

考試科目	資料結構及演算法	系所別	資訊科學系	考試時間	2 月 5 日(五) 第二節
<p>1. (10%) Prove or disprove the following statements (<math>n</math> is the size of the input array):</p> <ol style="list-style-type: none"> <li>(5%) The time complexity of any sorting algorithm is <math>O(n^2)</math>.</li> <li>(5%) The best-case time complexity of insertion sort is <math>\theta(n \log n)</math>.</li> </ol> <p>2. (10%) Assume <math>P \neq NP</math>. For each of the following statements, answer True or False. No explanation is needed.</p> <ol style="list-style-type: none"> <li>(5%) If a problem is in NP-hard, then the problem is in NP.</li> <li>(5%) If a problem is in NP, then the problem is in NP-complete.</li> </ol> <p>3. (15%) Use an example to explain why Dijkstra's shortest path algorithm cannot be applied to graphs with negative edge weights. In your example graph, please highlight the source and the destination.</p> <p>4. (10%) Give two advantages of red black trees over hash tables.</p> <p>5. (15%) An independent set <math>I</math> of an undirected graph <math>G = (V, E)</math> is a subset of <math>V</math> such that for any two vertices <math>u</math> and <math>v</math> in <math>I</math>, <math>u</math> and <math>v</math> are not adjacent, i.e., <math>(u, v) \notin E</math>. Design a dynamic program to find the largest independent set in a tree. ***Please explain the high-level idea of your answer in English or Chinese.***</p> <p>6. (15%) Consider an <math>n</math>-node sorted singly linked list <math>L</math>, where the first node (i.e., the head) stores the smallest data, the last node (i.e., the tail) stores the largest data, and the <math>i^{\text{th}}</math> node stores the <math>i^{\text{th}}</math> smallest data. Given the address of <math>L</math>'s head and a key, is it possible to search for the key in <math>L</math> in <math>O(\log n)</math> time? Please explain your answer.</p> <p>7. (15%) Design a polynomial-time algorithm for the following problem. Input: An <math>n</math>-node undirected graph <math>G</math>. Output: "Yes", if there is a cycle of length at most <math>n/2</math> (i.e., a cycle that has at most <math>n/2</math> nodes) in <math>G</math>; "No", otherwise. ***Please give the high-level idea of your algorithm and explain why it has polynomial running time.***</p> <p>8. (10%) Consider a 500-node binary heap, whose array representation is stored in Arr. Let <math>p</math> be a node in the binary heap. Further assume that <math>u</math> is <math>p</math>'s child node and <math>u</math>'s key is stored at Arr[305].</p> <ol style="list-style-type: none"> <li>(5%) How to find <math>p</math>'s key in Arr?</li> <li>(5%) Does <math>u</math> have a left child node in the binary heap? Please explain your answer.</li> </ol>					
備註	<p>一、作答於試題上者，不予計分。 二、試題請隨卷繳交。</p>				

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<p>1. Single choice (75%)      選擇題請在答案卡上作答，否則不予計分。</p> <p>(1) Which of the following is not a technology related to containers (e.g., Docker)? (A) cgroups (B) namespace (C) hypervisor (D) volume</p> <p>(2) When a process is being switched out of the CPU, which of the following situations does not cause the process is being added to the wait queue? (A) I/O request (B) create child process (C) time slice expired (D) wait for an interrupt</p> <p>(3) Which of the following is not a general function of a distributed operating system? (A) kernel migration (B) data migration (C) process migration (D) computation migration</p> <p>(4) In August 2018, a variant of WannaCry(WannaCrypt) forced TSMC to shut down several factories temporarily, this kind of virus is categorized as (A) spyware (B) ransomware (C) logic bomb (D) trap door</p> <p>(5) About NVM scheduling, which of the following is wrong? (A) in NVM, write time is usually not uniform whereas read is usually uniform (B) most NVM disks serve all read requests using C-SCAN (C) I/O performance of NVM is generally measured in IOPS (D) write performance in NVM depends on how full the device is.</p> <p>(6) Which of the following regarding the memory layout of a process is wrong? (A) the parameter values and the return address store in the stack area (B) the un-initialized global variables are stored in the bss area (C) the local variables are stored in the stack area (D) the heap area is also known as activation records</p> <p>(7) Given the following code segment, which of the following technologies is most possibly used by this program? (A) shared memory (B) pipe (C) RPC (D) MOM</p> <pre> int main() { ... close(fd[0]); write(fd[1], "test", 5); close(fd[1]); ... } </pre> <p>(8) Which of the following is false? (A) In asynchronous cancellation mode, a thread is terminated immediately after it receives a cancellation request (B) Most of the blocking system calls in POSIX are defined as cancellation</p>					

