行政院國家科學委員會專題研究計畫 成果報告

以群聚計算法求解叉狀型等候系統之穩態機率(I)
研究成果報告(精簡版)

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計 畫 主 持 人：陸行

報 告 附 件 ：出席國際會議研究心得報告及發表論文

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中 華 民 國 101 年 09 月 28 日

中 文 摘 要 ：　於計畫中進行調查一個新的計算方式於求解叉狀等候系統的穩態機率。我們初步證明此推導計算方式，縮減了以往複雜的矩陣計算。我們希望在這個研究計畫中可以演繹和推導此方法於一般服務時間的系統中。這種計算法的好處是在大型的矩陣計算時可以顯著地減少計算量，提供使用者快速而且正確的解。

中文關鍵詞：　類生死過程之馬可夫鏈，等候理論，穩態機率

英 文 摘 要 ：

英文關鍵詞：

行政院國家科學委員會補助專題研究計畫成果報告
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中　華　民　國　101　年　9　月　1　日

行政院國家科學委員會專題研究計畫成果報告
以群聚計算法求解叉狀型等候系統之穩態機率
Calculating Balance Equations of A Fork-Type Queue By State-Aggregate Approach

計畫編號: **NSC 100-2221-E-004-003**
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Abstract. Stationary probabilities are fundamental in response to various measures of performance in queueing networks. Solving stationary probabilities in Quasi-Birth-and-Death(QBD) type Markov Chain normally are dependent on the structure of the queueing network. In this paper, a new computing scheme is developed for attaining stationary probabilities in queueing networks of the fork-type. This scheme provides a general approach to reducing the complexity of computing algorithm. The result becomes more significant when a large buffer size is involved but cannot be ignored. The background theorem of this approach is proved and provided with an illustrated example in this paper.

1 System Description and Model Formulation

Consider a fork-type queueing model as shown in Figure 1. There are two work stations: the buffer space at the first station is fairly larger than that at the second station. Without loss of generality, we assume it is infinite at the first buffer but the size at the second buffer is B. All jobs after service at the first station are transferred immediately to the second station with probability q. With this simple model, we will show how to construct a computing procedure to obtain stationary probabilities with product form. In general, this scheme provides a direct approach to reducing the complexity of computing algorithm.

Assume that jobs arrive at the first station according to a Poisson process with rate λ and join an infinite waiting space at station 1. After service in station 1, with probability q, the job moves to a queue of finite capacity B at station 2. In other words, with probability $1-q$, the job leaves the system. Each queue is served by a single server with exponential service time of rate of μ_1 and μ_2. Note that the model belongs to the class of Markovian queueing network with blocking at the second station. Let $n(t)$ and $m(t)$ be the number of jobs present at time t in station 1 and 2, respectively. The process $\{(n(t), m(t)), t \geq 0\}$ is a Markov process on state space $\cup_{i\geq 0} l(i)$ with $l(i) = \{(i,0),\ldots,(i,B)\}$, $i \geq 0$. In the long run, let (n,m) present the system state and their possible transitions be given in the table 2.6.

The infinitesimal generator (transition rate matrix) $\mathbf{Q}$ has a block-tridiagonal structure, indicating a quasi-birth-and-death(QBD) process. Note that the last case when the second stage buffer is full or $m = B$ in the state-transition table, we assume that the job departed from station 1 leaves the system with probability 1 (i.e. the job leaves immediately when all buffer at station 2 is occupied) or we may say that the job which finds the buffer in station 2 is full gets lost. This assumption is obviously made for the purpose of the fork-type structure. Apparently, if the buffer size and the service rate at the second station are large enough, the probability of reaching this blocking state can be negligible. In fact, solving the stationary probability of states with $m = B$ is one of major concerns of this approach.

2 Model with special solutions

2.1 $M/M/c \to /M/d/B$

Consider a folk type queueing system where service times of each server are independent and identically distributed (i.i.d.) random variables subjecting to exponential distributions with service rate μ_k, $k = 1, 2$. Interarrival times of jobs are also i.i.d. random variables subjecting to an exponential distribution with arrival rate λ. Assume the service times and interarrival times are mutually independent. The service discipline is First come first served (FCFS). Under the stability condition of the system $\lambda < \mu_1$, the stationary state probabilities may be obtained and denoted by $\boldsymbol{\pi}_{n,m}$.

We consider the matrix $\mathbf{Q}$ namely

$$\mathbf{Q} = \begin{bmatrix} \mathbf{B}_1 & \mathbf{A}_0 & & & \\ \mathbf{A}_{21} & \mathbf{A}_{11} & \mathbf{A}_0 & & \\ & \mathbf{A}_{22} & \mathbf{A}_{12} & \mathbf{A}_0 & \\ & & \mathbf{A}_{23} & \mathbf{A}_{13} & \ddots \\ & & & \mathbf{A}_{24} & \ddots \\ & & & & \ddots \end{bmatrix}.$$

where $\mathbf{B}_1$, $\mathbf{A}_0$, $\mathbf{A}_1$ and $\mathbf{A}_2$ are $(B+1) \times (B+1)$ matrices, i.e.,

$$\mathbf{A}_0 = \begin{bmatrix} \lambda & & & & & \\ & \lambda & & & & \\ & & \ddots & & & \\ & & & \ddots & & \\ & & & & \ddots & \\ & & & & & \lambda \end{bmatrix};$$

