

Volatility trade-offs in exchange rate target zones

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Abstract

The volatility trade-offs (i.e. the negative relationships between exchange rate variability and the interest rate differential) exhibited in the Krugman [Krugman, P. (1991). Target zones and exchange rate dynamics. *Quarterly Journal of Economics*, 106, 669–682.] model depend on the assumption of uncovered interest rate parity (UIP). However, the bands for several economies in Latin America and Eastern Europe are substantially different from those within the European Monetary System (EMS), in that their parity relationship deviates from UIP and volatility trade-offs do not exist. This paper develops a graphical exposition and uses it to show that the degree of capital mobility may serve as a plausible vehicle to explain the empirical evidence found in Krugman's regime of exchange rate target zones. Based on a Fleming-type stochastic macro model, we find that when capital mobility is relatively low, exchange rate variability exhibits a positive relationship with the interest rate differential. This result can be regarded as a possible way of resolving the conflicting outcomes between Krugman's prediction and existing empirical observations.

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1. Introduction

In his pioneering contribution, Krugman (1991) sets up a stochastic model and finds that, in comparison with flexible exchange rates, an announcement of exchange rate target zones tends to lower the volatility of the exchange rate. This result is now dubbed the honeymoon effect. Another important implication of the Krugman (1991) model is that exchange rate variability (i.e. the deviations between the exchange rates and the central parity) is definitely negatively related to the differential between domestic and foreign interest rates.¹ However, evidence from empirical studies points towards a strong empirical rejection of this implication. The non-negative relationship between exchange rate variability and the interest rate differential does not seem to be restricted to countries or areas. Flood, Rose, and

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¹ See Svensson (1991) and Bertola and Svensson (1993) for a more detailed discussion.

Mathieson (1991), Svensson (1991), Bertola and Caballero (1992), Beetsma and Van der Ploeg (1994) and Kempa and Nelles (1999) find that the volatility trade-offs do not exist in the EMS. An analogous finding also emerges in Chile, Israel, Mexico (see, e.g., Helpman, Leiderman, and Bufman (1994)), and Sweden (see, e.g., Lindberg, Söderlind, and Svensson (1993)).

In order to achieve a better fit with the empirical evidence, some economists have attempted to extend Krugman's (1991) model and have instead proposed a variety of explanations. In the literature, Bertola and Caballero (1992), Bertola and Svensson (1993), Tristani (1994), and Werner (1995) consider the possibility of a time-varying realignment. Delgado and Dumas (1992), Beetsma and Van der Ploeg (1994), and Lewis (1995) appeal to the fact that the monetary authority often intervenes in the foreign exchange market in the interior of the exchange rate band (i.e. intra-marginal intervention), rather than at the edges. Kempa, Nelles, and Pierdzioch (1997), Kempa and Nelles (1999), Beetsma and Van der Ploeg (1998), and Belessakos and Giannikos (2002) allow for the sluggish adjustment of commodity prices. In particular, Belessakos and Giannikos (2002) show that when frictions (i.e. sticky prices) are present in the economy, the negative relationship between exchange rate variability and interest rate variability disappears.

Most of the available evidence regarding the exchange rate bands has been based on the experience of Western European (European Monetary System or EMS) economies that have operated their bands under a multilateral commitment to maintain and defend. However, as put forth by Helpman et al. (1994), the key aspect of the Krugman (1991) model, such as the negative relationship between exchange rate variability and the interest rate differential, is that it relies upon the assumption of uncovered interest rate parity (hence a time-invariant foreign exchange risk premium is present). Helpman et al. (1994) further point out that bands for other economies in Latin America and several in Eastern Europe are substantially different from those in the EMS, and their parity relation deviates from the uncovered interest rate parity proposed by Krugman (1991). "The reason is that capital mobility has been less-than-perfect, a consequence of the non-negligible information and transaction costs as well as of the remaining regulations that characterize financial markets." (Helpman et al., 1994, p. 264)

In addition, Reinhart and Reinhart (1999) have also studied the experiences of ten developing countries in Africa, Asia, Eastern Europe, and Latin America. They find that these countries often use reserve requirements to depress capital flow and mitigate the impact on the domestic money supply. Based on the International Monetary Fund's Annual Report on Exchange Arrangements and Exchange Restrictions, Alfaro (2004) develops a capital control index to measure the extent of capital mobility. He finds that in Southern European countries the capital control index exceeds 0.5 during the period 1966–1992, and that in Latin American countries the capital control index exceeds 0.5 during the period 1970–1994. These findings imply that in Southern European and Latin American countries imperfect capital mobility is a common fact.

Given the fact that imperfect capital mobility is observed in the developing countries, this paper sets up a Fleming (1962)-type target zone model that embodies imperfect capital mobility. Based on this model, we show that the degree of capital mobility will play an important role in determining the public's exchange rate expectations, and will hence further influence the relationship between exchange rate variability and the interest rate differential in exchange rate target zones.

It is a common belief that the complexity of the stochastic process is a frequent stumbling block for new readers of the target zone literature (Svensson, 1992). In departing from existing studies, this paper develops a graphical exposition, and uses it to provide an intuitive explanation of the empirical evidence, i.e. the positive relationship between exchange rate volatility and the interest rate differential.² It is important to emphasize that, due to the limitations of graphical analysis, we need to assume that the monetary authorities engage in finite-sized interventions discretely instead of infinitesimal interventions at the target zone boundaries. However, in their frequently cited paper, Flood and Garber (1991, p. 1371) point out that "[f]inite interventions may well be an important part of the story of real-world target zones." In that paper, Flood and Garber (1991) also intuitively show that the smooth pasting condition is present when the amount of discrete intervention approaches zero. This implies that our conclusion would be valid if discrete interventions were to be replaced by infinitesimal interventions.³

² Lai and Chang (2001) propose a graphical illustration to highlight the stabilizing characteristic of *inflation target zones*.