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<p>points (C) Asynchronous cancellation mode is not recommended in PThread (D) In Java, the deferred cancellation is implemented as the defer() method</p> <p>(9) Which of the following is true about the strategy that uses page fault frequency (PFF) to prevent thrashing? (A) A new page is allocated to a process if PFF is too high. (B) A page is deallocated from a process if the PFF is too low. (C) Easier to be implemented than Working Set (D) All of the above.</p> <p>(10) The cache that holds TLB in a CPU is called (A) TTE (B) TTB (C) TEB (D) TSB</p> <p>(11) Which of the following schemes is most suitable for a 64-bit computer system? (A) clustered page tables (B) inverted page table (C) hierarchical page tables (D) two-level paging</p> <p>(12) Which of the following statements is false (A) Banker's algorithm is applicable in a single-instance system (B) the drawback of using RAG algorithm is low resource utilization (C) most operation systems implements some form of deadlock prevention algorithms (D) livelock occurs when a thread continuously attempts an action that fails</p> <p>(13) Which of the following statements is wrong about semaphore implementation? (A) The acquireUninterruptibly() method in the Java semaphore is a non-busy waiting implementation (B) Use spinlock in multi-core system (C) Non-busy waiting implementation can only be used in a single-core system (D) disable interrupt in a single-core system.</p> <p>(14) Which of the following is not a technology for a Cloud Native system? (A) Kubernetes (B) Software Telemetry (C) MQTT (D) CQRS</p> <p>(15) Which of the following is not part of the middleware provided by the OS? (A) Threading service (B) Database service (C) Graphics service (D) Multimedia service</p> <p>(16) The reason that the interrupt vector is indexed by numbers is to increase _____ (A) Speed (B) Stability (C) Security (D) Extensibility</p> <p>(17) In Chrome, there are different types of process, which of the following renders web pages, deals with HTML, JavaScript? (A) browser process (B) renderer process (C) plug-in process (D) page process</p> <p>(18) Android has a system of importance hierarchy of process; the processes are classified as: 1. Background; 2. Service; 3. Foreground; 4. Visible. Please order the above categories from "the most important" to "the least important". (A) 3412 (B) 3421 (C) 4312 (D) 4123</p> <p>(19) Given a reference string: 4,3,2,1,4,3,5,4,2,1. How many page faults will occur using LRU? Assuming a 4-page frame. (A) 5 (B) 6 (C) 7 (D) 8</p>					

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<p>(20) About SSD (Solid-state disk), which of the following is correct? (A) SSD is a kind of nonvolatile memory device (B) FTL (Flash translation layer) is used to track logical blocks that contain valid data (C) SSD usually uses over-provisioning to improve read performance (D) Wear leveling is used to prevent certain blocks from being erased repeatedly.</p> <p>(21) Parent terminated without calling wait(), leaving the child alone is called (A) orphan (B) daemon (C) zombie (D) none of the above</p> <p>(22) Which of the following is false: (A) One-to-One threading is rarely used in current systems (B) Implicit threading usually use Many-to-Many threading model (C) One-to-One model is slow as creating a user-level thread also creates a kernel thread (D) JVM on Windows uses One-to-One threading model</p> <p>(23) Which of the following is true? (A) When the CPU receives a maskable interrupt, it stops what it is doing and immediately transfers execution to the interrupt handler (B) I/O controllers emit interrupts via interrupt-request lines (C) In critical sections, the CPU may disables the non-maskable interrupts (D) In an Intel processor, the interrupt No. 1 is a maskable interrupt</p> <p>(24) Which of the following page replacement algorithms is most similar to the one implemented in Linux. (A) FIFO (B) Least frequently used (C) LRU (D) LFU</p> <p>(25) In the Second readers-writers' problem, which of the following is false? (A) Only one writer can enter the critical section (B) many readers can access the critical section at the same time (C) When a reader acquires the mutex lock, the lock will not be released except all readers exits (D) Once a writer is ready, no new reader may start reading</p> <p>2. (5%) Please explain the motivation/purpose of using Condition Variables in a monitor?</p> <p>3. (5%) Given a paging memory system with 7-level page tables, assuming that the TLB hit rate is 90%, it takes 50 ns to access TLB and 600 ns to access memory. What is the EMAT of memory system?</p> <p>4. (15%) Terms</p> <p>(1) What is a “stack algorithm” for page replacement? What is the benefit of a stack algorithm?</p> <p>(2) What is a “dark web”?</p> <p>(3) What is Infrastructure as Code (IaC)? What are the benefits of IaC?</p>					
備 註	<p>一、作答於試題上者，不予計分。</p> <p>二、試題請隨卷繳交。</p>				

