Agent-Based Computational Macroeconomics: A Survey

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Summary. While by all standards the macroeconomic system is qualified to be a complex adaptive system, mainstream macroeconomics is not capable of demonstrating this feature. Recent applications of *agent-based modeling* to macroeconomics define a new research direction, which demonstrates how the macroeconomic system can be modeled and studied as a complex adaptive system. This paper shall review the development of agent-based computational modeling in macroeconomics.

Keywords. Complex adaptive system, Agent-based computational economics, Adaptive economic agents, Rational expectations equilibrium

1 Introduction

The growth of computer power enables us to study the complex economic processes and phenomena through computer simulation. Recently, attention has been paid to the issues of emerging behavioural patterns, structures, and social order (e.g., cooperation, coordination, networks, institutions, conventions, norms, the market, and its structure, etc.). Leigh Teafatsion in her homepage **Agent-Based Computational Economics** (**ACE**) claimed that "a central concern of **ACE** researchers is to understand the apparently spontaneous appearance of global regularities in economic processes, such as the unplanned coordination of trade in decentralized market economies that economists associate with Adam Smith's invisible hand." She continued this message by saying that the challenge is to explain these global regularities from the bottom up, in the sense that the regularities arise from the local interactions of autonomous agents channeled through actual or potential economic institutions rather than through fictitious top-down coordinating mechanisms such as a single representative consumer.

Axel Leijonhufvud, the founder of the Center for Computable Economics at UCLA, stated in his Distinguished Guest Lecture given at the 1992 Annual Meeting of the Southern Economic Association that "The economy is best conceived of as a network of interacting processors, each one with less capability to process information that would be required of a central processor set to solve the overall allocation problem for the entire system."(Leijonhufvud (1993), p.4)

With ever-increasing computer power, man can now have a better chance to simulate the evolution of a large population on a long-term scale from a few seconds to a few days in the electronic world. Through this power, people can raise and answer questions from an evolutionary perspective, and this is exactly one of the distinguishing features of agent-based computational economics: "Agent-based computational economics is roughly characterized as the computational study of economics modelled as evolving decentralized systems of autonomous interacting agents. (ACE website)"

The significance of studying economics from a perspective of simulated evolution is in fact well acknowledged not only by the economists in the western hemisphere, but also by those in the eastern hemisphere. For example, economists in Japan recently established the Japan Association for Evolutionary Economics (JAFEE). As it is well said in the chapter "Invitation for JAFEE", "The tasks of Evolutionary Economics are to elucidate the cognition that the coexistence and competition of multiple systems, organizations and technology is indispensable resource for future development, and propose a new direction of development of global society by analyzing the intrinsic dynamics of 'evolutionary process' in which each system, organization and technology are involved. (JAFFE website)"

The scope of both ACE and JAFEE touches an interdisciplinary term, known as *emergent properties*. In his keynote speech given at the 1997 Joint Conference of Information Sciences (JCIS'97), John Holland made an excellent remark on it. The talk, entitled "Emergence: Models, Metaphors and Innovation," points out that creative models exhibit emergent properties, so that "what comes out is more than what goes in." For example, a game like chess is defined by less than a dozen rules, yet it still rewards us with new insights and strategies after centuries of intensive study.

"What comes out is more than what goes in" is certainly not a general property shared by conventional economic models. Conventional economic models usually have a simple equilibrium (outcome) characterized by a *fixed point* or *a stationary distribution*. In other words, a typical economic model uses lots of rules or axioms to describe a system, while the outcome is pretty simple and regular. Therefore, in the conventional economic system, it is the opposite that holds: "what comes out is much less than what goes in."

With the advent of agent-based modeling, economists have begun to realize that economics can be more interesting and fruitful if economics is studied within the context of complex adaptive systems, as what was vividly described in Arthur (1992)'s artificial stock market.

> We find that early in the experiment the price settles to random noise about fundamental value. But after some time, mutually reinforcing trend-following or technical-analysis-like rules begin to appear in the predictor population.... Eventually a slowly chang-

ing "ecology" of hypothesis-predictors becomes established, with self-reinforcing technical trading rules very much part of the system....The system has changed....it coevolves and changes and transforms. It never settles. (Ibid, pp.23-24.)

In this paper we shall review the development of agent-based computational modeling in macroeconomics with a discussion of the following two complexity-related issues. First, from a microscopic perspective, *agent engineering*. How would agents behave when they are placed into a complex system in which their knowledge of the system is always incomplete? Would they evolve into complex and heterogeneous behavior in response to the surrounding co-evolving environment? Second, from a macroscopic perspective, *market dynamics*. What are the effects of agents' adaptation and interactions on the aggregate outcomes? Will the market exhibit complex dynamic behavior? Will the aggregate outcomes be emergent in the sense that some properties observed are not just the scaling-up of individual behavior? The review mainly covers production and price dynamics in the cobweb models, saving and inflation in the overlapping generations models, arbitraging and the exchange rate fluctuation in the foreign exchange rate market, and investment and the stock price in the stock market.

2 Production and Price Stability

The cobweb model is a familiar playground in which to investigate the effects of production decisions on price dynamics. In this model consumers base their decisions on the current market price, but producers decide how much to produce based on the past prices. Agricultural commodities serve as a good example of the cobweb model. This model plays an important role in macroeconomics, because it is the place in which the concept rational expectations originated (Muth 1961). Moreover, it is also the first neo-classical macroeconomic prototype to which an agent-based computational approach was applied (Arifovic 1994). This section will first briefly formulate the cobweb model and then review the work on agent-based modeling of the cobweb model.

Consider a competitive market composed of n firms which produce the same goods by employing the same technology and which face the same cost function described in Equation (1):

$$c_{i,t} = xq_{i,t} + \frac{1}{2}ynq_{i,t}^2 \tag{1}$$

where $q_{i,t}$ is the quantity supplied by firm i at time t, and x and y are the parameters of the cost function. Since at time $t-1$, the price of the goods at time t, P_t, is not available, the decision about optimal $q_{i,t}$ must be based on the expectation (forecast) of P_t, i.e., $P^e_{i,t}$. Given $P^e_{i,t}$ and the cost function $c_{i,t}$, the expected profit of firm i at time t can be expressed as follows:

$$\pi^e_{i,t} = P^e_{i,t} q_{i,t} - c_{i,t} \tag{2}$$

Given $P^e_{i,t}$, $q_{i,t}$ is chosen at a level such that $\pi^e_{i,t}$ can be maximized and, according to the first-order condition, is given by

$$q_{i,t} = \frac{1}{yn}(P^e_{i,t} - x) \tag{3}$$

Once $q_{i,t}$ is decided, the aggregate supply of the goods at time t is fixed and P_t, which sets demand equal to supply, is determined by the demand function:

$$P_t = A - B \sum_{i=1}^{n} q_{i,t}, \tag{4}$$

where A and B are parameters of the demand function.

Given P_t, the actual profit of firm i at time t is :

$$\pi_{i,t} = P_t q_{i,t} - c_{i,t} \tag{5}$$

The neo-classical analysis simplifies the cobweb model by assuming the homogeneity of market participants, i.e., a representative agent. In such a setting, it can be shown that the homogeneous rational expectations equilibrium price (P^*) and quantity (Q^*) are (Chen and Yeh 1996, p.449):

$$P^*_t = \frac{Ay + Bx}{B + y}, \quad Q^*_t = \frac{A - x}{B + y}. \tag{6}$$

2.1 **Agent-Based Cobweb Model**

2.1.1 Convergence to Rational Expectation Equilibrium

The neo-classical analysis based on homogeneous agents provides us with a limited understanding of the price dynamics or price instability in a real market, since firms' expectations of the prices and the resultant production decisions in general must be heterogeneous. Using genetic algorithms to model the adaptive behavior of firms' production, Arifovic (1994) gave the first agent-based model of the cobweb model. She applied two versions of GAs to this model. The basic GA involves three genetic operators: reproduction, crossover, and mutation. Arifovic found that in each simulation of the basic GA, individual quantities and prices exhibited fluctuations for its entire duration and did not result in convergence to the rational expectations equilibrium values, which is quite inconsistent with experimental results with human subjects.