$$\mathbf{A}_{1n} = \begin{bmatrix} -(\lambda + k\mu_1) & 0 & 0 & & & & \\ \mu_2 & -(\lambda + k\mu_1 + \mu_2) & 0 & & & & \\ & 2\mu_2 & -(\lambda + k\mu_1 + 2\mu_2) & & & & \\ & & \ddots & \ddots & & & \\ & & & d\mu_2 & -(\lambda + k\mu_1 + d\mu_2) & & \\ & & & & \ddots & \ddots & \\ & & & & & d\mu_2 & -(\lambda + k\mu_1 + d\mu_2) \end{bmatrix};$$

$$\mathbf{A}_{2n} = \begin{bmatrix} k\mu_1(1-q) & k\mu_1 q & 0 & & \\ & k\mu_1(1-q) & k\mu_1 q & & \\ & & \ddots & \ddots & \\ & & & k\mu_1(1-q) & k\mu_1 q \\ & & & & k\mu_1 \end{bmatrix};$$

where

$$k = \begin{cases} n & \text{if} \quad n < c; \\ c & \text{if} \quad n \geq c. \end{cases} \tag{2.1}$$

and

$$\mathbf{B}_1 = \begin{bmatrix} -\lambda & 0 & 0 & \cdots & 0 & 0 & \\ \mu_2 & -(\lambda+\mu_2) & 0 & \cdots & 0 & 0 & \\ 0 & 2\mu_2 & -(\lambda+2\mu_2) & \cdots & 0 & 0 & \\ & & & \ddots & \vdots & \vdots & \\ & & 3\mu_2 & & & & \\ & & & \ddots & & & \\ & & & & d\mu_2 & -(\lambda+d\mu_2) & \\ & & & & & \ddots & \ddots \\ & & & & & d\mu_2 & -(\lambda+d\mu_2) \end{bmatrix}$$

Let $\boldsymbol{\pi}$ be the stationary probability associated with $\mathbf{Q}$, namely, $\boldsymbol{\pi}\mathbf{Q} = \mathbf{0}$. Define $\boldsymbol{\pi} = (\boldsymbol{\pi}_0, \boldsymbol{\pi}_1, \boldsymbol{\pi}_2, \dots)$ where $\boldsymbol{\pi}_n$=$(\pi_{n,0}, \pi_{n,1}, \cdots, \pi_{n,B})$, for n=0,1,2,$\cdots$. At first, assume $d = 1$ in the following lemmas.

Lemma 2.1 *From state balance equations, we have the following equation by aggregately addition for each $n > 0$, i.e.,*
$-\lambda(\pi_{n-1,0} + \pi_{n-1,1} + \dots + \pi_{n-1,B}) + k\mu_1(\pi_{n,0} + \pi_{n,1} + \dots + \pi_{n,B}) = 0$
where $k = n$ if $1 \leq n < C$; $k = C$, otherwise.

Proof.
Let $\mathbf{e}$ be a column vector of accordingly size with every entry of 1. We consider

$$(\boldsymbol{\pi}_0\mathbf{B}_1 + \boldsymbol{\pi}_1\mathbf{A}_{21})\mathbf{e} = \mathbf{0}$$

which expresses the state balance equations of having 0 job at stage 1. It gives
$-\lambda\pi_{0,0} + \mu_2\pi_{0,1} + \mu_1(1-q)\pi_{1,0}+$
$-(\mu_2+\lambda)\pi_{0,1} + 2\mu_2\pi_{0,2} + \mu_1 q\pi_{1,0} + \mu_1(1-q)\pi_{1,1}+$
$-(2\mu_2+\lambda)\pi_{0,2} + 3\mu_2\pi_{0,3} + \mu_1 q\pi_{1,1} + \mu_1(1-q)\pi_{1,2} + ...$
$-((d-1)\mu_2+\lambda)\pi_{0,d-1} + d\mu_2\pi_{0,d} + \mu_1 q\pi_{1,d-2} + \mu_1(1-q)\pi_{1,d-1}+$
$-(d\mu_2+\lambda)\pi_{0,d} + d\mu_2\pi_{0,d+1} + \mu_1 q\pi_{1,d-1} + \mu_1(1-q)\pi_{1,d} + ...$
$-(d\mu_2+\lambda)\pi_{0,B-1} + d\mu_2\pi_{0,B} + \mu_1 q\pi_{1,B-2} + \mu_1(1-q)\pi_{1,B-1}+$
$-(d\mu_2+\lambda)\pi_{0,B} + \mu_1 q\pi_{1,B-1} + \mu_1\pi_{1,B} = 0.$

After a few simple algebra it is rewritten as concisely

$$-\lambda(\pi_{0,0}+\pi_{0,1}+...+\pi_{0,B})+\mu_1(\pi_{1,0}+\pi_{1,1}+....+\pi_{1,B}) = 0. \quad (2.2)$$

and the expressions of state balance equations when there are $n > 0$ jobs at stage 1, are

$$(\boldsymbol{\pi}_n\mathbf{A}_0 + \boldsymbol{\pi}_{n+1}\mathbf{A}_{1(n+1)} + \boldsymbol{\pi}_{n+2}\mathbf{A}_{2(n+2)})\mathbf{e} = \mathbf{0}.$$