³ In Appendix A, we broaden the assumption of finite-sized interventions to infinitesimal interventions, and make a detailed derivation for the mathematical solution. Our analysis indicates that the results with finite-sized interventions are parallel to those with infinitesimal interventions.

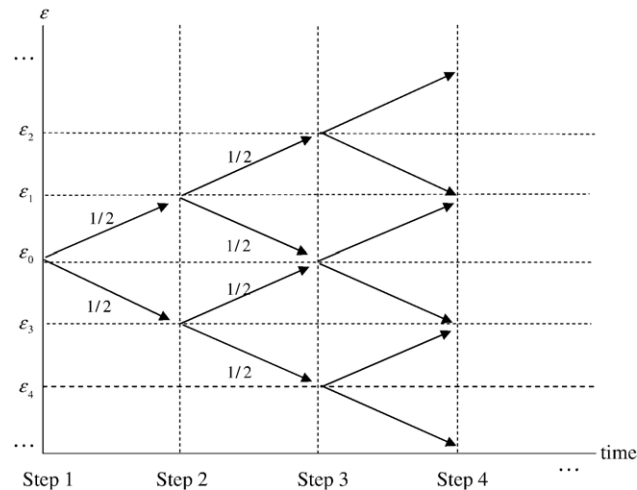


Fig. 1. Random walk representation.

The remainder of this paper is arranged as follows. The theoretical framework is outlined in Section 2. Section 3 analyzes how the degree of capital mobility will govern the ability of exchange rate target zones to stabilize exchange rates. Finally, Section 4 concludes the main findings of our analysis.

2. The theoretical model

In order to sharpen the salient feature of exchange rate target zones, the modeling strategy we adopt is to keep the model as simple as possible. Basically, the paper's theoretical model is an extension of the [Fleming \(1962\)](#)-type economy with imperfect capital mobility.⁴ Assuming that economic agents form their expectations rationally, we can use the following equations to represent this simple stochastic macro model:

$$y = \gamma y - \eta r + (\delta e - \theta y) + \varepsilon; \quad 0 < \gamma < 1, \eta, \delta, \theta > 0 \quad (1)$$

$$m = \phi y - \lambda r; \quad \phi, \lambda > 0 \quad (2)$$

$$(\delta e - \theta y) + \beta[r - r^* - E(de)/dt] = 0; \quad \delta, \theta, \beta > 0 \quad (3)$$

$$d\varepsilon = \sigma dZ. \quad (4)$$

With the exception of the domestic interest rate r and foreign interest rate r^* , all variables are expressed in natural logarithms. The variables are defined as follows: y =real output; e =the exchange rate (number of units of domestic currency per unit of foreign currency); m =the nominal money supply; and ε =random disturbances of commodity demand. In addition, E denotes the expectation operators and σ is the instantaneous standard deviation of the movement of ε .

Eq. (1) is the product market equilibrium condition, which indicates that product supply equals product demand. In Eq. (1) product demand includes consumption expenditure, investment, and the trade balance, where consumption expenditure is positively related to output, investment is negatively related to the interest rate, and the trade balance is specified as an increasing function of the exchange rate and a decreasing function of domestic output. Eq. (2) is the

⁴ Our model is closely related to that of [Belessakos and Giannikos \(2002\)](#). However, there are two differences between these articles with regard to the modeling framework. Firstly, while [Belessakos and Giannikos \(2002\)](#) assume that commodity prices are sticky and adjustable, our model assumes that commodity prices are sticky and disadjustable. Secondly, while [Belessakos and Giannikos \(2002\)](#) specify that capital is perfectly mobile, our model specifies that capital is imperfectly mobile.

money market equilibrium condition, stating that money supply equals money demand. Eq. (3) is the foreign market equilibrium condition, which says that in the exchange rate target zone regime, the overall balance of payments (i.e. the sum of the trade balance and net capital inflow) must be equal to zero. In line with the standard literature (e.g., Frenkel and Rodriguez (1982)), the net capital inflow is specified as a function of the difference between the return on domestic bonds r and the return on foreign bonds $r^* + E(de)/dt$. It should be noted that the coefficient β denotes the degree of capital mobility. Eq. (4) specifies that the stochastic product demand shock ϵ follows a Brownian motion process without drift.

The feature of ϵ as exhibited in Eq. (4) can be expressed in Fig. 1. Without loss of generality, we assume that the change in the product demand shock ϵ follows a discrete-state random walk. To be more specific, in each step the product demand shock ϵ either moves up or down by the same step-length with the same probability of 1/2. As exhibited in Fig. 1, at step 1 the product demand shock begins at a known level ϵ_0 and may move either up to ϵ_1 or down to ϵ_3 by the same distance (i.e. $\epsilon_1 - \epsilon_0 = -(\epsilon_3 - \epsilon_0)$), and each with a probability of 1/2. In addition, it is assumed that the probability of ϵ moving up or down in each step is independent of what happened in the previous steps. Analogously, at step 2 ϵ_1 will move up to ϵ_2 with a probability of 1/2, and will move down to ϵ_0 with a probability of 1/2.

It is clear from Fig. 1 that at any step the mean of the product demand shock ϵ is its initial level. For example, at step 1 the mean of ϵ at ϵ_0 is $\epsilon_0 (= \epsilon_1 \times 1/2 + \epsilon_3 \times 1/2)$; at step 2 the mean of ϵ at ϵ_1 is $\epsilon_1 (= \epsilon_2 \times 1/2 + \epsilon_0 \times 1/2)$. Accordingly, the expected change in ϵ at any step is zero. For example, at step 1 the expected change in ϵ at ϵ_0 is $(\epsilon_1 - \epsilon_0) \times 1/2 + (\epsilon_3 - \epsilon_0) \times 1/2 = 0$, and at step 2 the expected change in ϵ at ϵ_1 is $(\epsilon_2 - \epsilon_1) \times 1/2 + (\epsilon_0 - \epsilon_1) \times 1/2 = 0$.