Arifovic's second GA version, the augmented GA, includes the election operator in addition to reproduction, crossover, and mutation. The election operator involves two steps. First, crossover is performed. Second, the potential fitness of the newly-generated offspring is compared with the actual fitness values of its parents. Among the two offspring and two parents, the two highest fitness individuals are then chosen. The purpose of this operator is to overcome difficulties related to the way mutation influences the convergence process, because the election operator can bring the variance of the population rules to zero as the algorithm converges to the equilibrium values.

The results of the simulations show that the augmented GA converges to the rational expectations equilibrium values for all sets of cobweb model parameter values, including both stable and unstable cases, and can capture several features of the experimental behavior of human subjects better than other simple learning algorithms. To avoid the arbitrariness of choice of an adaptive scheme, Lucas (1986) suggested that comparison of the behavior of adaptive schemes with behavior observed in laboratory experiments with human subjects can facilitate the choice of a particular adaptive scheme. From this suggestion, the GA could be considered an appropriate choice to model learning agents in a complex system.

Arifovic (1994)'s finding, which is basically optimistic about the inherent stabilization force in the market, was was soon challenged and enriched by a series of follow-up studies (Dawid and Kopel 1998, Franke 1998). Dawid and Kopel (1998) complicated the simple cobweb model by including a term for fixed costs or overhead,

$$c_{i,t} = c_0 + xq_{i,t} + \frac{1}{2}ynq_{i,t}^2, \tag{7}$$

where c_0 denotes the short-term fixed costs of the firm. After the addition of the fixed costs, firms have to first decide whether they shall exit or stay in the market. This decision is crucial, because the fixed costs may cause negative profits for all quantities supplied ($q_{i,t}$), even if $q_{i,t} = 0$. Using genetic algorithms, Dawid and Kopel (1998) enhanced Arifovic (1994)'s string coding by one addition bit. The value of the bit shows the decision to exit or stay. Without this additional bit, they showed that the market state could be "locked in" a state where all firms make negative profits. However, by using this additional bit to implement a separation of the production decision into an exit and entry decision and a quantity determination, they found that the market converged to a heterogeneous rational expectations equilibrium, which describes an industry comprised of active and idle firms. This equilibrium is characterized by a market price which is supported by the number of firms who decide to exit the market. Therefore, extending Arifovic (1994)'s finding, Dawid and Kopel (1998)'s work showed how optimal market size (number of survivable firms) and quantities supplied can be simultaneously determined in such a biologically-inspired, agent-based model.

Franke (1998) complicated the simple cobweb model by first subjecting the demand side (Equation 4) to serially-correlated random shocks.

$$P_t = A - B \sum_{i=1}^{n} q_{i,t} + \mu_t, \tag{8}$$

where

$$\mu_t = \rho \mu_{t-1} + \nu_t. \tag{9}$$

ν_t are identically and independently drawn from a normal distribution with zero mean. The second complication is related to agent engineering. In both Arifovic (1994) and Dawid and Kopel (1998), the market only evolved one type of decision rule, namely, quantity decision or plus the exit-or-stay decision. Franke (1998) explored a higher level of heterogeneous agents: he considered four types of decision rules. The first one is the same as Arifovic (1994), but he also proposed three other classes of decision rules concerning different ways to model firms' expectations of price, $P^e_{i,t}$, namely, one class of adaptive expectations and two classes of regression strategies.

Firms characterized by these four classes of decision rules were competing with each other while they were evolving with genetic algorithms. This setting pushed the agent-based economic modeling to a new frontier. First, the diversity of the firms. One can ask whether there is one dominating class of decision rules. Is the leading positing stable over time? Second, the emergence of new rules and the survival time of them. How frequent or fast can the system generate new rules? Are there rules that are able to survive for a long time? If not, what is the general picture of the age structure of rules in a period of time? Third, behavior heterogeneity and stability. Suppose we remove one or a few class of rules out of the market and let the firms compete and evolve with the rest. Would that result in significantly different market dynamics?

Franke generally found that the GA did not lose the track of the homogeneous rational expectations equilibrium (HREE) even in the stochastic environment with the perturbations of demand. The macroeconomy outcome of this agent-based cobweb model may be described as an *near-equilibrium dynamics*. The actual market production deviated from the HREE production by a limited order of magnitude from 3.5% to 4.5%. The relatively small production deviation with respect to HREE is characterized by the *coevolution of strategies (decision rules)*. Coevolution has to be understood, not as a peaceful state of coexistence, but as an incessant struggle for survival where no strategy, and even any type of strategy, can be safe from being replaced in the near future. New strategies were spontaneously developed and old strategies were continually replaced. What kept the market functioning was this ongoing struggle of competing strategies. It was also found that a high degree of heterogeneity in strategy types is favorable to stability. Excluding selected strategy types from the initial distribution may result in a significantly higher deviation of aggregate output.

The application of genetic programming to the cobweb model started from Chen and Yeh (1996). Chen and Yeh (1996) compared the learning performance of GP-based learning agents with that of GA-based learning agents. They found that, like GA-based learning agents, GP-based learning agents also can learn the homogeneous rational expectations equilibrium price under both the stable and unstable cobweb case. However, the phenomenon of price euphoria, which did not happen in Arifovic (1994), does show up quite often at the early stages of the GP experiments. This is mainly because agents in their setup were initially endowed with very limited information as compared to Arifovic (1994). Nevertheless, GP-based learning can quickly coordinate agents' beliefs so that the emergence of price euphoria is only temporary. Furthermore, unlike Arifovic (1994), Chen and Yeh (1996) did not use the election operator. Without the election operator, the rational expectations equilibrium is exposed to potentially persistent perturbations due to agents' adoption of the new, but untested, rules. However, what shows up in Chen and Yeh (1996) is that the market can still bring any price deviation back to equilibrium. Therefore, the self-stabilizing feature of the market, known as the invisible hand, is more powerfully replicated in their GP-based artificial market.

The self-stabilizing feature of the market demonstrated in Chen and Yeh (1996) was furthered tested with two complications. In the first case, Chen and Yeh (1997) introduced a population of speculators to the market and examined the effect of speculations on market stability. In the second case, the market was perturbed with a structural change characterized by a shift in the demand curve, and Chen and Yeh (2000a) then tested whether the market could restore the rational expectations equilibrium. The answer to the first experiment is generally negative, i.e., speculators do not enhance the stability of the market. On the contrary, they do destabilize the market. Only in special cases when trading regulations, such as the transaction cost and position limit, were tightly imposed could speculators enhance the market stability. The answer for the second experiment is, however, positive. Chen and Yeh (2000a) showed that GP-based adaptive agents could detect the shift in the demand curve and adapt to it. Nonetheless, the transition phase was non-linear and non-smooth; one can observe slumps, crashes, and bursts in the transition phase. In addition, the transition speed is uncertain. It could be fast, but could be slow as well.

