade 7), as well as mathematics and science programmes for gifted children favouring Class D. More girls were in Classes A (low-increase; female: 52%) and C (middle-increase; 58%) than in Classes D (high-increase; 44%) and B (middle-flat; 50%), respectively. The results suggest that girls' mathematics ability tended to increase, despite their initial mathematics ability being at low- or middle-levels.

Parenting

No significant differences were observed in 5 of the 13 parenting measures, and those exhibiting significant differences had small effect sizes. Effective Grade 7 parenting was observed only in paternal and maternal monitoring. Effective Grade 11 parenting occurred in most paternal and maternal parenting practices (e.g. career discussions, listening, monitoring, and not comparing students with others). All significant positive parenting practices favoured students in all four classes, having the greatest effects on Class D, followed by C, B, and A, in descending order. The results imply that student positive growth trajectories relate to persistent, long-lasting, and positive parenting practices, even when students are in later adolescence (e.g. Grade 11).

Mathematics teaching

Class D students perceived higher degrees of clear and thorough lecturing, positive class interactions, and teachers' after-class attention promoting student learning. Class C, B and A students (in descending order) had positive perceptions of mathematics teaching.

Student behaviours and emotions

Class D (high-increase) students planned to study in higher education in regardless of field but exhibited particularly strong ambitions to study physical science (.14, a large effect size). They were, in decreasing order, less likely to be motivated to study life sciences (.07) or the social sciences and humanities (.05). Among the four classes, Class A students had the weakest ambitions to pursue higher education.

Class A students were less likely to reduce their leisure activities to prepare for tests, compared with Class C and D students in Grades 9 and 12. Class B students worked normally in Grade 9 compared with the other students, but they became more similar to Class A students in being less inclined to engage in fewer leisure activities in preparation for tests in Grade 12 relative to Class C and D students. The students who exhibited the largest reduction in inclination to participate in leisure activities to prepare for college entrance examinations in Grade 12 were those in Class D followed by those in Class C.

Students' emotions were represented as students' reflections on when they felt frustrated in learning mathematics in Grade 11. Compared with the other students, Class A and B students began experiencing frustration before Grade 4, whereas for Class C students, it began during Grades 5 and 6. The differences in frustration among the four classes increased during middle school (medium effect size) and gradually diminished from Grades 10 to 11, which may have been due to the interaction between students' mathematics ability, the tracking system (vocational, mixed, or academic curriculum), and student ambitions to pursue higher education. Among the four classes, Class D contained the highest number of students who never experienced frustration in mathematics.

Discussion

Four mathematics ability growth trajectories

Figure 3. The percentages in the four classes are consistent with Taiwan's mathematics scores in the PISA or TIMSS; that is, high means and large variations (Mullis et al. 2012; Organization for Economic Co-operation and Development 2014). Previous similar studies have identified four mathematics ability classes for kindergarten–primary-stage children (e.g. Chen, Hughes, and Kwok 2014). This study extends previous studies to adolescents along the 6 years of secondary education.

The identified classes have two unique phenomena. First, the middle-flat class (Class B) generally remains an unchanged development in mathematics ability along the 6 years of secondary education. Specifically, the middle-flat class has similar mathematics ability to the middle-increase class in Grade 7, but this ability has a slight decline, becoming similar to that of the low-increase class in Grade 12. The middle-flat class has not been observed in studies investigating preschool to primary school mathematics ability through GMM (Aunola et al. 2004; Chen, Hughes, and Kwok 2014; Hong and You 2012). All of the classes in these studies exhibited an increasing trend with the initial achievements (intercepts) appearing to have determined the long-term increasing trend though with slight fluctuations over time (i.e. generally positive slopes), which are consistent with the other three classes (Classes A, C, and D) identified in this study.

However, the unique pattern of the middle-flat class (Class B) is, for the most part, consistent with the stable middle-decrease class during grades 9–10 reported by Bowers and Sprott (2012). Examining the class differences in the contextual measures (Table 1) reveals that the middle-flat class has adequate (not the highest) family income, lives in urban areas, and can afford Grade 7 private schooling; however, this class has slightly low parent education, seldom speaks the official language at home, and experiences early frustration in mathematics learning.

The second unique phenomenon is that, except for the students in the high-increase class, those in the other three classes exhibit a decline in their mathematics ability from Grades 11 to 12, which has not been observed in previous studies. The reasons may be explained by the results of the contextual measure analysis (Table 1). The students in the high-increase class mostly study academic educational programmes in high school, enjoy desirable parenting practices, perceive positive mathematics teaching, have strong ambitions to pursue higher education (particularly in science), and disengage from some degrees of those contextual measures than the high-increase students. These two unique phenomena need to be validated by future research.