For notational simplicity, in the following graphical analysis $E(de)/dt$ is denoted by π^e . Substituting the relation $r = (\phi y - m)/\lambda$ in Eq. (2) into Eq. (1) yields:

$$e = \Omega_0 y - \frac{\eta}{\lambda \delta} m - \frac{1}{\delta} \epsilon, \quad (5)$$

where $\Omega_0 = [\lambda(1 - \gamma + \theta) + \eta\phi]/\eta\phi > 0$. From Eq. (5) we can trace the XX schedule, which represents the combinations of y and e that keep both the product and money market in equilibrium. As is evident, the slope of the XX line is:

$$\left. \frac{\partial e}{\partial y} \right|_{XX} = \Omega_0 > 0. \quad (5a)$$

By means of a similar procedure, substituting the relation $r = (\phi y - m)/\lambda$ (reported in Eq. (2)) into Eq. (3) gives:

$$e = \Omega_1 y + \frac{\beta}{\lambda \delta} m + \frac{\beta}{\delta} (\pi^e + r^*), \quad (6)$$

where $\Omega_1 = [(\lambda\theta - \beta\phi)/\lambda\delta]$. From Eq. (6) we can trace the AA schedule, which depicts the pairs of y and e that keep both the money and foreign market in equilibrium. Given $\Omega_1 = [(\lambda\theta - \beta\phi)/\lambda\delta]$, it is clear from Eq. (6) that the slope of the AA locus is:

$$\left. \frac{\partial e}{\partial y} \right|_{AA} = \Omega_1 \begin{cases} > 0; & \text{if } \beta < \frac{\lambda\theta}{\phi} \\ < 0; & \text{if } \beta > \frac{\lambda\theta}{\phi} \end{cases}. \quad (6a)$$

Eq. (6a) indicates that the slope of the AA locus may be positive or negative, depending upon the relative size of β and $\lambda\theta/\phi$. As is obvious, the AA schedule is positively-sloped if capital is relatively immobile (i.e. $\beta < \lambda\theta/\phi$), while a negatively-sloped AA schedule prevails if capital is relatively mobile.

3. The variability pertaining to exchange rate target zones

This section addresses the issue of whether exchange rate target zones stabilize the exchange rate when the economy experiences an aggregate demand shock. Due to the fact that the degree of capital mobility is crucial in determining the

variability of relevant macro variables, in what follows the discussion focuses on two cases: one where capital is relatively immobile ($\Omega_1 > 0$), and the other where capital is relatively mobile ($\Omega_1 < 0$).

3.1. Capital is relatively immobile ($\Omega_1 > 0$)

Assume that the monetary authorities announce that they stand ready to adjust foreign reserves (money supply) when the level of the exchange rate exceeds the upper bound \bar{e} or falls short of the lower bound \underline{e} . However, the monetary authorities do not alter the money stock when the exchange rate is in the interior of the band (\bar{e}, \underline{e}) . Before undertaking our graphical exposition, one point should be mentioned here. Due to the limitations of graphical analysis, we need to assume finite-sized interventions instead of infinitesimal interventions at the target zone boundaries. In Appendix A, we relax the assumption from finite-sized interventions to infinitesimal interventions, and provide a detailed derivation of the mathematical solution. Our analysis indicates that the results with finite-sized interventions are parallel to those with infinitesimal interventions.⁵

We now use a graphical presentation to address the stabilizing effect of exchange rate target zones. Fig. 2 depicts the diagram associated with $\Omega_1 > 0 > 0$, where the AA locus is upward sloping.⁶ For analytical simplicity, assume that initially the demand shock is ϵ_0 and the public's exchange rate expectations are nil (i.e. $\pi^e = 0$). In the upper panel of Fig. 2, the initial equilibrium is established at point E_0 , which is the intersection of the curves $XX(\epsilon_0)$ and $AA(\pi^e = 0)$. The initial output and exchange rate are y_0 and e_0 , respectively, and to make the analysis meaningful, we depict e_0 inside the band. In response to a rise in the demand shock from ϵ_0 to ϵ_1 , the $XX(\epsilon_0)$ curve shifts rightward to $XX(\epsilon_1)$. If the public does not change its expectations (i.e. $\pi^e = 0$), then $XX(\epsilon_1)$ intersects $AA(\pi^e = 0)$ at point E_1 , with y and e being y_1^{FF} and e_1^{FF} , respectively.

Similar to the Brownian motion process ϵ in the above, as illustrated in Fig. 1, two states may occur at the level of random walk ϵ_1 . First, with a probability of 1/2, ϵ_1 will decrease back to ϵ_0 . Second, with a probability of 1/2, ϵ_1 will increase to ϵ_2 . As indicated in the upper panel of Fig. 2, when ϵ_1 decreases back to ϵ_0 , the exchange rate will then fall from e_1^{FF} to e_0 . However, when ϵ_1 increases to ϵ_2 , the exchange rate will rise from e_1^{FF} to the upper edge of the band \bar{e} , rather than e_2 , because the monetary authorities will act to defend the target band.⁷ This implies that, when the shock is ϵ_1 , the public's exchange rate expectations under an exchange rate target zone (TZ) are given by $\pi_{TZ}^e = (\bar{e} - e_1^{FF})/2 + (e_0 - e_1^{FF})/2$. Given $-(e_0 - e_1^{FF}) > (\bar{e} - e_1^{FF})$, $\pi_{TZ}^e < 0$ is then true. The change in expectations from $\pi^e = 0$ to $\pi_{TZ}^e < 0$ will lead $AA(\pi^e = 0)$ to shift downward to $AA(\pi_{TZ}^e < 0)$.⁸ Accordingly, under an exchange target zone regime, the equilibrium in association with the level of random walk ϵ_1 is established at point E_3 , where $AA(\pi_{TZ}^e < 0)$ and $XX(\epsilon_1)$ intersect. At point E_3 , the level of output is y_3^{TZ} and the level of the exchange rate is e_3^{TZ} .