It may be rewritten elaborately as
$\lambda\pi_{n,0} - (\lambda + k\mu_1)\pi_{n+1,0} + \mu_2\pi_{n+1,1} + k^*\mu_1(1-q)\pi_{n+2,0} + \lambda\pi_{n,1} - (\lambda + k\mu_1 + \mu_2)\pi_{n+1,1} + \mu_2\pi_{n+1,2} + k^*\mu_1 q\pi_{n+2,0} + k^*\mu_1(1-q)\pi_{n+2,1} + \lambda\pi_{n,2} - (\lambda + k\mu_1 + \mu_2)\pi_{n+1,2} + \mu_2\pi_{n+1,3} + k^*\mu_1 q\pi_{n+2,1} + k^*\mu_1(1-q)\pi_{n+2,2} + ... + \lambda\pi_{n,B-1} - (\lambda + k\mu_1 + \mu_2)\pi_{n+1,B-1} + \mu_2\pi_{n+1,B} + k^*\mu_1 q\pi_{n+2,B-2} + k^*\mu_1(1-q)\pi_{n+2,B-1} + \lambda\pi_{n,B} - (\lambda + k\mu_1 + \mu_2)\pi_{n+1,B} + k^*\mu_1 q\pi_{n+2,B-1} + k^*\mu_1\pi_{n+2,B} = 0.$

where k and k^* are associated with $n+1$ and $n+2$ as defined in (2.1) respectively. By further calculation we obtain

$$\begin{aligned}&\lambda(\pi_{n,0} + \pi_{n,1} + ... + \pi_{n,B}) - (\lambda + k\mu_1)\\&(\pi_{n+1,0} + \pi_{n+1,1} + ... + \pi_{n+1,B})\\&+k^*\mu_1(\pi_{n+2,0} + \pi_{n+2,1} + ... + \pi_{n+2,B}) = 0.\end{aligned} \quad (2.3)$$

From (2.2) and (2.3) as $n = 0$, we have
$\lambda(\pi_{0,0} + \pi_{0,1} + ... + \pi_{0,B}) - (\lambda + \mu_1)(\pi_{1,0} + \pi_{1,1} + ... + \pi_{1,B}) + 2\mu_1(\pi_{2,0} + \pi_{2,1} + ... + \pi_{2,B}) = 0.$
and
$-\lambda(\pi_{0,0}+\pi_{0,1}+...+\pi_{0,B})+\mu_1(\pi_{1,0}+\pi_{1,1}+....+\pi_{1,B}) = 0$
together which implies $-\lambda(\pi_{1,0} + \pi_{1,1} + ... + \pi_{1,B}) + 2\mu_1(\pi_{2,0} + \pi_{2,1} + ... + \pi_{2,B}) = 0.$
By the similar manner, we can iteratively derive the expressions according to each $n > 1$. □

Lemma 2.2 *With each* m, $m = 0, 1, \cdots, B$, *we have* $\mu_2(\pi_{0,m+1}+\pi_{1,m+1}+...)-\mu_1 q(\pi_{1,m}+2\pi_{2,m}+3\pi_{3,m}+ ...) = 0.$

Proof.
Let $\mathbf{e}_p$ be a unit column vector with the pth entry of 1, for $1 \le p \le B+1$.
First, we consider the equations when there is 0 job at stage 2, namely,
$(\boldsymbol{\pi}_0\mathbf{B}_1 + \boldsymbol{\pi}_1\mathbf{A}_{21})\mathbf{e}_1 + \sum_{i=0}^{\infty}(\boldsymbol{\pi}_i\mathbf{A}_0 + \boldsymbol{\pi}_{i+1}\mathbf{A}_{1(i+1)} + \boldsymbol{\pi}_{i+2}\mathbf{A}_{2(i+2)})\mathbf{e}_1 = \mathbf{0}.$
It gives
$-\lambda\pi_{0,0} + \mu_2\pi_{0,1} + \mu_1(1-q)\pi_{1,0}+$
$\lambda\pi_{0,0} - (\lambda+\mu_1)\pi_{1,0} + \mu_2\pi_{1,1} + 2\mu_1(1-q)\pi_{2,0}+$
$\lambda\pi_{1,0} - (\lambda+2\mu_1)\pi_{2,0} + \mu_2\pi_{2,1} + 3\mu_1(1-q)\pi_{3,0} + ... = 0.$
By further aggregation of corresponding states, we have

$$\begin{aligned}&-\mu_1 q(\pi_{1,0} + 2\pi_{2,0} + ... + c\pi_{c,0} + ...) + \mu_2(\pi_{0,1} + \pi_{1,1}\\&+... + \pi_{c-1,1} + ...) = 0.\end{aligned} \quad (2.4)$$

Secondly, consider when there are $p-1$ jobs at stage 2, for $2 \le p \le B+1$.
It gives similar expressions as $(\boldsymbol{\pi}_0\mathbf{B}_1 + \boldsymbol{\pi}_1\mathbf{A}_{21})\mathbf{e}_p + \sum_{i=0}^{\infty}(\boldsymbol{\pi}_i\mathbf{A}_0 + \boldsymbol{\pi}_{i+1}\mathbf{A}_{1(i+1)} + \boldsymbol{\pi}_{i+2}\mathbf{A}_{2(i+2)})\mathbf{e}_p = 0.$
It results in
$-(\mu_2+\lambda)\pi_{0,n} + \mu_2\pi_{0,n+1} + \mu_1 q\pi_{1,n-1} + \mu_1(1-q)\pi_{1,n} + \lambda\pi_{0,n} - (\lambda + \mu_1 + \mu_2)\pi_{1,n} + \mu_2\pi_{1,n+1} + 2\mu_1 q\pi_{2,n-1} + 2\mu_1(1-q)\pi_{2,n} + \lambda\pi_{1,n} - (\lambda+2\mu_1+\mu_2)\pi_{2,n} + \mu_2\pi_{2,n+1} + 3\mu_1 q\pi_{3,n-1} + 3\mu_1(1-q)\pi_{3,n} + \cdots = 0.$
By further aggregation of corresponding states, we have