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I. 離散數學: 共 19 題(1-17, 26-27), 1-17 為選擇題, 26-27 為簡答題, 共 60%  
 II. 線性代數: 共 9 題(18-25, 28), 18-25 為選擇題, 28 為簡答題, 共 40%  
 非選擇題請書寫必要的解題過程, 僅書寫答案而缺乏必要的過程, 無法獲得該題滿分。可使用中文或英文作答, 力求書寫工整, 如字跡潦草, 無法閱讀, 將影響評分。

選擇題請在答案卡上作答, 否則不予計分。

- (3%) Let  $\phi$  denote an empty set. Which of the following statements is true?  
 (A)  $\phi \in \phi$  (B)  $\phi \subset \phi$  (C)  $\phi \subset \{\phi\}$  (D)  $|\phi| = 1$
- (3%) Let  $A, B, C$  be finite sets. Which of the following statements is false?  
 (A)  $|A \cup B| = |A| + |B| - |A \cap B|$  (B)  $P(A \cup B) = P(A) \cup P(B)$   
 (C)  $A - (B \cup C) = (A - B) - C$  (D)  $A \cap (B \oplus C) = (A \cap B) \oplus (A \cap C)$
- (3%) Let  $A, B, C$  be finite sets. Which of the following statements is true?  
 (A) If  $A \cup C = B \cup C$ , then  $A = B$  (B) If  $A \cap C = B \cap C$ , then  $A = B$   
 (C) If  $A - C = B - C$ , then  $A = B$  (D) If  $A \oplus C = B \oplus C$ , then  $A = B$
- (3%) Let  $p = 101$  be a prime and  $a = 2$  be a primitive root modulo  $p$ . Please determine the value of  $(a^7)^9 \pmod p$ .  
 (A) 88 (B) 89 (C) 90 (D) 91
- (3%) Let  $a, b, k, n$  be integers. Which of the following statements is not correct?  
 (A)  $a \equiv b \pmod n \Rightarrow ka \equiv kb \pmod n$   
 (B)  $a \equiv b \pmod{kn} \Rightarrow a \equiv b \pmod n$   
 (C)  $a \equiv b \pmod n \Rightarrow ka \equiv kb \pmod{kn}$   
 (D)  $a \equiv b \pmod n \Rightarrow a \equiv b \pmod{kn}$
- (3%) Which of the following statements is true?  
 (A)  $p \rightarrow q \equiv \neg(p \vee q)$  (B)  $\neg(p \leftrightarrow q) \equiv \neg p \leftrightarrow \neg q$   
 (C)  $\neg \exists x (Q(x) \wedge R(x)) \equiv \forall x (\neg Q(x) \vee R(x))$  (D)  $(p \wedge (p \rightarrow q) \rightarrow q) \equiv T$
- (3%) Which of the following statements is false?  
 (A)  $\forall x [Q(x) \wedge R(x)] \rightarrow \forall x Q(x) \wedge \forall x R(x)$  (B)  $\forall x [Q(x) \vee R(x)] \rightarrow \forall x Q(x) \vee \forall x R(x)$   
 (C)  $\exists x [Q(x) \wedge R(x)] \rightarrow \exists x Q(x) \wedge \exists x R(x)$  (D)  $\exists x [Q(x) \vee R(x)] \rightarrow \exists x Q(x) \vee \exists x R(x)$