Under the situation where the central bank does not set a specific target zone and allows the exchange rate to adjust freely, with a probability of 1/2 the exchange rate will fall from e_1^{FF} to e_0 when ϵ_1 decreases back to ϵ_0 . With a probability of 1/2, the exchange rate will rise from e_1^{FF} to e_2 when ϵ_1 increases to ϵ_2 . Given $-(e_0 - e_1^{FF}) = (e_2 - e_1^{FF})$, under a floating-exchange rate (FF) regime the public on average expects no change in the exchange rate (i.e. $\pi_{FF}^e = (e_0 - e_1^{FF})/2 + (e_2 - e_1^{FF})/2 = 0$). Accordingly, point E_1 (the intersection of both the $XX(\epsilon_1)$ locus and the $AA(\pi^e = 0)$ locus) is the equilibrium under a floating-exchange rate regime. As is obvious in the upper panel of Fig. 1, in response to a rise in the demand shock from ϵ_0 to ϵ_1 , the change in the exchange rate under a target zone ($e_3^{TZ} - e_0$) is less than that under a floating-exchange rate regime ($e_1^{FF} - e_0$). In addition, the change in output ($y_3^{TZ} - y_0$) under a target zone is also less than that under a floating-exchange rate regime ($y_1^{FF} - y_0$).

We now are ready to discuss domestic interest rate volatility. In the lower panel of Fig. 2, the LM curve traces the pairs of output and interest rates that satisfy the money market equilibrium condition reported in Eq. (2). From the upper panel of Fig. 2, we learn that, following a rise in the demand shock from ϵ_0 to ϵ_1 , output should increase from y_0

⁵ This point is raised by an anonymous referee, to whom we are grateful.

⁶ Given $\beta < \lambda\theta/\phi$, it is clear from Eqs. (5a) and (6a) that:

$$\partial e / \partial y|_{XX} = \Omega_0 = \theta/\delta + (1-\gamma)/\delta + \eta\phi/\lambda\delta > \theta/\delta - \beta\theta/\lambda\delta = \Omega_1 = \partial e / \partial y|_{AA}.$$

Accordingly, in Fig. 2 the XX line is steeper than the AA line.

⁷ This implies that the monetary authorities will sell foreign reserves in the foreign exchange market to keep the exchange rate at the upper edge of the band \bar{e} .

⁸ A reduction in the exchange rate expectations tends to improve the capital account, and results in an improvement in the balance of payments. In order to restore the foreign market equilibrium, other things being equal, the domestic currency should appreciate in response.

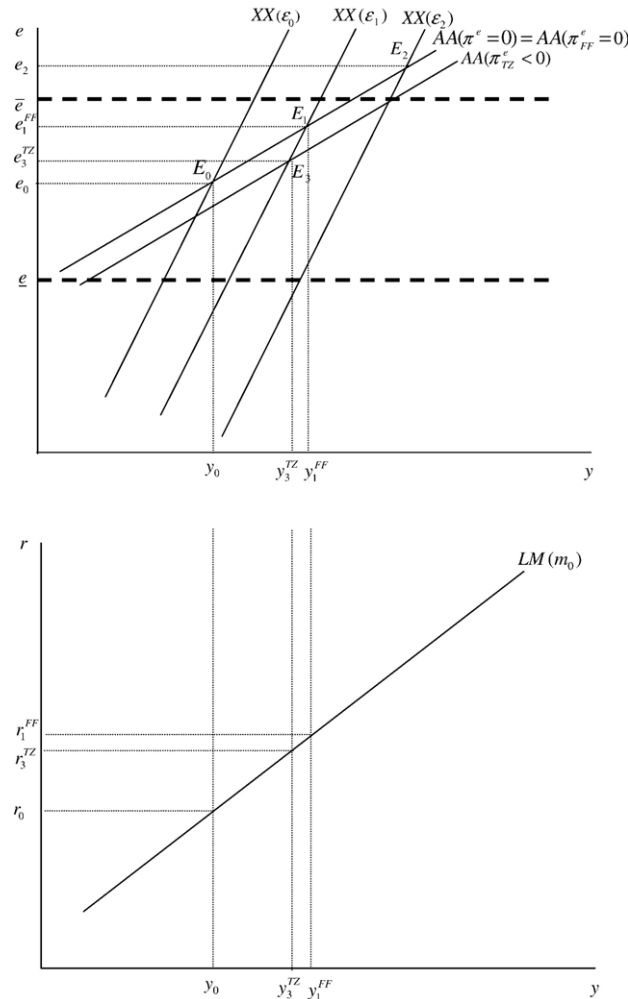


Fig. 2. Capital is relatively immobile.

to y_3^{TZ} under an exchange rate target zone regime. As exhibited in the LM curve, in order to satisfy the money market equilibrium condition, the domestic interest rate should rise from r_0 to r_3^{TZ} under an exchange rate target zone regime. Similarly, we can infer that the domestic interest rate should rise from r_0 to r_1^{FF} under a floating-exchange rate regime. As is evident, in response to a rise in the demand shock from ϵ_0 to ϵ_1 , a change in the domestic interest rate under a target zone ($r_3^{TZ} - r_0$) is less than that under a floating-exchange rate regime ($r_1^{FF} - r_0$). To be more precise, an announcement regarding exchange rate target zones will stabilize both output and the interest rate when capital mobility is relatively low.

In his pioneering model, [Krugman \(1991\)](#) predicts that the economy benefits from lower exchange rate variability at the expense of a higher interest rate differential between domestic and foreign interest rates. However, the [Krugman \(1991\)](#) model does not fit well in empirical work. Some empirical studies (e.g., [Bertola and Svensson \(1993\)](#) and [Kempa and Nelles \(1999\)](#)) indicate that exchange rate variability (i.e. the deviations between the exchange rates and the central parity) is often positively, and only occasionally negatively, related to the interest rate differential. Given that the foreign interest rate is fixed in our model, in comparison with a flexible exchange rate regime, in a target zone regime a reduction of changes in the domestic interest rate is associated with a decrease in the interest rate differential. Consequently, the result exhibited in [Fig. 2](#) reveals that when capital mobility is relatively low, exchange rate variability displays a positive relationship with the interest rate differential. This analytical result can be regarded as a plausible way of solving the conflicting outcome between Krugman's prediction and existing empirical observations in other EMS economies and some developing countries, such as Chile, Israel and Mexico.