This series of studies on the cobweb model enriches our understanding of the self-stabilizing feature of the market. The market has its limit, beyond which it can become unstable with crazy fluctuations. However, imposing trading regulations may relax the limit and enhance market stability. One is still curious to know where the self-stabilizing capability comes from in the first place. Economists have known for a long time that it comes from the free competition principle, or the survival-of-the-fittest principle. In GA or GP, this principle is implemented through *selection pressure*. Chen (1997) studied the role of selection pressure by replacing the usual proportionate selection scheme with the one based on the approximate uniform distribution, showing that if selection pressure is removed or alleviated, then the self-stabilizing feature is lost. In a word, selection pressure plays the role of the invisible hand in economics.

It is interesting to know whether the time series data generated by the artificial market can replicate some dynamic properties observed in the real market. Chen and Kuo (1999) and Chen and Yeh (2000a) started the analysis of the time series data generated from the artificial market. The time series data employed was generated by simulating the agent-based cobweb model with the presence of speculators. It was found that many stylized features well documented in financial econometrics can in principle be replicated from GP-based artificial markets, which include leptokutosis, non-IIDness, and volatility clustering. Furthermore, Chen and Yeh (2000a) performed a CUSUMSQ test, a statistical test for structural change, on the data. The test indicated the presence of structural changes in the data, which suggested that that the complex interaction process of these GP-based producers and speculators can even generate endogenous structural changes.

3 Saving and Inflation

While there are several approaches to introducing dynamic general equilibrium structures to economics, the overlapping generations model (hereafter, OLG) may be regarded as the most popular one in current macroeconomics. Over the last two decades, the OLG model has been extensively applied to studies of savings, bequests, demand for assets, prices of assets, inflation, business cycles, economic growth, and the effects of taxes, social security, and budget deficits. In the following, we shall first give a brief illustration of a simple OLG model of inflation, a *two-period* OLG model. We then present a *n-period* generalization of it.

3.1 Overlapping Generations Models

3.1.1 Two-Period OLG Model

A simple OLG model can be described as follows. It consists of overlapping generations of two-period-lived agents. At time t, N young agents are born. Each of them lives for two periods $(t, t+1)$. At time t, each of them is endowed with e^1 units of a perishable consumption good, and with e^2 units at time $t+1$ ($e^1 > e^2 > 0$). Presumably e^1 is assumed to be greater than e^2 in order to increase the likelihood (not ensure) that agents will choose to hold money from period 1 to 2 so as to push value forward. An agent born at time

t consumes in both periods. Term c_t^1 is the consumption in the first period (t), and c_t^2 the second period $(t+1)$. All agents have identical preference given by

$$U(c_t^1, c_t^2) = \ln(c_t^1) + \ln(c_t^2) \tag{10}$$

In addition to the perishable consumption good, there is an asset called *money* circulating in the society. The nominal money supply at time t, denoted by H_t, is exogenously determined by the government and is held distributively by the old generation at time t. For convenience, we shall define h_t to be $\frac{H_t}{N}$, i.e., the nominal per capita money supply.

This simple OLG gives rise to the following agent's maximization problem at time t:

$$\max_{(c_{i,t}^1, c_{i,t}^2)} \quad \ln(c_{i,t}^1) + \ln(c_{i,t}^2) \tag{11}$$

$$s.t. \;\; c_{i,t}^1 + \frac{m_{i,t}}{P_t} = e^1, \;\; c_{i,t}^2 = e^2 + \frac{m_{i,t}}{P_{t+1}}, \tag{12}$$

where $m_{i,t}$ represents the nominal money balances that agent i acquires at time period t and spends in time period $t+1$, and P_t denotes the nominal price level at time period t. Since P_{t+1} is not available at period t, what agents actually can do is to maximize their expected utility $E(U(c_t^1, c_t^2))$ by regarding P_{t+1} as a random variable, where $E(.)$ is the expectation operator. Because of the special nature of the utility function and budget constraints, the first-order conditions for this expected utility maximization problem reduce to the certainty equivalence form (13):

$$c_{i,t}^1 = \frac{1}{2}(e^1 + e^2 \pi_{i,t+1}^e) \tag{13}$$

where $\pi_{i,t+1}^e$ is agent i's expectation of the inflation rate $\pi_{t+1}(\equiv \frac{P_{t+1}}{P_t})$. This solution tells us the optimal decision of savings for agent i given her expectation of the inflation rate, $\pi_{i,t+1}^e$.

Suppose the government deficit G_t is all financed through seignorage and is constant over time ($G_t = G$). We can then derive the dynamics (time series) of nominal price $\{P_t\}$ and inflation rate $\{\pi_t\}$ from Equation (13). To see this, let us denote the savings of agent i at time t by $s_{i,t}$. Clearly,

$$s_{i,t} = e^1 - c_{i,t}^1 \tag{14}$$

From Equation (12), we know that

$$m_{i,t} = s_{i,t} P_t, \quad \forall i, t. \tag{15}$$

In equilibrium, the nominal aggregate money demand must equal nominal money supply, i.e.,

$$\sum_{i=1}^{N} m_{i,t} = H_t = H_{t-1} + G P_t, \quad \forall t. \tag{16}$$

The second equality says that the money supply at period t is the sum of the money supply at period $t-1$ and the nominal deficit at period t, GP_t. This equality holds, because we assume the government deficits are all financed by seignorage.

Summarizing Equations (15)-(16), we get

$$\sum_{i=1}^{N} s_{i,t}P_t = \sum_{i=1}^{N} s_{i,t-1}P_{t-1} + GP_t \tag{17}$$

The price dynamics are hence governed by the following equation:

$$\pi_t = \frac{P_t}{P_{t-1}} = \frac{\sum_{i=1}^{N} s_{i,t-1}}{\sum_{i=1}^{N} s_{i,t} - G} \tag{18}$$

Now suppose that each agent has perfect foresight, i.e.,

$$\pi_{i,t}^e = \pi_t, \quad \forall i, t. \tag{19}$$

By substituting the first-order condition (13) into Equation (17), the paths of equilibrium inflation rates under perfect foresight dynamics are then

$$\pi_{t+1} = \frac{e^1}{e^2} + 1 - \frac{2g}{e^2} - (\frac{e^1}{e^2})(\frac{1}{\pi_t}), \tag{20}$$

where $g = \frac{G}{N}$ is the real per capita deficit.

At steady state ($\pi_{t+1} = \pi_t$), Equation (20) has two real stationary solutions (fixed points), a low-inflation stationary equilibrium, π_L^*, and a high-inflation one, π_H^*, given by

$$\pi_L^* = \frac{1 + \frac{e^1}{e^2} - \frac{2g}{e^2} - \sqrt{(1 + \frac{e^1}{e^2} - \frac{2g}{e^2}) - 4\frac{e^1}{e^2}}}{2}, \tag{21}$$

$$\pi_H^* = \frac{1 + \frac{e^1}{e^2} - \frac{2g}{e^2} + \sqrt{(1 + \frac{e^1}{e^2} - \frac{2g}{e^2}) - 4\frac{e^1}{e^2}}}{2}. \tag{22}$$

3.1.2 N-Period OLG Model

Bullard (1992) studied the n-period generalization of the OLG model. Let n represents the number of periods in an agent's lifetime. At time t, there are n generation of agents alive, with each generation having a different birth date corresponding to times $t-n+1, t-n+2, ..., t-1, t$, and there is a single representative agent alive in every generation. The endowment profile is denoted by $\{e^1, e^2, ..., e^n\}$ and it is further assumed that

$$e^1 < e^2 < ... < e^n. \tag{23}$$

Agents in this economy can save only by holding fiat currency. Agents of all generations seek to maximize the same time separable logarithmic utility function:

$$U = \sum_{j=1}^{n} \ln^{c_{i,t}^{j}}, \tag{24}$$

subject to

$$c_{i,t}^{1} + \sum_{j=2}^{n} c_{i,t}^{j} \Pi_{k=0}^{j-2} \pi_{t+j} \leq e_{i,t}^{1} + \sum_{j=2}^{n} e_{i,t}^{j} \Pi_{k=0}^{j-2} \pi_{t+j}, \tag{25}$$

where $c_{i,t}^{j}$ denotes consumption by the agent born at time t in period $t+j-1$. and π_{t+j} denotes the gross inflation rate between dates between $t+j$ and $t+j+1$. Equation (25) is just *intertemporal budget constraint.*