$$\begin{aligned}&-\mu_2(\pi_{0,n} + \pi_{1,n} + ...) + \mu_2(\pi_{0,n+1} + \pi_{1,n+1} + ...)\\&-\mu_1 q(\pi_{1,n} + 2\pi_{2,n} + 3\pi_{3,n} + ...) + \mu_1 q(\pi_{1,n-1}+\\&2\pi_{2,n-1} + 3\pi_{3,n-1} + \cdots) = 0\end{aligned} \quad (2.5)$$

By (2.4) and (2.5) as $n = 1$, we have

$$-\mu_1 q(\pi_{1,1}+2\pi_{2,1}+3\pi_{3,1}+...)+\mu_2(\pi_{0,2}+\pi_{1,2}+...) = 0.$$

By the similar manner, we can iteratively derive the expressions according to each $m > 1$. Thus the stationary state probability is derived and the following lemma concludes the result. □

Lemma 2.3 *The solution has the form of* $\pi_{n,m} = \gamma(\prod_{i=1}^{n} \eta_{1,i})(\eta_2)^m$ *and* γ *is a normalization constant that makes the balance equations satisfied with the probability law.*

Proof.
By Lemma 2.1 we have $n \geq 0$
$\lambda \boldsymbol{\pi}_n \mathbf{e} = k\mu_1 \boldsymbol{\pi}_{n+1}\mathbf{e}$.
It produces
$\lambda \sum_{m=0}^{B} \gamma(\prod_{i=1}^{n} \eta_{1,i})(\eta_2)^m = k\mu_1 \sum_{m=0}^{B} \gamma(\prod_{i=1}^{n+1} \eta_{1,i})(\eta_2)^m$ implies $\eta_{1,n+1} = \frac{\lambda}{k\mu_1}$, where k is $n+1$ when $n+1 < C$; $k = C$ otherwise. To prove the form of $\eta_{1,i}$, We define for $0 \leq m \leq B$,
$P_m = \pi_{0,m} + \pi_{1,m} + ...$
$P_m^{'} = \pi_{1,m} + 2\pi_{2,m} + ...$
By Lemma 2.2, it derives $\mu_2 P_{m+1} = q\mu_1 P_m^{'}$.
Furthermore, it gives
$\mu_2 \sum_{n=0}^{\infty} \gamma(\prod_{i=1}^{n} \eta_{1,i})(\eta_2)^{m+1}$
$= q\mu_1 \sum_{n=1}^{\infty} \gamma(\prod_{i=1}^{n} \eta_{1,i})(\eta_2)^m$ implying,
$\mu_1 q(\frac{\lambda}{\mu_1} + 2\frac{\lambda}{\mu_1}\frac{\lambda}{2\mu_1} + 3\frac{\lambda}{\mu_1}\frac{\lambda}{2\mu_1}\frac{\lambda}{3\mu_1}...)$
$= \mu_2\eta_2(1 + \frac{\lambda}{\mu_1} + \frac{\lambda}{\mu_1}\frac{\lambda}{2\mu_1} + ...)$

Therefore, we have $\eta_2 = \frac{\lambda q}{\mu_2}$. Hence, we prove the form of η_2. □

Conclusively, it is easy to derive γ by $\sum_{n=0}^{\infty} \sum_{m=0}^{B} \pi_{n,m} = 1$ whose form is given in Theorem 2.1. Hopefully, the same pattern and proof of solutions may be applied for the model of $M/M/c \rightarrow /M/d/B$.

Theorem 2.1 *The stationary state probability for* $M/M/c \rightarrow /M/d/B$ *with its components which have the form of* $\pi_{n,m} = \gamma(\prod_{i=1}^{n} \eta_{1,i})(\prod_{j=1}^{m} \eta_{2,j})$, *where*

$$\eta_{1,i} = \begin{cases} \frac{\lambda}{i\mu_1} & \text{if} \quad 1 \leq i < c \\ \frac{\lambda}{c\mu_1} & \text{if} \quad i \geq c \end{cases}$$

$$\eta_{2,j} = \begin{cases} \frac{q\lambda}{j\mu_2} & \text{if} \quad 1 \leq j < d \\ \frac{q\lambda}{d\mu_2} & \text{if} \quad j \geq d \end{cases}$$

and

$$\gamma^{-1} = (1 + \sum_{i=1}^{c-1} (\frac{1}{i!})(\frac{\lambda}{\mu_1})^i + \sum_{i=c}^{\infty} (\frac{1}{c})^c (\frac{1}{c})^{i-c} (\frac{\lambda}{\mu_1})^i)$$
$$(1 + \sum_{j=1}^{d-1} (\frac{1}{j!})(\frac{q\lambda}{\mu_2})^j + \sum_{j=d}^{B} (\frac{1}{d})^d (\frac{1}{d})^{j-d} (\frac{q\lambda}{\mu_2})^j)$$