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8. (3%) How many bit strings are there containing exactly five 0s and fourteen 1s if every 0 must be immediately followed by two 1s?  
 (A) 112 (B) 124 (C) 126 (D) 132
9. (3%) Find the number of ways in which nine identical blocks can be given to four children A, B, C, D, where child A gets at most two blocks.  
 (A) 100 (B) 136 (C) 142 (D) 164
10. (3%) How many ways are there to put five temporary employees into four identical offices?  
 (A) 48 (B) 51 (C) 56 (D) 60
11. (3%) Let  $x = a + bk$  be the solution of the system of congruences  $x \equiv 7 \pmod{9}$ ,  $x \equiv 4 \pmod{12}$ , and  $x \equiv 16 \pmod{21}$ , where  $a$  and  $b$  are the smallest positive integer,  $k \in \mathbb{Z}$ . Which is the correct value of  $(2a + b) \pmod{11}$ ?  
 (A) 9 (B) 7 (C) 4 (D) 6
12. (3%) Let  $A$  be a set with  $n$  elements. Which of the followings is the number of binary relations on  $A$  which are symmetric and antisymmetric, and not irreflexive?  
 (A)  $2^n$  (B)  $2^n - 1$  (C)  $(2^n - 1) \cdot 3^{\frac{n^2-n}{2}}$  (D)  $(2^n - 2) \cdot 3^{\frac{n^2-n}{2}}$
13. (3%) Let  $A$  be a set with  $n$  elements. Which of the followings is the number of binary relations on  $A$  which are reflexive and not symmetric?  
 (A)  $2^{n^2} - 2^{n^2-n+1}$  (B)  $2^{n^2} \cdot 2^{\frac{n^2-n}{2}}$  (C)  $2^{n^2-n} - 2^{\frac{n^2-n}{2}}$  (D)  $(2^n - 1) \cdot 2^{\frac{n^2-n}{2}}$
14. (3%) Suppose that someone starts a chain letter. Each person who receives the letter is asked to send it to four other people. Let  $k$  be the number of people who have seen the letter, including the first person. Suppose no one receives more than one letter and the chain letter ends after there have been 100 people who read it but did not send it out. which is the correct value of  $k \pmod{11}$ ?  
 (A) 0 (B) 3 (C) 4 (D) 8
15. (3%) Let  $G$  be a connected bipartite planar simple graph with 12 vertices, and let  $e$  be the maximum value of the number of edges in  $G$ . Which of the followings is the value of  $e \pmod{11}$ ?

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(A) 3 (B) 7 (C) 9 (D) 10

16. (3%) Consider the following string aaabbcdddddeeeeeffgggggghhhiii. Suppose we at least require  $k$  bits to encode the string. Which of the followings is the value of  $k \bmod 11$ ?

(A) 1 (B) 6 (C) 5 (D) 10

17. (3%) Suppose the solution of the recurrence relation  $a_n = 7a_{n-1} - 16a_{n-2} + 12a_{n-3} + n4^n$  with  $a_0 = -2$ ,  $a_1 = 0$ , and  $a_2 = 5$ , is  $a_n = a \cdot 2^n + b \cdot n \cdot 2^{n-1} + c \cdot 3^n + (d_2n^2 + d_1n + d_0) \cdot 4^n$ . Which of the following is the value of  $(a + b + c + d_0 + d_1 + d_2) \bmod 11$ ?

(A) 9 (B) 8 (C) 7 (D) 6

18. (3%) Determine the number of solutions for the linear system 
$$\begin{cases} x + 2y - z = 2 \\ 2x + 5y - 3z = 1 \\ x + 4y - 3z = 3 \end{cases}$$

(A) no solution (B) a unique solution (C) exactly two solutions (D) infinite many solutions

19. (3%) If  $\det \begin{bmatrix} b_1 + c_1 & b_2 + c_2 & b_3 + c_3 \\ a_1 + c_1 & a_2 + c_2 & a_3 + c_3 \\ a_1 + b_1 & a_2 + b_2 & a_3 + b_3 \end{bmatrix} = k \cdot \det \begin{bmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{bmatrix}$ , what is the value of  $k$ ?

(A) 2 (B) 3 (C) 4 (D) 6

20. (3%) Which of the following statements is true?

(A) If none of the vectors in the set  $S = \{v_1, v_2, v_3\}$  in  $R_3$  is a multiple of one of the other vectors, then  $S$  is linearly independent.

(B) If  $A$  is a subspace, then its complement is also a subspace.

(C) If a square matrix  $A$  has independent columns, so does  $A^2$ .

(D) Every vector space has at least two distinct subspaces.

21. (3%) Which of the following statements is a linear transformation?