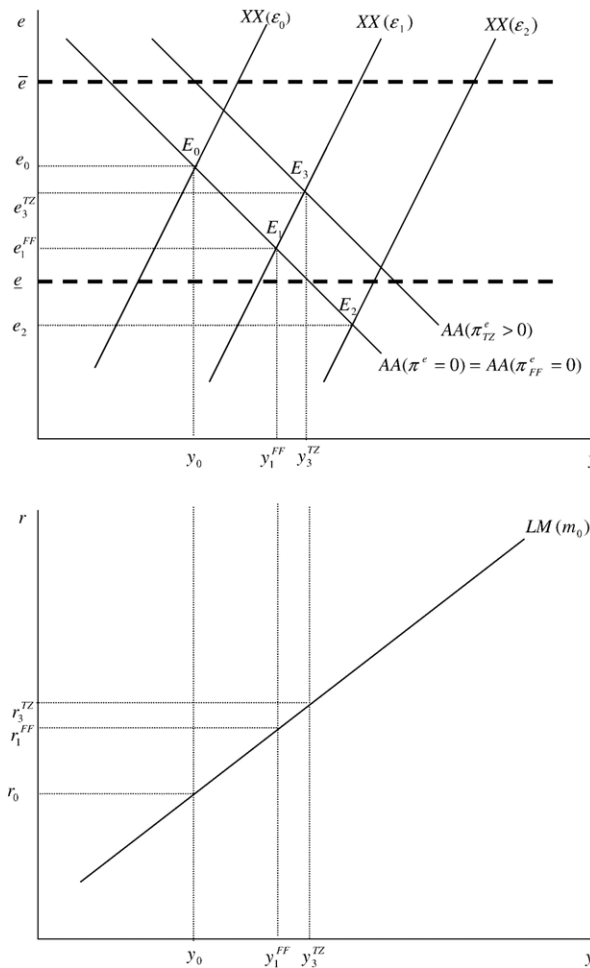


Fig. 3. Capital is relatively mobile.

3.2. Capital is relatively mobile ($\Omega_1 < 0$)

We now deal with the situation where capital mobility is relatively high. Fig. 3 depicts the diagram associated with $\Omega_1 < 0$, where the AA locus is downward sloping. Similar to the inference in Fig. 1, two states may occur at the level of random walk ϵ_1 . Firstly, when ϵ_1 decreases back to ϵ_0 (with a probability of 1/2), the exchange rate will rise from e_1^{FF} to e_0 . Secondly, when ϵ_1 increases to ϵ_2 (with a probability of 1/2), the exchange rate will fall from e_1^{FF} to the lower edge of the band \underline{e} , rather than to e_2 , because the monetary authorities will act to defend the target band. This implies that when the shock is ϵ_1 , the public's exchange rate expectations under an exchange rate target zone will be given by $\pi_{TZ}^e = (e_0 - e_1^{FF})/2 + (\underline{e} - e_1^{FF})/2$. Given $(e_1 - e_1^{FF}) > -(\underline{e} - e_1^{FF})$, $\pi_{TZ}^e > 0$ is thus true. As a consequence, under an exchange rate target zone regime the equilibrium occurs at point E_3 , where the $XX(\epsilon_1)$ locus intersects $AA(\pi_{TZ}^e > 0)$,⁹ with y and e being y_3^{TZ} and e_3^{TZ} , respectively.

The equilibrium under a floating-exchange rate regime is established at point E_1 , where the $XX(\epsilon_1)$ locus intersects $AA(\pi^e = 0)$, and with y and e being y_1^{FF} and e_1^{FF} , respectively. It is quite clear in Fig. 3 that, in response to a rise in the demand shock, the change in both output and the interest rate differential under a target zone is greater than that under a floating-exchange rate regime. Consequently, when capital mobility is relatively high, an exchange rate target zone will benefit from lower exchange rate variability at the expense of higher output and interest rate differential variability. For

⁹ A rise in the exchange rate expectations tends to worsen the capital account, and leads to a deterioration in the balance of payments. In order to restore the foreign market equilibrium, other things being equal, the domestic currency should depreciate in response.

some of the EMS countries with high capital mobility (e.g., France before the widening of the bands of the EMS), exchange rate variability is found to be negatively related to interest rate variability.¹⁰ Our theoretical prediction is thus conformable to the empirical observations in some EMS countries.

4. Concluding remarks

The prediction of Krugman's (1991) exchange rate target zone model and the observations of existing empirical studies reveal a conflicting outcome regarding the relationship between exchange rate variability and the interest rate differential. This puzzle has become a central issue in the exchange rate target zone literature. This paper sets up a Fleming-type model and highlights that the degree of capital mobility may serve as a plausible vehicle to reconcile the conflicting outcome between Krugman's (1991) prediction and the empirical reality.

Based on our model, we find that when capital mobility is relatively low, the expectations of the authorities' future interventions in the foreign exchange market generate a mechanism to lower the variability of the exchange rate, output, and the interest rate differential relative to a floating-exchange rate regime. To be more specific, exchange rate variability exhibits a positive relationship with the interest rate differential. However, when capital mobility is relatively high, an exchange rate target zone tends to lower the variability of the exchange rate, but raises the variability of output and the interest rate differential. This implies that an exchange rate target zone will benefit from lower exchange rate variability at the expense of higher output and interest rate differential variability if capital is highly mobile. These mixed results help us to explain the conflicting outcome between Krugman's (1991) prediction and the empirical observations.