Proof.
By the same approach used in Lemma 3.1 we have
$\lambda \pi_n \mathbf{e} = k\mu_1 \pi_{n+1}\mathbf{e}$
$\lambda \sum_{m=0}^{B} \gamma(\prod_{i=1}^{n} \eta_{1,i})(\prod_{j=1}^{m} \eta_{2,j})$
$= k\mu_1 \sum_{m=0}^{B} \gamma(\prod_{i=1}^{n+1} \eta_{1,i})(\prod_{j=1}^{m} \eta_{2,j})$. By comparing with each term for $n \geq 0$, it yields $\eta_{1,i} = \frac{\lambda}{k\mu_1}$ where $k = i$ if $i < c$ and $\eta_{1,0} = 1$; otherwise $k = c$. Thus, we prove the form of $\eta_{1,i}$.
Next, for a fixed m, we define:
$P_m = \pi_{0,m} + \pi_{1,m} + ...$
$P_m^{'} = \pi_{1,m} + 2\pi_{2,m} + ...$
The same approach as used in Lemma 3.2 it gives $k^{'}\mu_2 P_{m+1} = \mu_1 q P_m^{'}$. It can be rewritten as
$k^{'}\mu_2 \sum_{n=0}^{\infty} \gamma(\prod_{i=1}^{n} \eta_{1,i})(\prod_{j=1}^{m+1} \eta_{2,j})$
$= \mu_1 q \sum_{n=1}^{\infty} \gamma(\prod_{i=1}^{n} \eta_{1,i})(\prod_{j=1}^{m} \eta_{2,j})$.
It can be further simplified as
$k^{'}\mu_2\eta_{2,m+1}(1 + \frac{\lambda}{\mu_1} + \frac{\lambda}{\mu_1}\frac{\lambda}{2\mu_1} + ...) = \mu_1 q(\frac{\lambda}{\mu_1} + 2\frac{\lambda}{\mu_1}\frac{\lambda}{2\mu_1} + 3\frac{\lambda}{\mu_1}\frac{\lambda}{2\mu_1}\frac{\lambda}{3\mu_1}...)$
By comparing with each term for $B \geq m \geq 0$, it yields $\eta_{2,j} = \frac{\lambda q}{k^{'}\mu_2}$, where $k^{'}$ is j when $j < d < B$; otherwise $k^{'} = d$. Thus, we prove the form of $\eta_{2,j}$. By the same argument as described previously, it is easy to prove the total probability law holds for $\sum_{n=0}^{\infty} \sum_{m=0}^{B} \pi_{n,m} = 1$. □

Consider a folk-type queueing system as drawn in Figure 2 where there are c, d and g servers of service rates μ_1, μ_2 and μ_3 at station 1 ,2 and 3 respectively. The stationary probability is given in the following theorem.

Theorem 2.2 *The stationary state probability for* $M/M/c \rightarrow /M/d/B$ *with probability of* q *and* $M/M/c \rightarrow /M/g/H$ *with probability of* $1-q$ *and its components which have the form of*
$\pi_{n,m,p} = \gamma(\prod_{i=1}^{n} \eta_{1,i})(\prod_{j=1}^{m} \eta_{2,j})(\prod_{k=1}^{p} \eta_{3,k})$, *where*

$\eta_{1,i}$ *and* $\eta_{2,j}$ *are defined as before but*

$$\eta_{3,k} = \begin{cases} \frac{(1-q)\lambda}{k\mu_3} & if \quad 1 \leq k < g \\ \frac{(1-q)\lambda}{g\mu_3} & if \quad k \geq g \end{cases}$$

and

$$\gamma^{-1} = (1 + \sum_{i=1}^{c-1}(\frac{1}{i!})(\frac{\lambda}{\mu_1})^i + \sum_{i=c}^{\infty}(\frac{1}{c})^c(\frac{1}{c})^{i-c}(\frac{\lambda}{\mu_1})^i)$$
$$\times(1 + \sum_{j=1}^{d-1}(\frac{1}{j!})(\frac{q\lambda}{\mu_2})^j + \sum_{j=d}^{B}(\frac{1}{d})^d(\frac{1}{d})^{j-d}(\frac{q\lambda}{\mu_2})^j)$$
$$\times(1 + \sum_{k=1}^{g-1}(\frac{1}{k!})(\frac{(1-q)\lambda}{\mu_3})^k$$
$$+ \sum_{k=g}^{H}(\frac{1}{g})^g(\frac{1}{g})^{k-g}(\frac{(1-q)\lambda}{\mu_3})^j).$$

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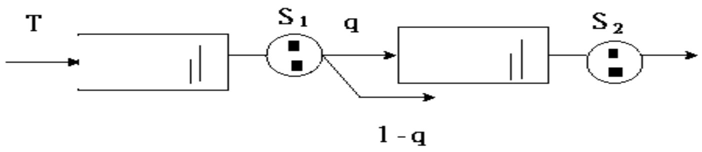

Figure 1: A simple fork-type queueing Model

From	To	Rate		Condition
(n,m)	$(n+1,m)$	λ	for	$n \geq 0$
(n,m)	$(n-1,m+1)$	$\mu_1 q$	for	$n > 0, 0 \leq m \leq B-1$
(n,m)	$(n-1,m)$	$\mu_1(1-q)$	for	$n > 0, 0 \leq m \leq B-1$
(n,m)	$(n,m-1)$	μ_2	for	$m \geq 1$
(n,m)	$(n-1,m)$	μ_1	for	$n > 0, m = B$