The solution to the maximization problem of the young agent born at time t yields a lifetime consumption and saving plan. Solving these problems for consecutive n generations, it is possible to construct the amount of aggregate savings (asset holdings) in the economy at any time t, S_t, by summing together the time t savings amounts of all individuals alive in the economy at time t. Bullard (1992) showed that this aggregate function is giving by

$$S_t = \sum_{i=1}^{n-1} e^{i} + \sum_{i=0}^{n-3} \sum_{j=1}^{n-2-i} e^{i+1} \Pi_{k=1}^{j} \pi_{t-k}^{-1} - E_1 - \sum_{i=1}^{n-2} \sum_{j=0}^{i} E_{1+i} \Pi_{k=1}^{i} \pi_{t-k}^{-1}, \tag{26}$$

where

$$E_k \equiv \frac{1}{n}[e^1 + \sum_{i=1}^{n-1} e^{i+1} \Pi_{j=k-1}^{k+i-2} \pi_{t+i}] \tag{27}$$

Since all savings must be held in the form of fiat currency, the market clearing condition for this economy is given by

$$S_t P_t = H_t. \tag{28}$$

As already state in Equation (17), an equilibrium law of motion for prices is

$$P_t = (\frac{S_{t-1}}{S_t - G}) P_{t-1}. \tag{29}$$

Bullard (1992) has shown, for the case where $G = 0$, that there are at most two steady-state equilibrium for π for any value of n; this result generalizes to a positive government deficit. Hence, the general n-period OLG model under perfect foresight is analogous to the more familiar two-period OLG economy with two steady-state inflation rates (Equations 21, 22).

3.1.3 Indeterminacy of the Inflation Rate

Despite its popularity, the OLG models are well known for their multiplicity of equilibria, in our case, the coexistence of two inflation equilibria: Equations (21) and (22). Things can be even more intriguing if these equilibria have different welfare implications. In our case, the one with a higher inflation rate is the Pareto-inferior equilibrium, whereas the one with a lower inflation rate is the Pareto-superior equilibrium.

3.2 Agent-Based OLG Models of Inflation

3.2.1 Self-Coordination of Heterogeneous Agents

To see whether decentralized agents are able to coordinate intelligently to single out a Pareto-superior equilibrium rather than be trapped in a Pareto-inferior equilibrium, Arifovic (1995) proposed the first agent-based modification of an OLG model of inflation. She applied genetic algorithms (GAs) towards modeling the learning and adaptive behavior of households. In her study, GA-based agents were shown to be able to select the Pareto-superior equilibrium. She further compared the simulation results based on GAs with those from laboratories with human subjects, concluding that GAs were superior to other learning schemes, such as the recursive least squares.

This line of research was further carried out in Dawid (1996), Bullard and Duffy (1998), Bullard and Duffy (1998), Bullard and Duffy (1999), and Birchenhall and Lin (2002). Bullard and Duffy (1999) made the distinction between two implementations of GA learning: depending on what to encode, GA learning can be implemented in two different ways, namely, learning how to optimize (Arifovic 1995) and learning how to forecast (Bullard and Duffy 1999). It was found that these two implementations lead to the same result: agents can indeed learn the Parato superior equilibrium. The only difference is the speed of convergence. The learning how to forecast version of genetic algorithm learning converges faster than the learning how to optimize implementation studied by Arifovic (1995). Nevertheless, a robust analysis showed that coordination was more difficult when the number of inflation values considered (search space) by agents was higher, when government deficits increased, and when agents entertained inflation rate forecasts outside the bounds of possible stationary equilibria.

Chen and Yeh (1999) generalized Bullard and Duffy (1999)'s learning how to forecast version of GA learning with GP. In Bullard and Duffy (1999), what learning agents learn is just a number of the inflation rate rather than a regularity about the motion of the inflation rate, which is a function. Chen and Yeh (1999) considered it too restrictive to learn just a number. From Grandmont (1985), if the equilibrium of an OLG is characterized by limit cycles or strange attractors rather than by fixed points, then what agents need to learn is not just a number, but a functional relationship, such as $x_t = f(x_{t-1}, x_{t-2}, ...)$. Chen and Yeh (1999) therefore generalized Bullard and Duffy (1999)'s evolution of "beliefs" from a sequence of populations of numbers to a sequence of populations of functions. Genetic programming serves as a convenient tool to make this extension.

The basic result observed in Chen and Yeh (1999) is largely consistent with Arifovic (1994) and Bullard and Duffy (1999), namely, agents being able to coordinate their actions to achieve the Pareto-superior equilibrium. Furthermore, their experiments showed that the convergence is not sensitive to the initial rates of inflation. Hence, the Pareto-superior equilibrium has a large domain of attraction. A test on a structural change (a change in deficit regime) was also conducted. It was found that GP-based agents were capable of converging very fast to the new low-inflationary stationary equilibrium after the new deficit regime was imposed. However, the basic result was not insensitive to the dropping of the survival-of-the-fittest principle. When that golden principle was not enforced, we experienced dramatic fluctuations of inflation and occasionally the appearance of super inflation. The agents were generally worse off.

Bullard and Duffy (1998) extended Arifovic (1995)'s two-period OLG model to an n-period one. As Bullard and Duffy (1999), agents based their saving decision their forecasts of the inflation rate. The forecast rules were encoded with strings of L binary bits. The first $L-1$ bits specify *whether or not to include certain lagged values* in either a linear least-squares auto-regression on past price level or past first differences in prices. The last bit specifies whether the auto-regression is to be performed using price *level* or *first difference* in price. What we have here is a way of *using genetic algorithms to encode a set of function forms*. A part of the strings is used to encode the *function* to be included, a part of the strings is used to decide which *variables* to be used, and the last part of the strings is the choice of a data preprocessor. This usage of GAs does not directly encode the objects. Instead, it gives them a number, and the bit strings are used to encoded the number only. This certainly enhance the expression power of GA, and is an alternative for GP.

The model parameters are estimated using the first half of the available data on past prices. The criterion used to assess forecast accuracy is the MSE between the model forecasts and the actual data over the second half of the price data set. It was found that for a relatively low value of n, the system is more likely to achieve coordination on the low inflation stationary perfect foresight equilibrium, which is consistent with the findings of the earlier analyses in two-period OLG economies. However, as n increases we see that persistent currency collapse outcomes become increasingly likely.

The pioneering works by Arifovic popularized a standard procedure, also known as *the augmented genetic algorithms*, to evolve a population of agents, namely,

$$\textbf{Reproduction} \rightarrow \textbf{Crossover} \rightarrow \textbf{Mutation} \rightarrow \textbf{Election},$$

or, written in a composite function,

$$\textbf{Election}\ (\textbf{Mutation}\ (\textbf{Crossover}\ (\textbf{Reproduction}))).$$

However, other variants also exist. Bullard and Duffy (1998) treated *imitation* (*reproduction*) and *innovation* (*crossover* and *mutation*) as two separate learning processes, and run a parallel procedure on both of them. Each process will produce one decision rule, and adaptive agent will decide which she should follow by the election operator. Therefore, their procedure is

$$\textbf{Election}\ ((\textbf{Reproduction}),\ (\textbf{Mutation}\ (\textbf{Crossover}))).$$

We see no particular reason why these two procedures could result in different outcomes. In particular, in both procedure, the election operator used as the last step gives the same protection against the disturbance from innovation.