Acknowledgements

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Appendix A

The simple stochastic macro model reported in Eqs. (1)–(4) can be rewritten as follows:

$$y = \gamma y - \eta r + (\delta e - \theta y) + \varepsilon; \quad 0 < \gamma < 1, \eta, \delta, \theta > 0 \quad (\text{A1})$$

$$m = \phi y - \lambda r; \quad \phi, \lambda > 0 \quad (\text{A2})$$

$$(\delta e - \theta y) + \beta[r - r^* - E(de)/dt] = 0; \quad \delta, \theta, \beta > 0 \quad (\text{A3})$$

$$d\varepsilon = \sigma dZ. \quad (\text{A4})$$

From Eqs. (A1)–(A3), we have the following matrix form:

$$\begin{pmatrix} (1 + \theta - \gamma) & -\delta & \eta \\ \phi & 0 & -\lambda \\ -\theta & \delta & \beta \end{pmatrix} \begin{pmatrix} y \\ e \\ r \end{pmatrix} = \begin{pmatrix} \varepsilon \\ m \\ \beta r^* + \beta(E(de)/dt) \end{pmatrix}. \quad (\text{A5})$$

Let $r^* = 0$ for simplicity. Using Cramer's rule, we derive the following "pseudo" reduced expressions:

$$y = \delta C(\beta + \eta)m + \lambda \delta C\varepsilon + \lambda \beta \delta C \frac{E(de)}{dt}, \quad (\text{A6})$$

$$e = \Pi m + C(\lambda \theta - \beta \phi)\varepsilon + \Psi \frac{E(de)}{dt}, \quad (\text{A7})$$

¹⁰ See Kempa and Nelles (1999) for a detailed description.

$$r = (\gamma - 1)\delta C m + \phi \delta C \varepsilon + \phi \beta \delta C \frac{E(\mathrm{d}e)}{\mathrm{d}t}, \quad (\text{A8})$$

where

$$C = 1/\delta[\phi(\beta + \eta) + \lambda(1 - \gamma)] > 0,$$

$$\Pi = C[(1 + \theta - \gamma)\beta + \theta\eta] > 0,$$

$$\Psi = \beta[\phi\eta + \lambda(1 + \theta - \gamma)]C > 0.$$

Eq. (A7) states that the level of the exchange rate is related to both fundamentals and expectations regarding the future value of the exchange rate. It follows from Eq. (A7) that the general solution for e is:

$$e = \Pi m + C(\lambda\theta - \beta\phi)\varepsilon + A\exp(s\varepsilon) + B\exp(-s\varepsilon), \quad (\text{A9})$$

where $s = \sqrt{2/\Psi\sigma^2}$ and A and B are undetermined parameters.

Comparing Eq. (A9) with Eq. (A7) yields the expectation of the exchange rate movement:

$$\frac{E(\mathrm{d}e)}{\mathrm{d}t} = \frac{1}{\Psi} [A\exp(s\varepsilon) + B\exp(-s\varepsilon)]. \quad (\text{A10})$$

By plugging Eq. (A10) into Eqs. (A6) and (A8), we can obtain a general solution for output and the interest rate exhibited within the target zone:

$$y = \delta C(\beta + \eta)m + \lambda\delta C\varepsilon + \frac{\lambda\beta\delta C}{\Psi} [A\exp(s\varepsilon) + B\exp(-s\varepsilon)], \quad (\text{A11})$$

$$r = (\gamma - 1)\delta C m + \phi\delta C\varepsilon + \frac{\phi\beta\delta C}{\Psi} [A\exp(s\varepsilon) + B\exp(-s\varepsilon)], \quad (\text{A12})$$

Assume that the authorities stand ready to adjust the money supply at the upper bound of exchange rate \bar{e} and the lower bound of exchange rate \underline{e} . While the exchange rate stays within the interior of the band, the monetary authorities do not alter the money stock. Based on this intervention rule, the dynamic locus of e can be expressed as:

$$e = \begin{cases} \bar{e}; & \bar{e}_+ \leq \bar{e} \\ \Pi m + (\lambda\theta - \beta\phi)C\varepsilon + [A\exp(s\varepsilon) + B\exp(-s\varepsilon)]; & \underline{e}_+ \leq \bar{e}_- \\ \underline{e}; & \bar{e} \leq \underline{e}_- \end{cases} \quad (\text{A13})$$

where \bar{e} and \underline{e} are the corresponding values to which the monetary authorities decrease and increase the money supply to defend exchange rate target zones, respectively. The terms \bar{e}_+ and \underline{e}_- represent the right-hand- and left-hand-side limits of \bar{e} , respectively, and \underline{e}_+ and \bar{e}_- represent the right-hand- and left-hand-side limits of \underline{e} , respectively.

We now proceed to solve the undetermined variables: A , B , \bar{e} , and \underline{e} . In line with [Froot and Obstfeld \(1991\)](#), these unknown parameters are determined by two continuity conditions and two smooth pasting conditions. Since the agents know that the monetary authorities will adjust the money stock when the exchange rate reaches the upper or lower bounds of the target zone, they will readjust their portfolio in advance. Thus, the continuity condition prevents the exchange rate from jumping discretely when the monetary authorities intervene in the money market. Furthermore, the smooth pasting condition means that the exchange rate dynamic locus is tangential to the horizontal lines at the band's edges. These conditions imply:

$$e_{\bar{e}_+} = e_{\bar{e}_-}, \quad (\text{A14})$$

$$e_{\underline{e}_+} = e_{\underline{e}_-}, \quad (\text{A15})$$

$$\frac{\partial e_{\bar{e}_-}}{\partial \varepsilon} = 0, \quad (\text{A16})$$

$$\frac{\partial e_{\underline{\varepsilon}+}}{\partial \varepsilon} = 0. \quad (\text{A17})$$

