5 Number of Bounded Regions

In this section, the number of bounded regions we discuss are in sense of maximizing the total regions made by k partitioner. We state the questions in the order of dimension in the following. And then, we will generalize them to higher dimensional space in the end of this section.

Number of Bounded Regions in Sense of k-max-point-drawing
How many bounded regions are there in sense of a k-max-point-drawing?

Number of Bounded Regions in Sense of k-max-line-drawing
How many bounded regions are there in sense of a k-max-line-drawing?

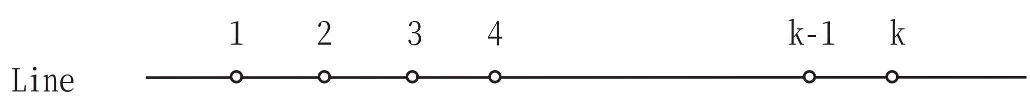
Number of Bounded Regions in Sense of k-max-plane-drawing
How many bounded regions are there in sense of a k-max-plane-drawing?

5.1 Number of Bounded Regions in Sense of k-max-point-drawing

How many bounded regions are there in sense of k-max-point-drawing?

Let ${a_k}^*$ be the number of bounded regions in sense of k-max-point drawing.

It is easy to see that the number of bounded regions ${a_k}^* = k - 1 = C_1^{k-1}$ in the following figure.



5.2 Number of Bounded Regions in Sense of k-max-line-drawing

How many bounded regions are there in sense of k-max-line-drawing?

Let ${b_k}^*$ be the number of bounded regions in sense of k-max-line-drawing.

The following figure 9 is a k-max-line-drawing. We can see that each of the more bounded regions made by the k^{th} line and marked by red bold line are below the k^{th} line and connected to each bounded regions created by the k^{th} line and the $k-1$ points since any three of the k lines will intersect with a common point to form a bounded region.

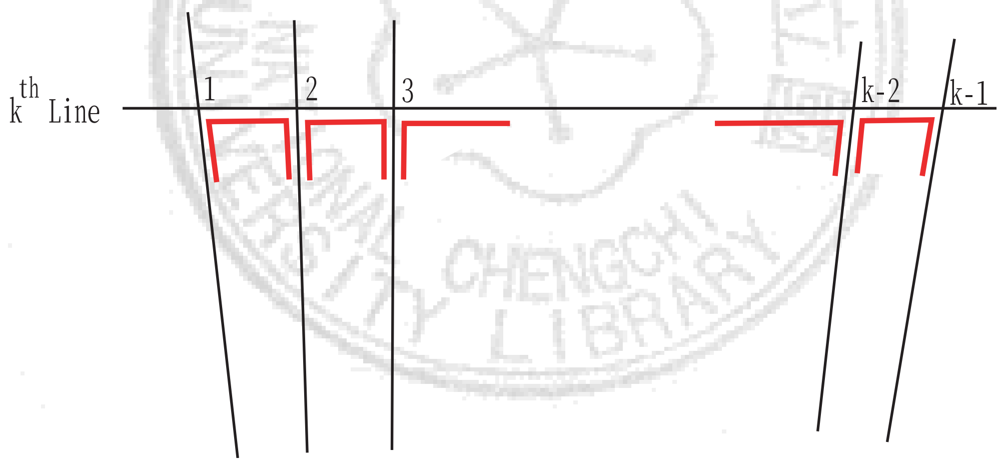


Figure 9: Recurrence Relation of Bounded Regions of k-max-line-drawing

Therefore, we get a recurrence relation in the following:

$${b_k}^* = {b_{k-1}}^* + {a_{k-1}}^*$$

Since this recurrence relation is of the same form as the recurrence relation of maximizing the total number of regions, we can use the similar way to obtain the formula that ${b_k}^* = C_2^{k-1}$.

5.3 Number of Bounded Regions in Sense of k-max-plane-drawing

How many bounded regions are there in sense of k-max-plane-drawing?

Let ${c_k}^*$ be the number of bounded regions in sense of k-max-plane-drawing.

We can easily see that the recurrence relation for this question is of the same form as the former one ${c_k}^* = {c_{k-1}}^* + {b_{k-1}}^*$, we can use the same way to get its formula that ${c_k}^* = C_3^{k-1}$, too.

5.4 Number of Bounded Regions of Higher Dimensional Space

Before generalizing these questions to n-dimensional space, we make a summary about these the results we have in the following:

Number of bounded regions in sense of k-max-point-drawing is $\underline{C_1^{k-1}}$

Number of bounded regions in sense of k-max-line-drawing is $\underline{C_2^{k-1}}$

Number of bounded regions in sense of k-max-plane-drawing is $\underline{C_3^{k-1}}$

Then, we generalize the question into the following one:

How many bounded regions are there in an n-dimensional space if k different $(n-1)$-dimensional spaces partition an n-dimensional space into the largest number of different parts?

Let ${P_{n,k}}^*$ be such a number of bounded regions in this question. Here, n means the n-dimensional space and k means a set of k partitioner in this space (i.e. k different $(n-1)$-dimensional spaces).

Although we don't have a clear definition about the bounded region in an n-dimensional space made by a set of $(n-1)$-dimensional spaces, we still can give an conjecture about the formula of ${P_{n,k}}^*$ by observing the relationship between the formulas and the similar form of recurrence relations of lower dimensional spaces.

First, we have the recurrence relation that ${P_{n,k}}^* = {P_{n,k-1}}^* + {P_{n-1,k-1}}^*$

And then, since the recurrence relation is of the same form as before, the rest things will be routine procedures. Using the preceding formulas of lower dimensional space, we can get that ${P_{n,k}}^* = C_n^{k-1}$.