

# Chapter 4

## Numerical Experiments

In this chapter, we provide the program of product-form method which is coded in Matlab. The program suits the  $C_k/C_m/1/N$  queueing systems with the case of simple roots and it needs the following features:

1. The capacity size of queueing system,  $N$ .
2. The number of phases of arrival process,  $k$ .
3. The number of phases of service process,  $m$ .
4. The arrival rate at each phase,  $\lambda_i$ ,  $i = 1, 2, \dots, k$ .
5. The service rate at each phase,  $\mu_j$ ,  $j = 1, 2, \dots, m$ .
6. The branching probability of arrival process  $p_i$ ,  $i = 1, 2, \dots, k$ .
7. The branching probability of service process  $q_j$ ,  $j = 1, 2, \dots, m$ .

After computing, the program would present the follows:

1. The expected system size,  $L_s$ .
2. The expected queue size,  $L_q$ .
3. The expected time spent in queue,  $W_q$ .
4. The expected time spent in system,  $W_s$ .
5. The idle probability of system,  $\pi_0$ .
6. The probability that a customer is lost,  $\pi_N$ .
7. The stationary probability,  $\pi_i$ ,  $i = 1, 2, \dots, N$ .

## 4.1 Using the Product-Form Method by Matlab

In this section, we want to introduce the program of the  $C_2/C_3/1/6$  system. The system has the following features:

$$N = 6,$$

$$\lambda_1 = \lambda_2 = 4, \quad p_1 = 0.5, \quad p_2 = 1,$$

$$\mu_1 = \mu_2 = \mu_3 = 5, \quad q_1 = 0.5 = q_2 = 0.5, \quad q_3 = 1.$$

**Step 1** Put the CD into the CD ROM.

**Step 2** Turn on the software of MATLAB.

**Step 3** Setting path:

Input the location of CD ROM on the space of "Current Directory". For example: D:\

**Step 4** Run program:

Input the name of program on "Command Window". For example:>> *productform*

**Step 5** Input the parameter:

The followings will show on "Command Window".

\*\*\*\*\*This is  $C_k/C_m/1/N$  Queueing System. \*\*\*\*\*

Please input the capacity size of Queueing System = 6

Please input the number of phases of arrival process = 2

Please input the number of phases of service process = 3

Please input the arrival rate at each phase = [4, 4]

Please input the service rate at each phase = [5, 5, 5]

Please input the branching probability of arrival process = [0.5, 1]

Please input the branching probability of service process = [0.5, 0.5, 1]

**Step 6** Numerical results:

The followings will show on "Command Window".

The expected system size is 2.7564

The expected queue size is 1.8990

The expected time spent in queue is 0.7935

The expected time spent in system is 1.1518

The idle probability of system is 0.151678

The probability that a customer is lost = 0.102557

The stationary probability is

$pi =$

0.15168 0.17526 0.16414 0.15083 0.13814 0.12649 0.10256

where

$$pi = (\pi_0, \pi_1, \dots, \pi_6)$$

**Step 7** Stationary probability in detail:

If we want to know  $\mathbf{P}_2$ , we can input " $n = 2$ ". The followings will show on "Command Window".

\*\*The following section is the probability of each phase of n in the system.\*\*

\*\*\*\*\*If you input  $n=10000$ , then it ends the program.\*\*\*\*\*

please input  $n = 2$

The probability of 2 customers in system is

$pn =$

0.058637 0.038696 0.014069 0.028508 0.016518 0.0077149

where

$$\mathbf{P}_n = (P_{n,1,1}, P_{n,1,2}, \dots, P_{n,1,m}, P_{n,2,1}, \dots, P_{n,i,j}, \dots, P_{n,k,m}).$$

**Step 8** End the program:

Input " $n = 10000$ " and then input " $0$ ". The following message will show "Command Window".