Birchenhall and Lin (2002) provided perhaps the most extensive coverage of robustness checks ever seen in agent-based macroeconomic models. Their work covers two different levels of GA designs: one is genetic operators, and the other is architecture. For the former, they consider different implementations of the four main GA operators, i.e., selection, crossover, mutation, and election. For the latter, they consider a single-population GA (social learning) vs. a multi-population GA (individual learning). They found that Bullard and Duffy's results are sensitive to two main factors: the election operator and architecture. Their experimental results in fact lend support to some early findings, e.g., the significance of the election operator (Arifovic 1994) and the different consequences of social learning and individual learning (Vriend 2001). What is particularly interesting is that individual learning reduces the rate of convergence to the same belief. This is certainly an important finding, because most studies on the convergence of GAs to Pareto optimality are based on the social learning version. For more discussion on the distinction between individual learning and social learning, see Chen (2002).

3.2.2 Cyclical Equilibria

Bullard and Duffy (1998) studied a more complicated version of the two-period OLG model based on Grandmont (1985). They consider the following utility function for the households,

$$U(c_t^1, c_t^2) = \frac{\ln(c_t^1)^{1-\rho_1}}{1-\rho_1} + \frac{\ln(c_t^2)^{1-\rho_2}}{1-\rho_2} \tag{30}$$

Under time-separable preferences and provided that the value of the coefficient of relative risk aversion for the old agent (ρ_2) is high enough and that of the young agents is low enough (ρ_1), Grandmont (1985) showed that stationary perfect-foresight equilibria also may exist in which the equilibrium dynamics are characterized either as *periodic* or *chaotic trajectories* for the inflation rate, and these complicated stationary equilibria are also Pareto optimal. To have these possibilities, they set ρ_2 equal to 2 and then increased the value of this preference parameter up to 16 by increments of 0.1, while fixed ρ_1 at 0.5 in all cases.

The forecast rule considered by Bullard and Duffy (1998) is to use the price level that was realized $k+1$ periods in the past as the forecast of next period's price level, namely,

$$P^e_{i,t} = P_{t-k-1}, \quad k \in [0, \bar{k}]. \tag{31}$$

In their case, $\bar{k}$ was set to 256, which allows the agents to take actions consistent with a periodic equilibrium of an order as high as 256. Alternatively, agent i's forecast of the gross inflation factor between dates t and $t+1$ is given by

$$\pi^e_{i,t} = \frac{P_{t-k-1}}{P_{t-1}}. \tag{32}$$

As usual, the lifetime utility function was chosen as the fitness function to evaluate the performance of a particular forecast rule. Instead of roulette wheel selection, tournamnet selection was applied to create the next generation.

It was found that the stationary equilibria on which agents coordinate were always relatively simple - either a steady state or a low-order cycle. For low values of ρ_2, in particular, those below 4.2, they observed convergence to the monetary steady state in every experiment, which is the same prediction made by the limited backward perfect-foresight dynamics. As ρ_2 was increased further, the limiting backward perfect foresight dynamics displayed a bifurcation, with the monetary steady state losing stability and never regaining it for values of $\rho_2 \geq 4.2$. However, in their system with learning, the monetary steady state was always a limit point in at least 1 of the 10 experiments conducted for each different value of ρ_2. Also, for $\rho_2 \geq 4.2$, their system often converged, in at least one experiment, to a period-2 stationary equilibrium, even in cases in which that equilibrium, too, had lost its stability in the backward perfect-foresight dynamics.

It is difficult, however, for an economy comprised of optimizing agents with initial heterogeneous beliefs to coordinate on especially complicated stationary equilibria, such as the period-k cycles where $k \geq 3$. In particular, the period-3 cycle that is stable in the backward perfect-foresight dynamics for values $\rho_2 \geq 13$ was never observed in their computational experiments. Interesting enough, three is the last entry of *Sarkovskii's ordering*, whereas one, two and four are first few entries.

They also found that the time it took agents to achieve coordination tended to increase with the relative risk aversion of the old agents over a large portion of the parameter space. Usually, it was the case when the system converged to the period-2 cycle. Moreover, when cycles exist, the transient dynamics of their systems could display qualitatively complication dynamics for long periods of time before eventually to relatively simple, low-periodicity equilibria.

3.2.3 Sunspots

A related phenomenon to cyclical equilibria is *sunspot equilibria*. The sunspot variable is the variable which has no intrinsic influence on an economy, i.e., it has nothing to do with an economy's fundamentals. Sunspot equilibria exist if the sunspot variable can impact the economy simply because a proportion of agents believe so and act accordingly to their belief. Azariadis and Guesnerie (1986) showed that the connection between cyclical and sunspot equilibria are very close. They proved that a two-state stationary sunspot equilibrium exists if and only if a period-2 equilibrium exists. Dawid (1996) started with an OLG model of inflation comparable to Bullard and Duffy (1998).

He studied an economy whose households have the following utility function,

$$U(c_t^1, c_t^2) = 0.1[c_t^1]^{0.9} + 10 - [\frac{10}{1 + c_t^2}]^2 \tag{33}$$

This utility function has the property that the concavity with respect to c_t^1 is much smaller than the concavity with respect to c_t^2, which is necessary for the existence of a periodic equilibrium (Grandmont 1985).

He first found that in cases where periodic equilibria exist, households' beliefs were successfully coordinated to the period-2 cycle rather than the steady state. He then assumed all households to be sunspot believers and showed that households' beliefs converged to the sunspot equilibrium. In that case, the observed values of the price levels are completely governed by something which has nothing to do with the economy's fundamentals. Finally, he relaxed the assumption by simulating an explicit contest between sunspot believers and sunspot agnostics. The simulation showed that in most cases, the population consisted, after a rather short period, only of households whose actions depended on the value of the sunspot variable.

4 Arbitrage and Foreign Exchange Rate Fluctuations

Another popular class of OLG models to which an agent-based approach is applied is the the OLG model of foreign exchange rates, which is a version of the two-country OLG model with fiat money (Kareken and Wallance 1981).

4.1 **The OLG Model of the Exchange Rate**

There are two countries in the model. The residents of both countries are identical in terms of their preferences and lifetime endowments. The basic description of each country is the same as the single-country OLG model. Each household of generation t is is endowed with e^1 units of a perishable consumption good at time t, and e^2 of the good at time $t+1$, and consumes c_t^1 of the consumption good when young and c_t^2 when old. Households in both countries have common preferences given by

$$U(c_t^1, c_t^2) = \ln(c_t^1) + \ln(c_t^2). \tag{34}$$

A government of each country issues its own unbacked currency, $H_{1,t}$ and $H_{2,t}$. Households can save only through acquiring these two currencies. There are no legal restrictions on holdings of foreign currency. Thus, the residents of both countries can freely hold both currencies in their portfolios. A household at generation t solves the following optimization problem at time t:

$$\max_{(c_{i,t}^1, m_{i,1,t})} \quad \ln(c_{i,t}^1) + \ln(c_{i,t}^2) \tag{35}$$

$$s.t. \;\; c_{i,t}^1 + \frac{m_{i,1,t}}{P_{1,t}} + \frac{m_{i,2,t}}{P_{2,t}} = e^1, \;\; c_{i,t}^2 = e^2 + \frac{m_{i,1,t}}{P_{1,t+1}} + \frac{m_{i,2,t}}{P_{2,t+1}}, \tag{36}$$

where $m_{i,1,t}$ is household i' nominal holdings of currency 1 acquired at time t, $m_{i,2,t}$ is household i' nominal holdings of currency 2 acquired at time t, $P_{1,t}$ is the nominal price of the good in terms of currency 1 at time t, and $P_{2,t}$ is the nominal price of the good in terms of currency 2 at time t. The savings of household i at time t by $s_{i,t}$ is

$$s_{i,t} = e^1 - c_{i,t}^1 = \frac{m_{i,1,t}}{P_{1,t}} + \frac{m_{i,2,t}}{P_{2,t}}. \tag{37}$$