Contexts of different mathematics ability growth trajectories

Longitudinal, repeated, and one-item contextual measures justify the use of MANOVA, rather than regression, to obtain an unbiased complete understanding of the four identified classes. The MANOVA results reveal that class differences among the four sets of contextual measures are all significant but differ in their effect sizes (or the percentages of the class differences explained): demographics (large effect size), student behaviours and emotions (large effect size), mathematics teaching (small effect size), and parenting (small effect size), in descending order.

Demographics

Despite the small effect sizes, the trend of gender differences in the four classes is that girls gradually increase their mathematics ability along the 6 years of secondary education although girls tend to be low or middle mathematics achievers in Grade 7. Boys are generally middle or high mathematics achievers in Grade 7, but they may give up on mathematics if they find that they are not good at it or work harder if they find that they are good at it. The analogy could be made that, in developing their mathematics ability, girls are like a tortoise (walk slowly but surely) and boys are like a hare (either sleep or run). Educators must consider gender differences in the patterns of mathematics ability growth trajectories. Misconceptions of girls having low mathematics ability could be replaced with an awareness of girls' special growth pattern for mathematics learning, which may help minimise the stereotyping of girls in terms mathematics ability and career development (Good, Aronson, and Harder 2008).

Apart from gender, the other demographic measures are generally related to SES. The SES-related measures tend to positively relate to the desirability of mathematics ability trajectories, with family income having a small effect size, parent education having a medium one, and educational programme choices having a large one. The view is that the students in the high-increase class have parents with a high level of education, have desirable family incomes, live in urban areas (where living expenses are high), and speak the official language at home. Their parents send them to private middle schools for because they attain desirable high school entrance examination results, they can enter the public academic track or educational programmes for gifted children. The view of the students in the low-increase class appears to be the reverse of those in the high-increase class. The results are consistent with previous findings that high mathematics ability relates to high SES and academic educational programmes (e.g. Trautwein et al. 2002; Yoshino 2012). Considering only the demographic factors may suggest that the link between SES and mathematics ability is inevitable. The following three sets of contextual measures may break this apparent link.

Parenting

Despite the small effect sizes, parental monitoring is the most effective and long-lasting parenting practice for differentiating the four classes among the parenting measures examined in this study. Another critical finding with educational implications is that Grade 11 parenting practices play a more prominent role in differentiating student mathematics ability growth trajectories than Grade 7 practices do.

The results suggest that positive parenting practices must be persistently applied, even when adolescents are in high school. The parenting practices included in this study are generally viewed as monitoring or caring parenting practices (career discussions, listening, and monitoring). Caring and monitoring parenting practices are sufficient for achieving desirable development in adolescent academic ability, a result that is consistent with previous findings (e.g. Bodovski and Youn 2010; Hsu et al. 2011). The emphasis on the long-lasting effects of parental monitoring practices for adolescents from middle school to high school is a novel finding of this study.

Parental social comparison behaviour relates to achievement development in a ne[illegible] direction. This result suggests that social comparison is an undesirable parenting behaviour, as suggested by literature on the relationship between academic self-concept and achievement using social comparison as a theoretical basis (Chiu 2012; Marsh et al. 2014). Parental social comparison behaviour appears to be a new concept in the literature, may be a culturally specific phenomenon, and needs to be examined by future research.

Mathematics teaching

classes in both Grades 9 and 12 though with small effect sizes. This result suggests that student perceptions of teaching relate to student mathematics ability. A salient case is the middle-flat class, in which positive perceptions decline, becoming similar to those of the low-increase class by Grade 12. However, the survey methodology cannot confirm a cause-and-effect relationship, so the reasons for the results should be discussed from diverse perspectives or at least consider the possibility that mathematics teaching and ability affect one other.

Firstly, consider mathematics teaching as the cause for the result in the Taiwan context, where mathematics teachers for Grades 9 and 12 differ, all Grade 9 students learn under the same curriculum, Grade 12 students are tracked into different schools and several types of curricula (i.e. academic, mixed and vocational). Despite the generally high-quality mathematics teaching in secondary education in Taiwan (Laschke 2013), the results cannot rule out the possibility that high-achieving students experience better mathematics teaching including by tracking inside schools (especially in Grade 9). If this is the case, inequality in mathematics teaching remains an issue for educators and policymakers to address even if innovative measures for increasing educational equality continuously develop in Taiwan (Chiu 2018b).