please input  $n = 10000$

The probability of 10000 customers in system is 0

\*\*\*\*\*End.\*\*\*\*\*

The  $C_k/C_m/1/N$  problem is (1) continuous (0) end

Please choose:0

\*\*\*\*\*The  $C_k/C_m/1/N$  problem is end!\*\*\*\*\*

## 4.2 Case 1: $C_k/C_m/1/4$

The following tables are constructed by the product-form method, except the case of  $\rho = 1$ . The case of  $\rho = 1$  is constructed by the product-form method with small adjustment in (2.11).  $L_q(pd)$  means the expected queue length which is calculated from the equations (2.21)~(2.25).  $L_q(td)$  means the expected queue length which is calculated from the equations (2.2)~(2.6). The difference comes from approximation used by vector product-form approach. The mean service time is 0.1, i.e.  $\mu = 10$ .

In the Table 1, we list the stationary probabilities of  $M/E_2/1/4$  queuing system, and  $\mu_1 = \mu_2 = 20$ .

Table 1:  $M/E_2/1/4$

$\rho$	$\lambda_1$	$\pi_0$	$\pi_1$	$\pi_2$	$\pi_3$	$\pi_4$	$L_q(pd)$	$L_q(td)$
0.2	2	0.8001	0.168	0.0273	0.0040	0.0005	0.0370	0.0370
0.5	5	0.5097	0.2867	0.1294	0.0549	0.0193	0.2971	0.02972
0.8	8	0.2748	0.2638	0.2093	0.1587	0.0935	0.8071	0.8071
1	10	0.1698	0.2122	0.2228	0.2255	0.1698	1.1830	1.1832
2	20	0.0097	0.0372	0.1143	0.3142	0.5141	2.2851	2.2687
3	30	0.0017	0.0107	0.0546	0.2639	0.6686	2.5883	2.5850

In the Table 2, we list the stationary probabilities of  $M/E_3/1/4$  queuing system, and  $\mu_1 = \mu_2 = \mu_3 = 30$ .

Table 2:  $M/E_3/1/4$ 

$\rho$	$\lambda_1$	$\pi_0$	$\pi_1$	$\pi_2$	$\pi_3$	$\pi_4$	$L_q(pd)$	$L_q(td)$
0.2	2	0.8001	0.1709	0.0274	0.0035	0.0004	0.0354	0.0330
0.5	5	0.5076	0.2985	0.1317	0.0510	0.0152	0.2792	0.2746
0.8	8	0.2661	0.2747	0.2192	0.1605	0.0827	0.7883	0.7842
1	10	0.1579	0.2163	0.2339	0.2365	0.1579	1.1804	1.1774
2	20	0.0096	0.0354	0.1115	0.3383	0.5055	2.3047	2.3019
3	30	0.0010	0.0071	0.0449	0.2799	0.6671	2.6062	2.6057

In the Table 3, we list the stationary probabilities of  $E_2/E_2/1/4$  queuing system, and  $\mu_1 = \mu_2 = 20$ .

 Table 3:  $E_2/E_2/1/4$ 

$\rho$	$\lambda_1 = \lambda_2$	$\pi_0$	$\pi_1$	$\pi_2$	$\pi_3$	$\pi_4$	$L_q(pd)$	$L_q(td)$
0.2	4	0.8	0.1885	0.0110	0.0005	0	0.0121	0.0121
0.5	10	0.5017	0.3565	0.1073	0.0283	0.0062	0.1824	0.1843
0.8	16	0.2347	0.3123	0.2219	0.1450	0.0702	0.7225	0.7662
1	20	0.1165	0.2132	0.2309	0.2406	0.1988	1.3085	1.3085
2	40	0.0024	0.0146	0.0601	0.2607	0.6608	2.5639	2.5678
3	60	0.0002	0.0019	0.0177	0.1785	0.8018	2.7800	2.7800

In the Table 4, we list the stationary probabilities of  $E_2/E_3/1/4$  queuing system,  $\mu_1 = \mu_2 = \mu_3 = 30$ .