The exchange rate e_t between the two currencies is defined as $e_t = P_{1,t}/P_{2,t}$. When there is no uncertainty, the return on the two currencies must be equal,

$$R_t = R_{1,t} = R_{2,t} = \frac{P_{1,t}}{P_{1,t+1}} = \frac{P_{2,t}}{P_{2,t+1}}, \quad t \geq 1, \tag{38}$$

where $R_{1,t}$ and $R_{2,t}$ are the gross real rate of return between t and $t+1$, respectively. Rearranging (38), we obtain

$$\frac{P_{1,t+1}}{P_{2,t+1}} = \frac{P_{1,t}}{P_{2,t}} \quad t \geq 1. \tag{39}$$

From equation (39) it follows that the exchange rate is constant over time:

$$e_{t+1} = e_t = e, \quad t \geq 1. \tag{40}$$

Savings demand derived from household's maximization problem is given by

$$s_{i,t} = \frac{m_{i,1,t}}{p_{1,t}} + \frac{m_{i,2,t}}{p_{2,t}} = \frac{1}{2}[e^1 - e^2\frac{1}{R_t}]. \tag{41}$$

Aggregate savings of the world at time period t, S_t, are equal to the sum of their savings in terms of currency 1, $S_{1,t}$, and in terms of currency 2, $S_{2,t}$. With the homogeneity assumption, we can have

$$S_{1,t} = \sum_{i=1}^{2N} \frac{m_{i,1,t}}{P_{1,t}} = \frac{2Nm_{1,t}}{P_{1,t}}, \tag{42}$$

and

$$S_{2,t} = \sum_{i=1}^{2N} \frac{m_{i,2,t}}{P_{2,t}} = \frac{2Nm_{2,t}}{P_{2,t}}. \tag{43}$$

The equilibrium condition in the loan market requires

$$S_t = S_{1,t} + S_{2,t} = N[e^1 - e^2\frac{P_{1,t+1}}{P_{1,t}}] = \frac{H_{1,t} + H_{2,t}e}{P_{1,t}}. \tag{44}$$

4.1.1 Indeterminacy of the Exchange Rate

Equation (44) only informs us of the real saving in terms of the real world money demand. This equation alone cannot determine the household real demands for each currency. Hence, this equation cannot uniquely determine a set of price $(P_{1,t}, P_{2,t})$, and leave the exchange rate indeterminate as well. This is known as the famous *indeterminacy of exchange rate proposition*. The proposition says that if there exists a monetary equilibrium in which both currencies are valued at some exchange rate e, then there exists a monetary equilibrium at any exchange rate $\hat{e} \in (0, \infty)$ associated with a different price sequence $\{\hat{P}_{1,t}, \hat{P}_{2,t}\}$ such that

$$R_t = \frac{P_{1,t}}{P_{1,t+1}} = \frac{P_{2,t}}{P_{2,t+1}} = \frac{\hat{P}_{1,t}}{\hat{P}_{1,t+1}} = \frac{\hat{P}_{2,t}}{\hat{P}_{2,t+1}}, \tag{45}$$

and

$$S_t = \frac{H_{1,t} + H_{2,t}e}{P_{1,t}} = \frac{H_{1,t} + H_{2,t}\hat{e}}{\hat{P}_{1,t}}, \tag{46}$$

where

$$\hat{P}_{1,t} = \frac{H_{1,t} + \hat{e}H_{2,t}P_{1,t}}{H_{1,t} + eH_{2,t}}, \quad \hat{P}_{2,t} = \frac{\hat{P}_{1,t}}{\hat{e}}. \tag{47}$$

4.1.2 Indeterminacy of the Price Level and the Portfolio

Rearranging Equation (44), one can derive the law of motion of $P_{1,t}$.

$$P_{1,t+1} = \frac{e^1}{e^2} P_{1,t} - \frac{H_{1,t} + eH_{2,t}}{Ne^2} \tag{48}$$

For any given exchange rate e, this economy with constant supplies of both currencies, H_1 and H_2, has a steady-state equilibrium, namely,

$$P_{1,t+1} = P_{1,t} = P_1^* = \frac{H_1 + eH_2}{N(e^1 - e^2)} \tag{49}$$

Like e, the level of P_1^* is also indeterminate. In addition, since households are indifferent between the currencies that have the same rates of return in the homogeneous-expectations equilibrium, the OLG model in which agents are rational does not provide a way to determine the portfolio $\lambda_{i,t}$, which is the fraction of the savings placed into currency 1.

4.2 Agent-Based OLG Models of the Exchange Rate

4.2.1 Exchange Rate Dynamics

In order to examine the behavior of the exchange rate and the associated price dynamics, Arifovic (1996) initiated the agent-based modeling of the exchange rate in the context of the OLG model. In the OLG model of the exchange rate, households have two decisions to make when they are young, namely, saving ($s_{i,t}$) and portfolio ($\lambda_{i,t}$). These two decisions were encoded by concatenation of two binary strings, the first of which encoded $s_{i,t}$, whereas the second of which encoded $\lambda_{i,t}$. The single-population augmented genetic algorithm was then applied to evolve these decision rules. The length of a binary string, l, is 30: The first 20 elements of a string encode the first-period consumption of agent i of generation t; the remaining 10 elements encode the portfolio fraction of agent i.

$$\underbrace{010100...110}_{20 \quad bits: s_{i,t}} \; \underbrace{101..001}_{10 \quad bits: \lambda_{i,t}}$$

While Equation (40) predicts the constancy of the exchange rate, genetic algorithm simulations conducted by Arifovic (1996) indicated no sign of the setting of the exchange rate to a constant value. Instead, they showed persistent fluctuations of the exchange rate. Adaptive economic agents in this model can, in effect, endogenously generate *self-fulfilling arbitrage opportunities*, which in turn make exchange rates continuously fluctuate.

The fluctuating exchange rate was further examined using formal statistical tests in both Arifovic (1996) and Arifovic and Gencay (2000). First, in

Arifovic (1996), the stationarity test (the Dickey-Fuller test) was applied to examine whether the exchange rate series is nonstationary. The result of the test did not indicate nonstationarity. Second, Arifovic and Gencay (2000) analyzed the statistical properties of the exchange rate returns, i.e., the logarithm of e_t/e_{e-1}. The independence tests (the Ljung-BOx-Pierce test and the BDS test) clearly rule out the lack of persistence (dependence) in the return series. Third, they plotted the phase diagrams of the return series and found that there is a well-defined attractor for all series. The shapes of the attractor are robust to the changes in the OLG model parameters as well as to the changes in the GA parameters. Fourth, to verify that this attractor is chaotic, the largest two Lyapunov exponents were calculated. The largest Lyapnov exponent is positive in all series, which supports that attractors under investigation are chaotic. Finally, volatility clustering was also found to be significant in the return series. This series of econometric examinations confirms that agent-based modeling is able to replicate some stylized facts known in financial markets.