Secondly, when considering the relationship in the opposite direction, there is a possibility that high-ability students tend to be more perceptive of high-quality mathematics teaching compared with low-ability students. In other words, student perceptions of teaching may be irrelevant to mathematics teaching approaches, as indicated by the findings of previous research regarding the uncertain relationships between mathematics teaching and student mathematics ability (e.g. Byrnes and Miller 2007). This reasoning suggests that students project their mathematics ability onto teachers' teaching quality and implies that students' perceptions or assessments of teacher or teaching quality may be biased by their ability. Future research and educational policies may need to consider student ability or achievements when assessing teaching quality.

Student behaviours and emotions

The view of student behaviours and emotions is that students in the high-increase class disengage from leisure activities when preparing for high school or college entrance studying the vocational curriculum and enjoying leisure activities as usual; however, they experience frustration with mathematics early (starting from before Grade 4 and becoming exacerbated during middle school). To simplify this view, all students' mathematics frustration or happiness becomes similar if their ability matches their study goals or curricula. Tailored curricula matching students' ability must be developed. The educational design in Taiwan must reduce constraints and provide flexibility to allow for student regulation between personal abilities and educational designs (Chiu 2016).

Limitations of this study and suggestions for future research

This study selected contextual measures from the TEPS, which provides fruitful contextual measures for a specific culture. The TEPS, however, was mainly designed on the basis of sociology, and the measures of demographics, parenting, and student behaviours and emotions are not closely related to mathematics learning. Future research may need to validate and elaborate the findings through longitudinal databases focusing on mathematics education or experimental studies in other cultures. Secondly, many of the contextual measures in the TEPS were measured dichotomously. Self-reported measures using a Likert-type scale can provide more information. Thirdly, this study uses a survey as the data collection method, which cannot be used to make claims about cause-and-effect relationships. Although the contextual measures (e.g. mathematics teaching) imply causes for ability, only experimental designs can address a cause-and-effect relationship. Finally, the TEPS collected sufficient data in relation to after-school teaching that permeates Asian education. After-school teaching, however, is a complicated issue in terms of its interplay with contextual measures such as those investigated in this study, and future research could provide further insights into this issue.

Conclusion

The ecological approach proposed in this study emphasises four contexts relating to student mathematics ability development. The growth context is addressed by GMM, which obtains four distinct classes of mathematics ability growth trajectories: low-increase, middle-flat, middle-increase, and high-increase. The biosocial, sociocultural, differences, whereas mathematics teaching and parenting explain small percentages. Demographics create what may appear to be an inevitable link between SES and ability growth trajectories; however, the findings regarding parenting and student behaviours and emotions suggest that there are positive measures for breaking this link. High-achieving students perceive better mathematics teaching than low-achieving students do; the cause-and-effect relationship between mathematics teaching and ability remain an unresolved issue because this study uses data derived from a survey.

In summary, the finding of this study provides the following takeaway messages for educators, educational researchers, and policymakers to improve their practices.

1. Adolescents (secondary school students) start their mathematics ability at high, middle, and low levels. Although most students increase their mathematics abilities over the six years of secondary education, there is a group of students (4%, Class B), starting from middle ability, that may face stagnation and decline in later mathematics ability development. These students are unique and a new contribution to the literature, and the issue deserves further investigation, understanding, and educational designs.

2. Girls and boys have slightly different patterns of mathematics ability growth trajectories. Girls tend to experience low-increase or middle-increase development in mathematics ability, whereas boys tend to experience high-increase and middle-flat development.

3. SES with later educational choices (e.g. private schools; vocational, mixed or academic high schools; and mathematics or science gifted education programmes) relates to mathematics ability development. Higher SES relates to more desirable mathematics ability development.

4. Monitoring or caring parenting practices (e.g. career discussions, listening, and monitoring) relate to desirable mathematics ability development, especially in Grade 11. Social comparison may be an undesirable parenting behaviour, which is a new contribution to the literature and deserves future research.

5. Higher-quality mathematics teaching through lecturing, interaction, and after-class attention relates to desirable mathematics ability development.

activities and goal setting than students with other ability growth patterns do.

7. Before and during middle school, frustration in mathematics learning relates to mathematics ability. Near the end of high school, all adolescents' frustration in mathematics learning becomes similar perhaps due to their tracking into different types of high schools or curricula.

Acknowledgments

This work was supported by the Ministry of Science and Technology, Taiwan (MOST 108-2511-H-004-002). The funder only provides financial support and does not substantially influence the entire research process, from study design to submission. The authors are fully responsible for the content of the paper.

Disclosure statement

No potential conflict of interest was reported by the author.

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Funding

This work was supported by the Ministry of Science and Technology, Taiwan [MOST 108-2511-H-004-002].

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