 Table 4:  $E_2/E_3/1/4$ 

$\rho$	$\lambda_1 = \lambda_2$	$\pi_0$	$\pi_1$	$\pi_2$	$\pi_3$	$\pi_4$	$L_q(pd)$	$L_q(td)$
0.2	4	0.8000	0.1908	0.0115	0.0003	0.0000	0.0122	0.0095
0.5	10	0.5009	0.3750	0.1055	0.0216	0.0035	0.1595	0.1532
0.8	16	0.2270	0.3354	0.2363	0.1408	0.0577	0.6911	0.717
1	20	0.1021	0.2157	0.2446	0.2541	0.1865	1.3123	1.3087
2	40	0.0005	0.0067	0.0436	0.2699	0.6792	2.6210	2.6157
3	60	0.0000	0.0006	0.0100	0.1779	0.8115	2.8003	2.8001

In the Table 5, we list the stationary probability of  $E_2/C_2/1/4$  queuing system. After phase 1, the service time comes to an end with probability 0.2, and it enters the next phase with probability 0.8, i.e.  $q_1 = 0.2$ . And  $\mu_1 = \mu_2 = 18$ .

Table 5:  $E_2/C_2/1/4$

$\rho$	$\lambda_1 = \lambda_2$	$\pi_0$	$\pi_1$	$\pi_2$	$\pi_3$	$\pi_4$	$L_q(pd)$	$L_q(td)$
0.2	4	0.8	0.1870	0.0123	0.0007	0	0.0137	0.0131
0.5	10	0.5023	0.3455	0.1118	0.0324	0.0079	0.2004	0.2029
0.8	16	0.2393	0.2984	0.2168	0.1472	0.0775	0.7436	0.7945
1	20	0.1252	0.2098	0.2247	0.2337	0.2067	1.3122	1.3122
2	40	0.0041	0.0203	0.0699	0.2519	0.6512	2.5273	2.5340
3	60	0.0004	0.0036	0.0247	0.1766	0.7947	2.7619	2.7619

In the table 6, we list the stationary probabilities of  $E_2/C_3/1/4$  queuing system. After phase 1, the service time come to an end with probability 0.2, and enter the next phase with probability 0.8. After phase 2, the service time comes to an end with probability 0.5, and it enters the next phase with probability 0.5, i.e.  $p_1 = q_1 = 0.2$  and  $q_2 = 0.5$ . And  $\mu_1 = \mu_2 = \mu_3 = 22$ .

Table 6:  $E_2/C_3/1/4$

$\rho$	$\lambda_1 = \lambda_2$	$\pi_0$	$\pi_1$	$\pi_2$	$\pi_3$	$\pi_4$	$L_q(pd)$	$L_q(td)$
0.2	4	0.8000	0.1876	0.0140	0.0007	0.0000	0.0155	0.0131
0.5	10	0.5020	0.3485	0.1156	0.0316	0.0072	0.2003	0.1968
0.8	16	0.2382	0.3011	0.2229	0.1487	0.0758	0.7477	0.7897
1	20	0.1228	0.2088	0.2283	0.2371	0.2053	1.3183	1.3155
2	40	0.0037	0.0196	0.0689	0.2523	0.6527	2.5316	2.5383
3	60	0.0004	0.0036	0.0246	0.1762	0.7951	2.7624	2.7625

In the Table 7, we list the stationary probabilities of  $C_2/C_2/1/4$  queuing system. After phase 1, the interarrival time and service time come to an end with probabilities 0.2, and enter the next phase with probabilities 0.8, i.e.  $p_1 = q_1 = 0.2$ . And  $\mu_1 = \mu_2 = 18$ .

Table 7:  $C_2/C_2/1/4$ 

$\rho$	$\lambda_1 = \lambda_2$	$\pi_0$	$\pi_1$	$\pi_2$	$\pi_3$	$\pi_4$	$L_q(pd)$	$L_q(td)$
0.2	3.6	0.8000	0.1805	0.0178	0.0016	0.0001	0.0213	0.0213
0.5	9	0.5038	0.3262	0.1187	0.0398	0.0114	0.2326	0.2345
0.8	14.4	0.2489	0.2878	0.2146	0.1513	0.0846	0.7711	0.8037
1	18	0.1369	0.2110	0.2240	0.2309	0.1972	1.2774	1.2774
2	36	0.0058	0.0254	0.0808	0.2647	0.6188	2.4666	2.4778
3	54	0.0006	0.0049	0.0302	0.1919	0.7723	2.7308	2.7310