(A)  $L(x, y, z) = (x + y, y^2, 2z)$

(B)  $L \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2x - 3y \\ 3y - 2z \\ 2z + 1 \end{bmatrix}$

(C)  $L(p(x)) = \begin{bmatrix} \int_0^1 p(x) d(x) \\ p'(x) \end{bmatrix}$ ,  $p(x) \in P_2$

(D)  $L(x, y, z) = (4x, \frac{y^2}{z^2})$

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22. (3%) Which of the following statements is true?
- (A) A square matrix with linearly independent column vectors is diagonalizable.  
 (B) If  $A$  and  $B$  are diagonalizable, so is  $AB$ .  
 (C) If  $A$  be diagonalizable and  $AB = BA$ , then  $B$  is diagonalizable.  
 (D) If  $A$  is a  $3 \times 3$  matrix with distinct eigenvalues  $0, 1, 2$ , then the matrix  $(A + I)$  must be invertible.
23. (3%) Which of the following statements is true?
- (A) Let  $S$  be a subset of a vector space. Then  $S = (S^\perp)^\perp$   
 (B) Let  $V$  be a subset of  $\mathbb{R}^n$ , and if  $V$  is orthogonal to  $W$ , then  $V^\perp$  is orthogonal to  $W^\perp$ .  
 (C) If  $u$  and  $v$  are vectors in  $\mathbb{R}^n$ , and if distance from  $u$  to  $v$  equals the distance from  $u$  to  $-v$ , then  $u$  and  $v$  are orthogonal.  
 (D) If  $u, v$  and  $w$  are vectors in  $\mathbb{R}^n$ , and if  $u$  is orthogonal to  $v + w$ , then  $u$  is orthogonal to  $v$  and  $w$ .
24. (3%) Given the following data points  $\{(-1, 1), (1, 3), (2, 3)\}$ , suppose that the linear function  $y = \frac{b}{a} + \frac{d}{c}x$  is the approximation to fit the data in the least squares sense, please find the value of  $(a + b + c + d) \bmod 11$ .  
 (A) 1 (B) 3 (C) 7 (D) 10
25. (3%) Let  $S = \text{span}\{(1 \ 3 \ 1 \ 1)^T, (1 \ 1 \ 1 \ 1)^T, (-1 \ 5 \ 2 \ 2)^T\}$  be a subspace of  $\mathbb{R}^4$ . Suppose  $w_1 = \left(\frac{a_1}{\sqrt{12}} \ \frac{a_2}{\sqrt{12}} \ \frac{a_3}{\sqrt{12}} \ \frac{a_4}{\sqrt{12}}\right)^T$ ,  $w_2 = \left(\frac{b_1}{2} \ \frac{b_2}{2} \ \frac{b_3}{2} \ \frac{b_4}{2}\right)^T$ ,  $w_3 = \left(\frac{c_1}{\sqrt{6}} \ \frac{c_2}{\sqrt{6}} \ \frac{c_3}{\sqrt{6}} \ \frac{c_4}{\sqrt{6}}\right)^T$  is an orthonormal basis of  $S$ . Please find the value of  $(a_1 + a_2 + a_3 + a_4) + (b_1 + b_2 + b_3 + b_4) + (c_1 + c_2 + c_3 + c_4)$ ?  
 (A) 5 (B) 6 (C) 7 (D) 8
26. (4%) Use generating functions to prove Pascal's identity:  $C(n, r) = C(n-1, r) + C(n-1, r-1)$  when  $n$  and  $r$  are positive integers with  $r < n$ . [Hint: Use the identity  $(1+x)^n = (1+x)^{n-1} + x(1+x)^{n-1}$ .]
27. (5%) Let  $(x_i, y_i)$ ,  $i = 1, 2, 3, 4, 5$ , be a set of five distinct points with integer coordinates in the  $xy$  plane. Show that there is at least one of the midpoints of the segment joining of these points which has integer coordinates.
28. (16%)  
 Let  $T: P_2 \rightarrow P_2$  be given by
- $$T(p(x)) = p(x-1)$$
- Consider the two ordered bases  $\beta = \{x^2, x, 1\}$  and  $\gamma = \{x, x+1, x^2-1\}$ .



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- (a) (6%) Find  $[T]_{\beta}$  and  $[T]_{\gamma}$ .
- (b) (6%) Find the matrix  $S$  such that  $[T]_{\gamma} = S[T]_{\beta}S^{-1}$ .
- (c) (4%) If the basis is  $\gamma$ , find the dimension of  $\ker(T)$  and the basis of  $\ker(T)$ .



備

註

- 一、作答於試題上者，不予計分。
- 二、試題請隨卷繳交。