Substituting Eq. (A13) into Eqs. (A14) (A15) (A16) (A17) yields:

$$\bar{e} = \Pi m + C(\lambda\theta - \beta\phi)\bar{e} + A\exp(s\bar{e}) + B\exp(-s\bar{e}), \quad (\text{A14a})$$

$$\underline{e} = \Pi m + C(\lambda\theta - \beta\phi)\underline{e} + A\exp(s\underline{e}) + B\exp(-s\underline{e}), \quad (\text{A15a})$$

$$C(\lambda\theta - \beta\phi) + A\exp(s\bar{e}) + B\exp(-s\bar{e}) = 0, \quad (\text{A16a})$$

$$C(\lambda\theta - \beta\phi) + A\exp(s\underline{e}) + B\exp(-s\underline{e}) = 0, \quad (\text{A17a})$$

It follows from Eqs. (A16a) and (A17a) that the smooth pasting conditions can be solved for A and B as functions of $\bar{\varepsilon}$ and $\underline{\varepsilon}$:

$$A = A(\bar{\varepsilon}, \underline{\varepsilon}) = \frac{(\lambda\theta - \beta\phi)C[\exp(-s\bar{e}) - \exp(-s\underline{e})]}{\exp[s(\bar{\varepsilon} - \underline{\varepsilon})] - \exp[s(\underline{\varepsilon} - \bar{\varepsilon})]}, \quad (\text{A18})$$

$$B = B(\bar{\varepsilon}, \underline{\varepsilon}) = \frac{(\lambda\theta - \beta\phi)C[\exp(s\bar{e}) - \exp(s\underline{e})]}{\exp[s(\bar{\varepsilon} - \underline{\varepsilon})] - \exp[s(\underline{\varepsilon} - \bar{\varepsilon})]}, \quad (\text{A19})$$

Assume that the bounds of the band are symmetric around zero (i.e. $e = -\bar{e}$) and $m=0$ initially. With these relations and Eqs. (A18) and (A19), the continuity conditions in Eqs. (14a) and (15a) can be rewritten as:

$$\bar{e} = C(\lambda\theta - \beta\phi)\bar{e} + [A(\bar{\varepsilon}, \underline{\varepsilon})\exp(s\bar{e}) + B(\bar{\varepsilon}, \underline{\varepsilon})\exp(-s\bar{e})], \quad (\text{A14b})$$

$$-\bar{e} = C(\lambda\theta - \beta\phi)\underline{e} + [A(\bar{\varepsilon}, \underline{\varepsilon})\exp(s\underline{e}) + B(\bar{\varepsilon}, \underline{\varepsilon})\exp(-s\underline{e})], \quad (\text{A15b})$$

Substituting Eqs. (A18) and (A19) into (A14b) and (A15b), we can infer that:

$$\underline{\varepsilon} = -\bar{\varepsilon}. \quad (\text{A20})$$

Eq. (A20) reveals an important implication: when the random market fundamentals follow a Brownian motion without drift and $m=0$ initially, the symmetrical exchange rate bounds can be alternatively expressed by the symmetrical market fundamental bounds.

Substituting $\underline{\varepsilon} = -\bar{\varepsilon}$ into Eqs. (A18) and (A19), we have:

$$A = -B = -\frac{1}{2} \left[\frac{(\lambda\theta - \beta\phi)C}{[\cosh(s\bar{e})]} \right]. \quad (\text{A21})$$

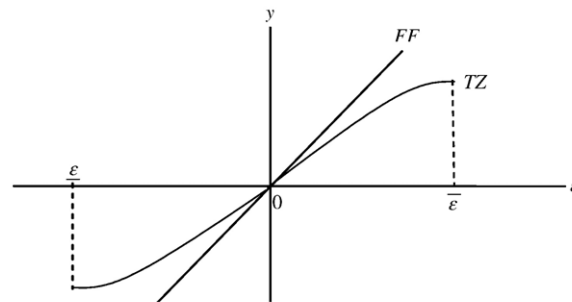
Combining Eq. (A21) with (A9), (A11) and (A12) and recalling that $m=0$ will initially yield the closed dynamic loci of output, the exchange rate and the interest rate within the bands:

$$y = \lambda\delta C e^{-\frac{\lambda\beta\delta C(\lambda\theta - \beta\phi)[\sinh(s\bar{e})]}{\Psi[\cosh(s\bar{e})]}}, \quad (\text{A22})$$

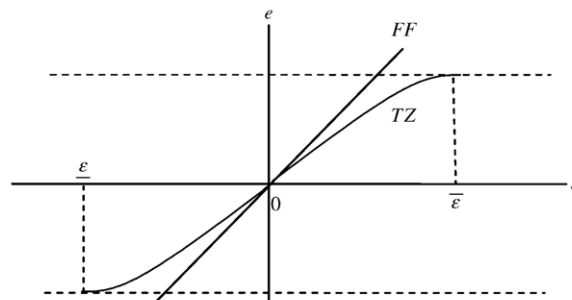
$$e = C(\lambda\theta - \beta\phi)\varepsilon - \frac{C(\lambda\theta - \beta\phi)[\sinh(s\varepsilon)]}{[\cosh(s\bar{\varepsilon})]}, \quad (\text{A23})$$

$$r = \phi\delta C\varepsilon - \frac{\phi\beta\delta C(\lambda\theta - \beta\phi)[\sinh(s\varepsilon)]}{\Psi[\cosh(s\bar{\varepsilon})]}. \quad (\text{A24})$$

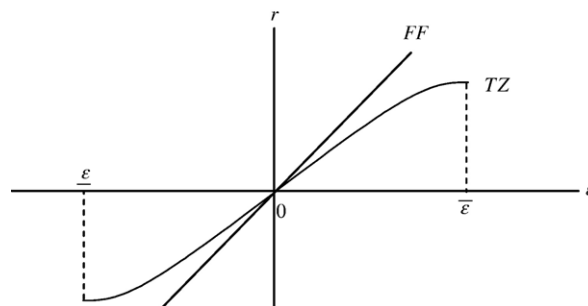
If the monetary authorities do not set an exchange rate band, implying that $\bar{e} \rightarrow \infty$ and $\underline{e} \rightarrow -\infty$, then the edges of the market fundamental have the properties $\bar{\varepsilon} \rightarrow \infty$ and $\underline{\varepsilon} \rightarrow -\infty$. With this relation, from Eqs. (A18) and (A19) we have $A=B=0$. It should be noted that under this situation the monetary authorities do not intervene in the money market to alter the money stock, and hence the regime of a floating-exchange rate is equivalent to that of a pegged monetary



Panel (a)



Panel (b)



Panel (c)

Fig. A1. The dynamic loci of y , e and r when capital is relatively immobile ($\beta < \lambda\theta / \phi$).