In the Table 8, we list the stationary probabilities of  $C_2/C_3/1/4$  queuing system. After phase 1, the interarrival time and service time come to an end with probabilities 0.2, and enter the next phase with probabilities 0.8. After phase 2, the service time comes to an end with probability 0.5, and it enters the next phase with probability 0.5, i.e.  $p_1 = q_1 = 0.2$  and  $q_2 = 0.5$ . And  $\mu_1 = \mu_2 = \mu_3 = 22$ .

 Table 8:  $C_2/C_3/1/4$ 

$\rho$	$\lambda_1 = \lambda_2$	$\pi_0$	$\pi_1$	$\pi_2$	$\pi_3$	$\pi_4$	$L_q(pd)$	$L_q(td)$
0.2	3.6	0.8000	0.1811	0.0192	0.0016	0.0001	0.0227	0.0206
0.5	9	0.5035	0.3287	0.1218	0.0392	0.0106	0.2320	0.2291
0.8	14.4	0.2477	0.2900	0.2196	0.1529	0.0830	0.7744	0.7995
1	18	0.1346	0.2104	0.2273	0.2342	0.1955	1.2820	1.2795
2	36	0.0051	0.0247	0.0797	0.2655	0.6201	2.4711	2.4824
3	54	0.0006	0.0049	0.0300	0.1916	0.7729	2.7318	2.7319

In the Table 9, we list the stationary probabilities of  $C_4/C_3/1/4$  queuing system. After phase  $i$ ,  $i = 1, 2, 3$ , the interarrival time come to an end with probability 0.2, and enter the next phase with probability 0.8. After phase  $j$ ,  $j = 1, 2$ , the service time comes to an end with probability 0.5, and it enters the next phase with probability 0.5, i.e.  $p_i = 0.2$ ,  $i = 1, 2, 3$  and  $q_j = 0.5$ ,  $j = 1, 2$ . And  $\mu_1 = \mu_2 = \mu_3 = 17.5$ .

Table 9:  $C_4/C_3/1/4$ 

$\rho$	$\lambda_i, i = 1, \dots, 4$	$\pi_0$	$\pi_1$	$\pi_2$	$\pi_3$	$\pi_4$	$L_q(pd)$	$L_q(td)$
0.2	5.905	0.8000	0.1769	0.0225	0.0025	0.0002	0.0281	0.0258
0.5	14.76	0.5046	0.3175	0.1242	0.0443	0.0140	0.2548	0.2509
0.8	23.616	0.2542	0.2810	0.2133	0.1533	0.0943	0.8028	0.8316
1	29.52	0.1420	0.2059	0.2146	0.2207	0.2169	1.3066	1.3066
2	59.04	0.0068	0.0278	0.0765	0.2060	0.6574	2.4606	2.5095
3	88.56	0.0010	0.0061	0.0288	0.1315	0.8083	2.7168	2.7642

### 4.3 Case 2: $C_k/C_m/1/6$

The following tables are constructed by the product-form method, but the case of  $\rho = 1$ . The case of  $\rho = 1$  is constructed by the product-form method with small adjustment in (2.11).  $L_q(pd)$  means the expected queue length which is calculated from the equations (2.21)~(2.25).  $L_q(td)$  means the expected queue length which is calculated from the equations (2.2)~(2.6). The difference comes from approximation used by vector product-form approach. The mean of service time distribution is 0.1, i.e.  $\mu = 10$ .

In the Table 10, we list the stationary probabilities of  $M/E_2/1/6$  queuing system, and  $\mu_1 = \mu_2 = 20$ .

