

# Chapter 1

## Introduction

We consider a  $C_k/C_m/1/N$  open queueing model which restrains a finite number of customers in the system. It is assumed that both interarrival and service time are Coxian distributions with  $k$  and  $m$  phases respectively. This system is a single server queueing system. Customers are served under the First-come First-served discipline (FCFS). The  $C_k/C_m/1/N$  queueing system and the  $PH/PH/1/N$  queueing system are the same.

The examples of  $C_k/C_m/1$  can be found in many applications. In Neuts[8], stationary probabilities was obtained in matrix-geometric form. The matrix-geometric method relies on determining the minimal nonnegative matrix solution  $R$  of a matrix-quadratic equation; the invariant vector is expressed in terms of powers of  $R$ . Bertsimas [2] studied a  $C_k/C_m/s$  queue. He showed that the equilibrium probabilities for unboundary states are geometric in the number of waiting customers by using a generating function technique, we will take a different approach in this thesis.

Le Boudec[5] studied a  $PH/PH/1$  queue. He showed that the stationary probability is a linear combination of product-forms which can be expressed in terms of roots of the associated characteristic polynomial. He showed that all eigenvectors used in the expression of the stationary probability of  $PH/PH/1$  are Kronecker

products and gave a simple formula for computing the stationary probability of the number of customers in the system. Luh[6] used a similar approach to derive stationary probabilities in terms of linear combinations of product-forms in studying a system of two stations in tandem.

Wang[10] considered a  $PH/PH/1/N$  open queueing system containing finite number of customers  $N$ . She showed that the number of roots of the associated characteristic polynomial depends on the utilization factor,  $\rho$ , but independently of  $N$ . Liu[7] established a procedure for solving stationary probabilities. It is easy to construct the product-form when the  $m$  roots of the characteristic polynomial are distinct, each vectors used in the expression of stationary probabilities are described in terms of Kronecker products. One may refer to [7] for details.

Although, Le Boudec[5], Wang[10], have provided the product-form method to solve the stationary probabilities, they do not list the numerical results in detail and provide a solution procedure. In this thesis, our goal is to calculate the stationary probabilities of  $C_k/C_m/1/N$  numerically by using the product-form method and compare it with a traditional method. We modify the product-form method by perturbation when the utilization factor,  $\rho$ , equals to one. The Matlab software is the computing tool to solve the stationary probability. In this study, we focus on discussion the implementation of the product-form method. Precision and stability involutes numerical concept and schemes which would distract one's attention from the solution procedure itself. Thus, it is ignored in this study. The results of this study may be helpful to students and teachers who want to have a numerical solution of stationary probabilities of  $C_k/C_m/1/N$  queueing system, since there is no existing close-form solution of such systems.

This thesis is organized in the following. In chapter 2, we introduce the model of  $C_k/C_m/1/N$  queues and the vector product-form solution. Chapter 3 establish an algorithm for solving stationary probabilities and give two examples to illustrate the stationary probabilities. In chapter 4, we describe the operational procedures of program and give the tables of numerical results of  $C_k/C_m/1/4$  and  $C_k/C_m/1/6$ .

The numerical results of the case of  $\rho = 1$  is calculated by using the product-form method with small adjustment of constant which would give a satisfactory result from our experiments.