Table 1: State transition table

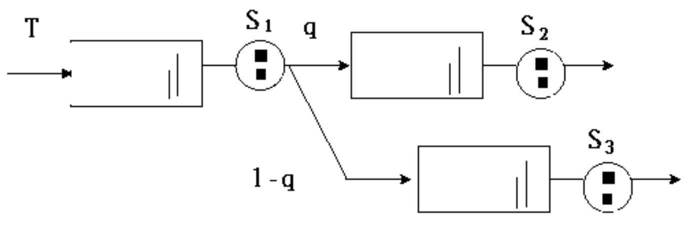

Figure 2: A folk-type tandem queue model with a state dependent probability

行政院國家科學委員會補助國內專家學者出席國際學術會議報告

101 年　04 月　30 日

報告人姓名	陸行	服務機構及職稱	政治大學應用數學系教授
時間 會議地點	2012/4/21-2012/4/23 Chicago, USA	本會核定補助文號	
會議名稱	（中文） （英文）The 2012 POMS Annual Conference		
發表論文題目	（中文） （英文）A two-stage security-check system at border-crossing stations		

一、　參加會議經過

在參加洛利之「1st International IBM Cloud Academy Conference」後，緊接著參加此次會議。會議在芝加哥的 Marriott 的飯店舉行，為期四天。

這個會議有超過 1000 篇的論文發表，是大型的會議，專家學者來自世界各地的，如美國、法國、挪威、瑞典、以色列、意大利、韓國、日本、英國和。大會的子題共有 27 個，詳見附錄。

研究報告是「A two-stage security-check system at border-crossing stations」內容是關於分析安全服務系統，這一類的服務系統中存在兩個主要目標：最大化安全檢查的有效性，和最大限度地減少客戶的等待時間。研究發現，這兩個目標可以是一致的。或根據對容量狀態調整系統的衝突，主要是建立二階段式的選擇性安全檢查系統，研究中將此系統建成數學模

型，以一個兩階段的排隊過程分析其等候時間。每一位旅客都需經過第一階段的督察檢查，然後選出一個比例的客戶通過第二階段作進一步檢查。這個進一步檢查的比例，可以數學證明確定保有足夠的安檢水平，成為一個檢查約束。另外，也提供一階段與二階段之檢查比較分析，基於兩者之性能，以精確和近似方法計算隊列長度和等候時間以表示其服務能力。討論在一個"安全有利的"，"安全不利"或"安全不可行"的狀態，根據到達率和必要的安全水平調查在美國和加拿大邊界過境站之安全檢查的排隊系統。我們發現，兩個階段的安全檢查系統優於一階段安全檢查系統。雖然這些研究結果證實單一服務器模型，但是這些特性也保持在一個更一般性多服務器設置。

筆者也參加其他的議題報告內含如下列議題

- Observations from OASIS Violations Data for Border Inspection
- A Meta-Heuristic Optimization for Scheduling Heat-Treatment Furnace in Steel Casting Industry
- QRA and RBD techniques to evaluate the cost-effectiveness of transport security systems
- The Demand Weighted Vehicle Routing Problem
- A serial scheme for resource-constrained scheduling within Microsoft Project 2010
- Scheduling of Multi-skilled Staff Across Multiple Locations
- Bicriteria Scheduling with Batch Deliveries
- Approximation Schemes for Parallel Machine Scheduling with Availability
- Single machine batch scheduling with release times and delivery costs
- Solution Approaches for a Joint Production and Transportation Planning Problem
- Optimal delivery time quotation in supply chains to minimize tardiness and delivery costs
- Bi-criteria Scheduling Subject to Machine Availability
- Design of Characteristic Functions for Cooperative Games in Operations Planning
- Production Scheduling with Subcontracting: The Subcontractor's Pricing Game

經過進一步的討論，筆者獲得許多寶貴的經驗，對未來的研究和研究議題提供深遠的影響。

二、 與會心得

筆者能參加此次會議，純屬因緣巧合。原因受到議題主持人的邀請，雖然此次會議並不是筆者過去長期參與的會議，但是參與後發現許多值得學習的題目和研究方法。同時也可以將筆者過去研究出之評估效能的數學模型方法介紹給從事智慧型製造研究的學者專家。也希望能在會議中能吸取其他的工業事例，藉此運用數學模型解決他們的問題。

三、 攜回資料名稱及內容

光碟會議集 (Proceedings of POMS 2012 Conference Program)。

四、 附錄

會議 Topics

Behavioral Operations
Capacity Management
Empirical Research in OM
Finance and OM Interface
Healthcare Operations Management
Humanitarian Operations and Disaster Management
Inventory Management
Manufacturing Operations
Operations Management in China and East Asia
OM in India / SE Asia
OM in Latin America and the Caribbean
Operations Management and Economic Models
OM – Marketing Interface
OM-Practice
OM in Travel, Tourism and Hospitality
Product Innovation and Technology Management
Product Management
Production Planning and Scheduling

Retail Operations Management
Scheduling and Logistics
Service Operations
Supply Management
Supply Chain Management
Supply Chain Risk Management
Sustainable Operations
Teaching in OM
Vendor and Supply Contracts

After 911

Waiting at Security Screening point at an airport

Border Line-ups - *South-bound AM hours – to the USA*

http://www.th.gov.bc.ca/ATIS/index.htm

BORDER POLICY RESEARCH INSTITUTE

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Mission

The Border Policy Research Institute (BPRI) is a multi-disciplinary research institute at Western Washington University in Bellingham, Washington. The BPRI focuses on research that informs policy-makers on matters related to the Canada - U.S. border. Priority focus areas are trade and transportation, economics, environment, immigration, and border security. The University established the BPRI to further a mission of promoting research, academic programs, and public programming on critical policy issues affecting the Pacific Northwest. The BPRI works closely with cognate programs at the University and collaborates with many public and private entities within the Pacific Northwest.