Table 10:  $M/E_2/1/6$ 

$\rho$	$\lambda_1$	$\pi_0$	$\pi_1$	$\pi_2$	$\pi_3$	$\pi_4$	$\pi_5$	$\pi_6$	$L_q(pd)$	$L_q(td)$
0.2	2	0.8000	0.1680	0.0273	0.0040	0.0006	0.0001	0.0000	0.0375	0.0375
0.5	5	0.5016	0.2822	0.1274	0.0540	0.0224	0.0092	0.0032	0.3557	0.3557
0.8	8	0.2356	0.2262	0.1795	0.1361	0.1019	0.0761	0.0446	1.2846	1.2846
1	10	0.1169	0.1461	0.1534	0.1552	0.1557	0.1558	0.1169	2.1386	2.1386
2	20	0.0015	0.0054	0.0164	0.0447	0.1183	0.3107	0.5030	4.2185	4.2065
3	30	0.0001	0.0005	0.0024	0.0115	0.0551	0.2636	0.6668	4.5793	4.5788

In the Table 11, we list the stationary probabilities of  $M/E_3/1/6$  queuing sys-

tem, and  $\mu_1 = \mu_2 = \mu_3 = 30$ .

Table 11:  $M/E_3/1/6$

$\rho$	$\lambda_1$	$\pi_0$	$\pi_1$	$\pi_2$	$\pi_3$	$\pi_4$	$\pi_5$	$\pi_6$	$L_q(pd)$	$L_q(td)$
0.2	2	0.8000	0.1709	0.0274	0.0035	0.0004	0.0000	0.0000	0.0358	0.0333
0.5	5	0.5010	0.2946	0.1300	0.0503	0.0189	0.0071	0.0021	0.3260	0.3210
0.8	8	0.2296	0.2370	0.1891	0.1385	0.0999	0.0719	0.0370	1.2385	1.2344
1	10	0.1071	0.1468	0.1587	0.1605	0.1607	0.1607	0.1071	2.1404	2.1379
2	20	0.0010	0.0038	0.0121	0.0366	0.1108	0.3351	0.5006	4.2611	4.2606
3	30	0.0000	0.0002	0.0012	0.0072	0.0450	0.2798	0.6667	4.6030	4.6029

In the Table 12, we list the stationary probabilities of  $E_2/E_2/1/6$  queuing system, and  $\mu_1 = \mu_2 = 20$ ,  $\lambda_1 = \lambda_2$ .

Table 12:  $E_2/E_2/1/6$

$\rho$	$\lambda_i$	$\pi_0$	$\pi_1$	$\pi_2$	$\pi_3$	$\pi_4$	$\pi_5$	$\pi_6$	$L_q(pd)$	$L_q(td)$
0.2	4	0.8000	0.1885	0.0110	0.0005	0.0000	0.0000	0.0000	0.0121	0.0121
0.5	10	0.5001	0.3554	0.1070	0.0282	0.0072	0.0018	0.0004	0.1940	0.1942
0.8	16	0.2137	0.2844	0.2021	0.1321	0.0849	0.0544	0.0262	1.0695	1.0939
1	20	0.0795	0.1453	0.1566	0.1587	0.1598	0.1645	0.1356	2.2897	2.2897
2	40	0.0002	0.0009	0.0036	0.0146	0.0600	0.2605	0.6602	4.5559	4.5559
3	60	0.0000	0.0000	0.0002	0.0019	0.0177	0.1785	0.8017	4.7795	4.7795

In the Table 13, we list the stationary probabilities of  $E_2/E_3/1/6$  queuing system,  $\mu_1 = \mu_2 = \mu_3 = 30$ ,  $\lambda_1 = \lambda_2$ .