4.2.2 *Currency Attacks and Collapse*

Arifovic (2002) considered a different applications of GAs to modeling the adaptive behavior of household. Instead of savings and portfolio decision rules, she turned to the forecasting behavior of households. The forecasting models of exchange rates employed by agents are simple moving-average models. They differ in the rolling window size, which are endogenously determined and can be time-variant. What is encoded by GAs is the size of the rolling window rather than the usual savings and portfolio decision. Simulations with this new coding scheme resulted in the convergence of the economies to a single-currency equilibrium, i.e., the collapse of one of the two currencies. This result was not found in Arifovic (1996). This study therefore shows that different implementations of GA learning may have non-trivial effects on the simulation results. In one implementation, one can have persistent fluctuation of the exchange rate (Arifovic (1996)); in another case, one can have a single-currency equilibrium.

Following the design of Franke (1998), Arifovic (2002) combined two different applications of GA learning. In addition to the original population of agents, who are learning how to forecast, she added another population of agents, who are learning how to optimize. Nevertheless, unlike Franke (1998), these two population of agents did not compete with each other. Instead, they underwent separate genetics algorithm updating. Simulations with these two separate evolving populations did not have the convergence to single currency equilibrium, but were characterized instead by persistent fluctuation.

A different scenario of the currency collapse is also shown in Arifovic (2001), which is an integration of the OLG model of exchange rate with the

OLG model of inflation. In this model, the governments of both countries have constant deficits ($G_i, i = 1, 2$) which were financed via seignorage,

$$G_i = \frac{H_{i,t} - H_{i,t-1}}{P_{i,t}}, \quad i = 1, 2. \tag{50}$$

Combining Equations (44) and (50) gives the condition for the monetary equilibrium in which both governments finance their deficits via seignorage:

$$G_1 + G_2 = S_t - S_{t-1}R_{t-1}. \tag{51}$$

This integrated model inherits the the indeterminacy of the exchange rate from the OLG model of the exchange rate and the indeterminacy of the inflation rate from the OLG model of inflation. Any constant exchange rate e ($e \in (0, \infty)$) is an equilibrium that supports the same stream of government deficits (G_1, G_2), and the same equilibrium gross rate of return (and thus the same equilibrium savings). The existence of these equilibrium exchange rates indicates that the currencies of both countries are valued despite the difference of the two countries' deficits. In fact, in equilibrium the high-deficit country and the low-deficit county experience the same inflation rate, and hence so do their currencies' rates of return. Nonetheless, since the high-deficit country has a higher money supply, if both currencies are valued, then the currency of the high-deficit country will eventually drive the currency of the low-deficit country out of households' portfolios. Given this result, it might be in the interest of a country with lower deficits to impose a degree of capital control.

Arifovic (2001) showed that agent-based dynamics behave quite different from the above homogeneous rational expectations equilibrium analysis. In her agent-based environment, the evolution of households' decision rules of savings and portfolios results in a flight away from the currency used to finance the larger of the two deficits. In the end, households hold all of their savings in the currency used to finance the lower of the deficits. Thus, the economy converges to the equilibrium in which only the low-deficit currency is valued. The currency of the country that finances the larger of the two deficits become valueless, and we have a single-currency equilibrium again.

5 Artificial Stock Markets

Among all applications of the agent-based approach to macroeconomic modeling, the most exciting one is the *artificial stock market*. By all standards, the stock market is qualified to be a complex adaptive system. However, conventional financial models are not capable of demonstrating this feature. On the contrary, the famous no-trade theorem shows in equilibrium how inactive this market can be (Tirole 1982). It was therefore invigorating when John Holland and Brian Arthur established an economics program at the Santa Fe Institute in 1988 and chose artificial stock markets as their initial research project. The SFI artificial stock market is built upon the standard asset pricing model (Grossman 1976, Grossman and Stiglitz 1980). What one can possibly learn from this novel approach was well summarized in Palmer et al. (1994), which is in fact the first journal publication on an agent-based artificial stock market. A series of follow-up studies materialized the content of this new fascinating frontier in finance.

5.1 Agent Engineering and Trading Mechanisms

Agent-based artificial stock markets have two main stays: agent engineering and institution (trading mechanism) designs. Agent engineering mainly concerns the construction of the financial agents. Tayler (1995) showed how to use genetic algorithms to encode trading strategies of traders. A genetic fuzzy approach to modeling trader's behavior was shown in Tay and Linn (2001), whereas the genetic neural approach was taken by LeBaron (2001). To simulate the agent-based artificial stock market based on the standard asset pricing model, the AI-ECON Research Center at National Chengchi University developed software known as the **AI-ECON artificial stock market** (**AIE-ASM**). The AIE artificial stock market differs from the SFI stock market in the computational tool that is employed. The former applies genetic programming, while the latter has genetic algorithms. In AIE-ASM, genetic programming is used to model agents' expectations of the price and dividends. A menu-like introduction to AIE-ASM Ver. 2 can be found in Chen, Yeh and Liao (2002).

In Chan et al. (1999) and Yang (2001) we see a perfect example of bringing different learning schemes into the model. The learning schemes incorporated into Chan et al. (1999) include empirical Bayesian traders, momentum traders, and nearest-neighbor traders, where those included in Yang (2001) are neural networks traders and momentum traders. LeBaron (1999) gave a more thorough and general discussion of the construction of artificial financial agents. In addition to models, data is another dimension of agent engineering. What can be addressed here is the issue of stationarity that the series traders are looking at. Is the entire time series representative of the same dynamic process, or have things changed in the recent past? LeBaron (2001) studied traders who are initially heterogeneous in perception with different time horizons, which characterize their interpretation of how much of the past is relevant to the current decision making.

Chen and Yeh (2001) contributed to agent engineering by proposing a modified version of social learning. The idea is to include a mechanism, called the *business school*. Knowledge in the business school is open for everyone.

Traders can visit the business school when they are under great survival pressure. The social learning version of genetic programming is applied to model the evolution of the business school rather than directly on traders. Doing it this way, one can avoid making an implausible assumption that trading strategies, as business secrets, are directly imitable. Yeh and Chen (2001a) further combined this modified social learning scheme with the conventional individual learning scheme in an integrated model. In this integrated model a more realistic description of traders' learning behavior is accomplished: the traders can choose to visit the business school (learning socially), to learn exclusively from their experience (learning individually), or both. In their experiments, based on the effectiveness of different learning schemes, traders will switch between social learning and individual learning. Allowing such a competition between these two learning styles, their experiment showed that it is the individual learning style which won the trust of the majority. To the best of our knowledge, this is the only study which leaves the choice of the two learning styles to be endogenously determined. Other aspects of agent engineering studied include search intensity, psychological pressure, and prudence. (Chen and Yeh 2000b, Chen and Yeh 2000c)

The second component of agent-based stock markets is the institutional design. An institutional design should answer the following five questions: who can trade, when and how can orders be submitted, who may see or handle the orders, how are orders processed, and how are prices eventually set. Trading institutional designs in the conventional SFI artificial stock market either follow the Walrasian tatonnement scheme or the rationing scheme. Chan et al. (1999) and Yang (2001), however, considered a double auction mechanism. This design narrows the gap between artificial markets and the real market, and hence makes it possible to compare the simulation results with the behavior of real data, e.g., tick-by-tick data. Since stock market experiments with human subjects were also conducted within the double auction framework (Smith, Suchanek and Williams 1988), this also facilitates the conversation between the experimental stock market and the agent-based artificial stock market.

Based on agent engineering and trading mechanism designs, agent-based artificial stock markets can generate various market dynamics, including price, trading volumes, the heterogeneity and complexity of traders' behavior, and wealth distribution. Among them, price dynamics is the one under the most intensive study. This is not surprising, because ever since the 1960s price dynamics has been the focus of studies on random walks, the efficient market hypothesis, and market rationality (the rational expectations hypothesis). With the advancement of econometrics, it further became the focus of the study of non-linear dynamics in the 1980s.