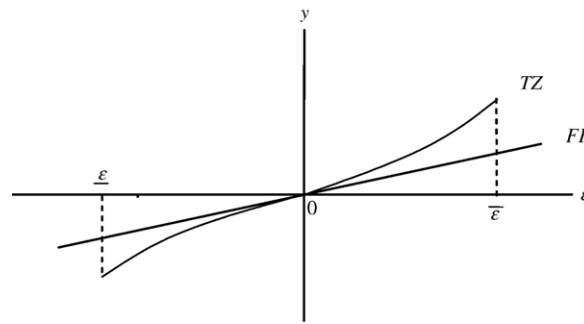
stock. It then follows from Eqs. (A9), (A11) and (A12) that the dynamic behavior of y , e , and r in the regime of a pegged monetary stock is:

$$y = \lambda \delta C \varepsilon, \quad (\text{A22a})$$

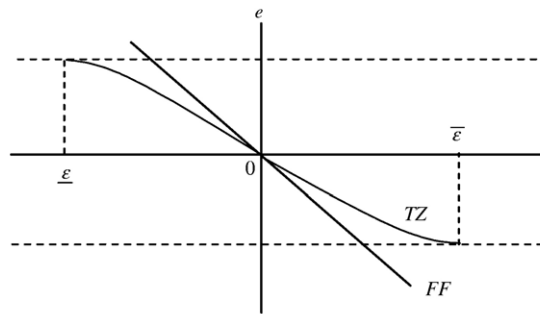
$$e = C(\lambda \theta - \beta \phi) \varepsilon, \quad (\text{A23a})$$

$$r = \phi \delta C \varepsilon. \quad (\text{A24a})$$

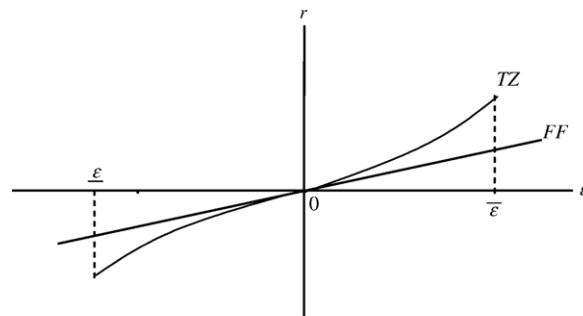
Eqs. (A22a) (A23a) (A24a) reveal that, if the monetary authorities do not set any edge for the exchange rate, then public agents will expect that the instantaneous change in the exchange rate is zero. From Eqs. (A6) (A7) (A8), y , e , and r are then completely determined by market fundamentals.



Panel (a)



Panel (b)



Panel (c)

Fig. A2. The dynamic loci of y , e and r when capital is relatively immobile ($\beta > \lambda \theta / \phi$).

Eqs. (A22) (A23) (A24) indicate that whether $(\lambda\theta - \beta\phi)$ is greater than zero or not will govern the dynamic behavior of y , e and r within the target zones. Accordingly, in what follows the discussion is classified according to either of two cases: capital is relatively immobile ($\beta < \lambda\theta/\phi$) and capital is relatively mobile ($\beta > \lambda\theta/\phi$).

A.1. Capital is relatively immobile ($\beta < \lambda\theta/\phi$)

Based on Eqs. (A22) (A23) (A24) with capital being relatively immobile ($\beta < \lambda\theta/\phi$), we can graph the output, exchange rate, and interest rate loci within the bands, which are labeled “TZ” to the relevant variables in panel (a), panel (b) and panel (c) of Fig. A1, respectively. Similarly, according to Eqs. (A22a) (A23a) (A24a), we can graph the dynamic loci of y , e , and r under the floating-exchange rate regime, which are labeled “FF” to the relevant variables in panel (a), panel (b) and panel (c) of Fig. A1, respectively.

In Fig. A1, for a given fluctuation in ϵ within the interval between $\bar{\epsilon}$ and $\underline{\epsilon}$, all of the output, exchange rate, and interest rate volatilities under the regime of an exchange rate target zone are smaller than those under the regime of a floating-exchange rate. Hence, the commitment of the monetary authorities to defending a zone will stabilize y , e and r , which is the well-known “honeymoon effect” in the target zone literature. In addition, Fig. A1 also provides an important policy implication in that, when the monetary authorities implement an exchange rate target zone policy with relatively immobile capital, exchange rate variability will exhibit a positive relationship with the interest rate differential.

A.2. Capital is relatively mobile ($\beta > \lambda\theta/\phi$)

According to Eqs. (A22) (A23) (A24) with $\beta > \lambda\theta/\phi$ and (A22a) (A23a) (A24a), we can similarly depict the dynamic loci of y , e and r under the two different regimes. The dynamic loci of the relevant variables are labeled “TZ” and “FF” in panel (a), panel (b) and panel (c) in Fig. A2, respectively. It is quite clear in panels (a)–(c) of Fig. A2 that, if the economy faces an aggregate demand shock, then the exchange rate target zone policy will stabilize the exchange rate, but will destabilize both output and the interest rate. Fig. A1 also indicates that, when the monetary authorities implement an exchange rate target zone policy with relatively mobile capital, exchange rate variability will be negatively related to the differential between the domestic and foreign interest rates.

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