Table 13:  $E_2/E_3/1/6$

$\rho$	$\lambda_i$	$\pi_0$	$\pi_1$	$\pi_2$	$\pi_3$	$\pi_4$	$\pi_5$	$\pi_6$	$L_q(pd)$	$L_q(td)$
0.2	4	0.8	0.1908	0.0115	0.0003	0.0000	0.0000	0.0000	0.0122	0.0095
0.5	10	0.5000	0.3744	0.1054	0.0216	0.0043	0.0008	0.0001	0.1654	0.1582
0.8	16	0.2089	0.3088	0.2175	0.1296	0.0762	0.0448	0.0184	0.9763	0.9869
1	20	0.0685	0.1446	0.1633	0.1646	0.1654	0.1705	0.1252	2.2965	2.2939
2	40	0.0000	0.0002	0.0014	0.0078	0.0443	0.2695	0.6768	4.6118	4.6114
3	60	0.0000	0.0000	0.0000	0.0006	0.0100	0.1779	0.8114	4.8001	4.8001

In the Table 14, we list the stationary probabilities of  $E_2/C_2/1/6$  queuing system. After phase 1, the service time comes to an end with probability 0.2, and it enters the next phase with probability 0.8, i.e.  $q_1 = 0.2$ . And  $\mu_1 = \mu_2 = 18$ ,  $\lambda_1 = \lambda_2$ .

Table 14:  $E_2/C_2/1/6$

$\rho$	$\lambda_i$	$\pi_0$	$\pi_1$	$\pi_2$	$\pi_3$	$\pi_4$	$\pi_5$	$\pi_6$	$L_q(pd)$	$L_q(td)$
0.2	4	0.8	0.1870	0.0123	0.0007	0.0000	0.0000	0.0000	0.0137	0.0137
0.5	10	0.5002	0.3441	0.1113	0.0323	0.0091	0.0025	0.0006	0.2161	0.2164
0.8	16	0.2168	0.2705	0.1965	0.1334	0.0892	0.0594	0.0311	1.1236	1.1531
1	20	0.0861	0.1442	0.1536	0.1556	0.1569	0.1613	0.1423	2.2921	2.2921
2	40	0.0004	0.0018	0.0060	0.0203	0.0698	0.2515	0.6503	4.5134	4.5135
3	60	0.0000	0.0001	0.0005	0.0036	0.0247	0.1765	0.7945	4.7606	4.7606

In the Table 15, we list the stationary probabilities of  $E_2/C_3/1/6$  queuing system. After phase 1, the service time comes to an end with probability 0.2, and it enters the next phase with probability 0.8. After phase 2, the service time comes to an end with probability 0.5, and it enters the next phase with probability 0.5. And  $\mu_1 = \mu_2 = \mu_3 = 22$ ,  $\lambda_1 = \lambda_2$ .

Table 15:  $E_2/C_3/1/6$

$\rho$	$\lambda_i$	$\pi_0$	$\pi_1$	$\pi_2$	$\pi_3$	$\pi_4$	$\pi_5$	$\pi_6$	$L_q(pd)$	$L_q(td)$
0.2	4	0.8	0.1876	0.0140	0.0007	0.0000	0.0000	0.0000	0.0155	0.0131
0.5	10	0.5001	0.3472	0.1152	0.0315	0.0083	0.0022	0.0005	0.2141	0.2086
0.8	16	0.2156	0.2727	0.2019	0.1347	0.0885	0.0580	0.0295	1.1161	1.1384
1	20	0.0842	0.1430	0.1555	0.1573	0.1583	0.1627	0.1408	2.2996	2.2973
2	40	0.0003	0.0017	0.0057	0.0196	0.0688	0.2520	0.6519	4.5188	4.5189
3	60	0.0000	0.0001	0.0005	0.0036	0.0246	0.1762	0.7950	4.7613	4.7612

In the Table 16, we list the stationary probabilities of  $C_2/C_2/1/6$  queuing system. After phase 1, the interarrival time and service time come to an end with

probabilities 0.2, and enter the next phase with probabilities 0.8, i.e.  $p_1 = q_1 = 0.2$ .

And  $\mu_1 = \mu_2 = 18$ ,  $\lambda_1 = \lambda_2$ .