5.2 Mispricing

Agent-based artificial stock markets make two important contributions to our understanding of the behavior of stock prices. First, they enable us to understand what may cause the price to deviate from rational equilibrium price or the so-called fundamental value.

Both Yang (2001) and Chan et al. (1999) discussed the effect of momentum traders on price deviation. Yang (2001) found that the presence of momentum traders can drive the market price away from the homogeneous rational equilibrium price. Chan et al. (1999) had a similar finding: adding momentum traders to a population of empirical Bayesian has an adverse impact on market performance, although price deviation decreased as time went on. LeBaron (2001) inquired whether agents with a long-horizon perception can learn to effectively use their information to generate a relatively stable trading environment. The experimental results indicated that while the simple model structure with fixed long horizon agents replicates the usual efficient market results, the route to evolving a population of short horizon agents to long horizons may be difficult. Arthur et al. (1997) and LeBaron, Arthur and Palmer (1999) found that when the speed of learning (the length of a genetic updating cycle) decreased (which forces agents to look at longer horizon features), the market approached the REE.

Chen and Liao (2002a) is another study devoted to price deviation. They examined how well a population of financial agents can track the equilibrium price. By simulating the artificial stock market with different dividend processes, interest rates, risk attitudes, and market sizes, they found that the market price is not an unbiased estimator of the equilibrium price. Except in a few extremely bad cases, the market price deviates from the equilibrium price moderately from minus four percent to positive sixteen percent. The pricing errors are in fact not patternless. They are actually negatively related to market sizes: a thinner market size tends to have a larger pricing error, and a thicker market tends to have a smaller one. For the thickest market which they have simulated, the mean pricing error is only 2.17%. This figure suggests that the new classical simplification of a complex world may still provide a useful approximation if some conditions are met, such as in this case, the market size.

5.3 Complex Dynamics

As to the second contribution, agent-based artificial stock markets also enhance our understanding of several stylized features well documented in financial econometrics, such as fat tails, volatility clusters, and non-linear dependence. LeBaron, Arthur and Palmer (1999) showed that the appearance of the ARCH effect and the non-linear dependence can be related to the speed of learning. Yang (2001) found that the inclusion of momentum traders generates a lot of stylized features, such as excess volatility, excess kurtosis (leptokurtotic), lack of serial independence of return, and high trading volume.

Another interesting line is the study of emergent properties within the context of artificial stock markets. Emergence is about "how large interacting ensembles exhibit a collective behavior that is very different from anything one may have expected from simply scaling up the behavior of the individual units" (Krugman (1996); p.3). Consider the efficient market hypothesis (EMH) as an example. If none of the traders believe in the EMH, then this property will not be expected to be a feature of their collective behavior. Thus, if the collective behavior of these traders indeed satisfies the EMH as tested by standard econometric procedures, then we would consider the EMH as an emergent property. As another example, consider the rational expectations hypothesis (REH). It would be an emergent property if all our traders are boundedly rational, with their collective behavior satisfying the REH as tested by econometrics.

Chen and Yeh (2002) applied a series of econometric tests to show that the EMH and the REH can be satisfied with some portions of the artificial time series. However, by analyzing traders' behavior, they showed that these aggregate results cannot be interpreted as a simple scaling-up of individual behavior. The main feature that produces the emergent results may be attributed to the use of genetic programming, which allows us to generate a very large search space. This large space can potentially support many forecasting models in capturing short-term predictability, which makes simple beliefs (such as that where the dividend is an iid series, or that when the price follows a random walk) difficult to be accepted by traders. In addition to preventing traders from easily accepting simple beliefs, another consequence of a huge search space is the generation of sunspot-like signals through mutually-reinforcing expectations. Traders provided with a huge search space may look for something which is originally irrelevant to price forecasts. However, there is a chance that such kinds of attempts may mutually become reinforced and validated. The generation of sunspot-like signals will then drive traders further away from accepting simple beliefs.

Using Granger causality tests, Chen and Yeh (2002) found that dividends indeed can help forecast returns. By their experimental design, the dividend does not contain the information of future returns. What happens is a typical case of mutually-supportive expectations that make the dividend eventually contain the information of future returns.

As demonstrated in Chen and Yeh (2001) and Chen and Yeh (2002), one of the advantages of agent-based computational economics (the bottom-up approach) is that it allows us to observe what traders are actually thinking and doing. Are they martingale believers? Are they sunspot believers? Do they believe that trading volume can help predict returns? By counting the number of traders who actually use sunspots or trading volumes to forecast returns, one can examine whether sunspots' effects and the causal relation between stock returns and trading volume can be two other emergent properties (Chen and Liao 2002b, Chen and Liao 2002c).

5.4 Market Diversity and Market Efficiency

Yeh and Chen (2001b) examined another important aspect of agent engineering, i.e., *market size* (number of market participants). Few studies have addressed the significance of market size on the performance of agent-based artificial markets. One good exception is Bell and Beare (2002), whose simulation results showed that the simple tradable emission permit scheme (an auction scheme) can be the most effective means for pollution control when the number of participants is small. However, as the number of participants increases, its performance declines dramatically and becomes inferior to that of the uniform tax scheme. Another exception is Bullard and Duffy (1999). In most studies, the number of market participants is usually determined in an arbitrary way, mainly constrained by the computational load. Arifovic (1994), however, justified the number of participants from the viewpoint of search efficiency. She mentioned that the minimal number of strings (agents) for an effective search is usually taken to be 30 according to artificial intelligence literature. Nonetheless, agent-based artificial markets have different purposes and concerns.

Related to market size is *population size*. In the case of social learning (single-population GA or GP), market size is the same as population size. However, in the case of individual learning (multi-population GA or GP), population size refers to something different, namely, the number of solution candidates each trader has. Like market size, population size is also arbitrarily determined in practice.

Yeh and Chen (2001b) studied the effect of market size and population size upon market efficiency and market diversity under social and individual learning styles. Their experimental results obtained can be summarized as two effects on market efficiency (price predictability), namely, the *size effect* and the *learning effect*. The size effect says that the market will become efficient when the number of traders (market size) and/or the number of models (GP trees) processed by each trader (population size) increases. The learning effect says that the price will become more efficient if traders' adaptive behavior becomes more independent and private. Taking a look at market diversity, we observe very similar effects except for population size: market diversity does not go up with population size. These findings motivate us to search for a linkage between market diversity and market efficiency. A "theorem" may go as follows: a larger market size and a more independent learning style will increase the diversity of traders' expectations, which in turn make the market become more active (high trading volume) and hence more efficient (less predictable). Their simulation results on trading volumes also supported this "theorem". They further applied this "theorem" to explain why the U.S stock market behaves more efficient than Taiwan's stock market.

6 Concluding Remarks

The agent-based approach to macroeconomic modeling has a one-decade history. Its impacts on the mainstream macroeconomics are increasing, and it should play a much more important role in the 21st-century. In this survey article we review the development of the agent-based approach to macroeconomic modeling. We witness how the agent-based approach has revolutionized the conventional macroeconomic general equilibrium analysis built upon the unconvincing assumption of homogeneous agents. Artificial adaptive agents are introduced to model the adaptive behavior of agents placed in a complex environment which may naturally arise when the homogeneity assumption is dropped. The resultant dynamic aggregate behavior can be much more complex than just the homogeneous rational expectations equilibrium. The complex aggregate dynamics are characterized by persistent fluctuations with chaotic or non-linear stochastic properties, which are frequently observed in a real macroeconomic time series. What co-evolves with these complex aggregate complex dynamics is a great diversity of adapting agents, who are continuously reviewing and revising their decision rules.

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