Announcements

Summer 2010 Border Policy Brief Now Available

Titled "WHTI, the Recession, and Cross-Border Travel," this Brief looks at recent cross-border travel trends across the Mexico-US and the Canada-US borders, seeking to understand the impacts attributable to the implementation of the Western Hemisphere Travel Initiative (WHTI) and the economic "meltdown" of late 2008. Download Policy Brief.

Research Report No. 10 Now Available

A two-stage security-check system at border-crossing stations

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National Chengchi University

Outline

- Introduction
- Modeling and Analysis
- Numerical Illustrations and Model validation
- Practical Value of the Study
- Conclusion

I. Introduction – main issues

- Two Main Goals:
  - To achieve sufficient security-level.
  - To offer good customer service (or less waiting time).
- Questions:
  - Does increasing required security level always lead to increasing customer waiting cost?
  - Which configuration of security-check facility performs better? Two-stage or One-stage inspection?

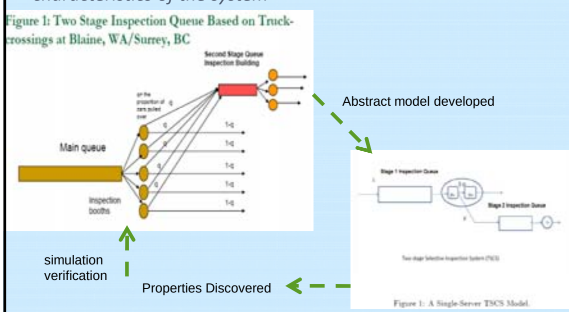
Our approach

- Develop a stylized two-stage queueing model to capture the main features of the system.
- Discover the performance characteristics and identify the major impacts of the key decision variable.
- Validate these properties discovered via simulation model

Related Literature – a sample

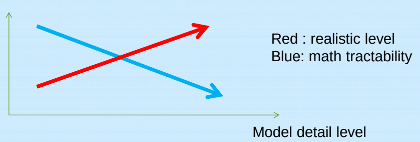
- Queueing models – El-Taha and Maddah (2006), Zhang (2009), Whitt (2008, 2009), etc.
- Security-Check Models – Kobza and Jacobson (1997),
- Jacobson et al. (2001), Lee et al (2009), Babu et al (2006), and Zhang (2009).

II. Modeling - *An abstract model to capture the main characteristics of the system*

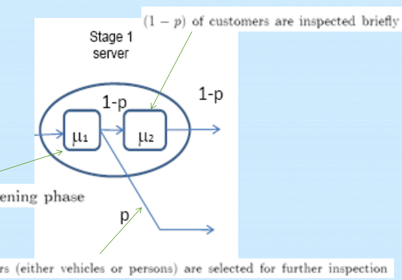


Determine the Abstract Level

- Modeling the critical factors (components)
- Ignoring the non-significant details
- Developing a realistic model to represent the practical system
- Keeping the mathematical tractability and simplicity (both exact and approximation approaches)



Here is an example: Modeling Primary (Stage 1) Inspection Operations at a border crossing station



Exact Analysis v.s Approximation Approach

- ***If the exact (but complex) analysis is feasible, do we need the approximation approach?***

It depends..

- Purpose of the study
  - – performance evaluation/optimization for systems with predictable general behaviors;
  - - insights generation for systems with unknown behaviors.
- Computational complexity of the exact approach
- Robustness and accuracy of the approximation approach.

Exact Analysis

- Computational approach – algorithm-based and no-closed form formulas.
- More restrictive assumptions.
- Computational complexity v.s Realistic modeling.

But…

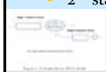
- Can be used to verify the approximations in special cases.

When is an approximation approach worthwhile for insight generation?

- Captures the main characteristics of the system
- Needs realistic conditions
  - e.g. Heavy traffic condition
- Is accurate and robust enough
- Has Relatively simple computations
- Reveals the fundamental relationship between the main performance measure(s) and the key decision variable(s) for a given set of system parameters.

An example of good approximation: In security-check queues, the key decision variable is *p* – proportion of selected customers for further inspection – *(this section is technical and details may be omitted in presentation)*

- 2^{nd} stage queue performance (as the 1^{st} stage is trivial) based on bounds:



$$E(W_2^q)_{\text{approx}} = \frac{1}{2}\left(\frac{r_0}{\nu(1-r_0)} + \frac{\lambda p}{\nu(\nu-\lambda p)}\right).$$

solve the functional equation of $A(\nu(1-z)) = z$ for the root, denoted by r_0.

- Key approximation – LST of Inter-arrival times for 2^{nd} Stage Queue(based on heavy traffic condition):

$$A(s) = \frac{pX(s)\,[\rho_1 + (1-\rho_1)I(s)]}{1-(1-p)\,[\rho_1 + (1-\rho_1)I(s)]\,Y(s)},$$

- Further approximation: $r_0 = a + bp$

$$E(T_2)_{approx} = \frac{1}{2}\left(\frac{1}{\nu(1-a-bp)} + \frac{1}{\nu-\lambda p}\right), \quad (13)$$

where a and b are the regression constants for a given parameter set of $\lambda, \mu_1, \mu_2,$ and ν, and is also increasing and convex in p.