Table 16:  $C_2/C_2/1/6$

$\rho$	$\lambda_i$	$\pi_0$	$\pi_1$	$\pi_2$	$\pi_3$	$\pi_4$	$\pi_5$	$\pi_6$	$L_q(pd)$	$L_q(td)$
0.2	3.6	0.8000	0.1805	0.0178	0.0016	0.0001	0.0000	0.0000	0.0213	0.0213
0.5	9	0.5004	0.3239	0.1179	0.0395	0.0129	0.0042	0.0012	0.2581	0.2585
0.8	14.4	0.2215	0.2561	0.1910	0.1347	0.0936	0.0648	0.0361	1.1808	1.2012
1	18	0.0942	0.1451	0.1534	0.1554	0.1565	0.1595	0.1358	2.2508	2.2508
2	36	0.0006	0.0026	0.0081	0.0254	0.0807	0.2644	0.6181	4.4494	4.4496
3	54	0.0000	0.0001	0.0008	0.0049	0.0302	0.1918	0.7721	4.7290	4.7290

In the Table 17, we list the stationary probabilities of  $C_2/C_3/1/4$  queuing system. After phase 1, the interarrival time and service time come to an end with probabilities 0.2, and enter the next phase with probabilities 0.8. After phase 2, the service time comes to an end with probability 0.5, and it enters the next phase with probability 0.5, i.e.  $p_1 = q_1 = 0.2$  and  $q_2 = 0.5$ . And  $\mu_1 = \mu_2 = \mu_3 = 22$ ,  $\lambda_1 = \lambda_2$ .

Table 17:  $C_2/C_3/1/6$

$\rho$	$\lambda_i$	$\pi_0$	$\pi_1$	$\pi_2$	$\pi_3$	$\pi_4$	$\pi_5$	$\pi_6$	$L_q(pd)$	$L_q(td)$
0.2	3.6	0.8000	0.1810	0.0192	0.0016	0.0001	0.0000	0.0000	0.0227	0.0206
0.5	9	0.5003	0.3267	0.1211	0.0390	0.0121	0.0037	0.0010	0.2555	0.2509
0.8	14.4	0.2203	0.2580	0.1954	0.1360	0.0933	0.0638	0.0345	1.1748	1.1891
1	18	0.0923	0.1442	0.1551	0.1570	0.1579	0.1609	0.1341	2.2568	2.2547
2	36	0.0005	0.0024	0.0077	0.0247	0.0796	0.2653	0.6196	4.4554	4.4555
3	54	0.0000	0.0001	0.0008	0.0049	0.0300	0.1915	0.7727	4.7301	4.7299

In the Table 18, we list the stationary probabilities of  $C_4/C_3/1/4$  queuing system. After phase  $i$ ,  $i = 1, 2, 3$ , the interarrival time come to an end with probability 0.2, and enter the next phase with probability 0.8. After phase  $j$ ,  $j = 1, 2$ , the service

time comes to an end with probability 0.5, and it enters the next phase with probability 0.5, i.e.  $p_i = 0.2$ ,  $i = 1, 2, 3$  and  $q_j = 0.5$ ,  $j = 1, 2$ . And  $\mu_1 = \mu_2 = \mu_3 = 17.5$ ,  $\lambda_1 = \lambda_2 = \lambda_3 = \lambda_4$ .

Table 18:  $C_4/C_3/1/6$

$\rho$	$\lambda_i$	$\pi_0$	$\pi_1$	$\pi_2$	$\pi_3$	$\pi_4$	$\pi_5$	$\pi_6$	$L_q(pd)$	$L_q(td)$
0.2	5.905	0.8000	0.1769	0.0225	0.0025	0.0003	0.0000	0.0000	0.0283	0.0259
0.5	14.76	0.5005	0.3150	0.1232	0.0439	0.0152	0.0052	0.0016	0.2859	0.2800
0.8	23.616	0.2244	0.2481	0.1884	0.1354	0.0961	0.0681	0.0418	1.2289	1.2469
1	29.52	0.0982	0.1434	0.1491	0.1511	0.1519	0.1545	0.1511	2.2805	2.2805
2	59.04	0.0009	0.0036	0.0100	0.0275	0.0753	0.2055	0.6544	4.3850	4.4728
3	88.56	0.0000	0.0003	0.0013	0.0060	0.0280	0.1297	0.8079	4.6552	4.7609