- Then, we can establish the convexity of the performance measures in the key decision variable.

Minimum p – required security level

(*The details of this section may be omitted in presentation*)

In realty, the actual proportion of customers for further inspection is determined by (a) screening stanrdard, and (b) random number generation.

- Proportion in (a), denoted by p_c, is not controllable and proportion in (b), denoted by p_d, is controllable.
- Actual proportion $p = p_c + p_d$ is also controllable via p_d.

Define the events A =The inspection system gives an alarm; and T =The customer carries a threat. Denote the event of further inspection (or no further inspection) by FI (or FI^c). For a given $P(FI) = p$, We have some useful probabilities defined as follows: $\theta_{FI} = P(A|T \cap FI), \theta_{FI^c} = P(A|T \cap FI^c), \tau = P(T), \alpha(p) = P(T|FI), \beta(p) = P(T|FI^c)$. Since the inspection procedure for further inspection is stricter than that of stage 1's primary inspection, we have $\theta_{FI} > \theta_{FI^c}$. In addition, for an effective initial screeining performed in stage 1, it is reasonable to assume $\alpha(p) = P(T|FI) > \tau = P(T) > \beta(p) = P(T|FI^c)$.

Sufficient security level

	Carrying a threat	No threat
Alarm	TA	FA
Clear	FC	TC

Ho: a customer does not carry a threat;
Ha: a customer carries a threat.
Rejecting Ho is equivalent to generating an "alarm"
Accepting Ho is equivalent to generating a "clear"

P(FC)=P(Type 2 Error) - fatal; P(FA)=P(Type 1 Error) – increasing congestion level

$$P(TA) = f(p) = \theta_{FI}\alpha(p)p + \theta_{FI^c}\beta(p)(1-p).$$

Minimize P(FC) is equivalent to Maximize P(TA) for a given P(T)=P(TA)+P(FC).

$P(TA)$ is an increasing function of p.

In practice, the minimum $p = p_0$ is determined by ensuring a sufficiently high $P(TA)$ (or a sufficiently low $P(FC)$) is achieved.

An Example

For a given initial screening procedure, we have $\alpha(p) = p_d\tau + (1-p_d)\gamma$, where $\gamma = P(T|\text{selected for further inspection by initial screening procedure})$. For example, suppose that $\tau = P(T) = 0.02, \theta_{FI} = 0.99, \theta_{FI^c} = 0.80, \gamma = 0.048, p_c = 0.05$ (these values are in the same magnitutde as the data provided by the Bureau of Transportation Statistics 2006) and If we want the probability of false clear (Type II error) is no more than 0.002 or $P(FC) \leq 0.002$, a required security level, using this data set, $\tau = \alpha(p)p + \beta(p)(1-p), p = p_d + p_c, \alpha(p) = p_d\tau + (1-p_d)\gamma$ and the $P(TA)$ expression we can find $p_0 = 0.2479$. This means that to ensure the security level to be achieved, at least 24.79% of customers should be subject to further inspection. Since $p = p_d + p_c \geq p_0 = 0.2479$ and $p_c = 0.05$, we have $p_d \geq 0.1979$. In other words, as long as the we use random number generation to select at least 19.79% of customers for futher inspection, the security level can be reached.

Cost of Raising Security Level

- Higher p => higher $P(T^c|FI) = 1-\alpha(p)$

=> More non-threat-carrying customers need to go through further inspection..

- Question: If you are a customer crossing the border, do you like a higher p or a lower p?

- Any answers??

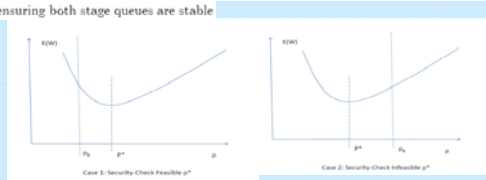
After p_0 is determined, E(WC) should be minimized

$$\min_p E(WC) = (1-p)E(T_1)h_1 + p[E(W_1^q) + 1/\mu_1 + E(T_2)]h_2$$
$$\approx E(WC)_{\text{approx}} = (1-p)E(T_1)h_1 + p[E(W_1^q) + 1/\mu_1 + E(T_2)_{\text{approx}}]h_2$$

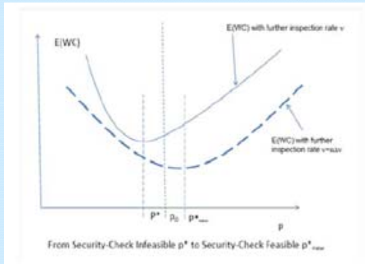
Subject to

$$\max\{p_0, p_{\min}\} < p < p_{\max}$$

where p_0 is the required security-check level (the minimum proportion of customers needed to go through further inspection) and $p_{\max} = \nu/\lambda$ and $p_{\min} = 1 - \mu_2(1/\lambda - 1/\mu_1)$ which are determined by ensuring both stage queues are stable



Implication of capacity status

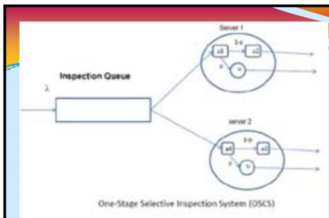


III Numerical Illustrations and Model Validation

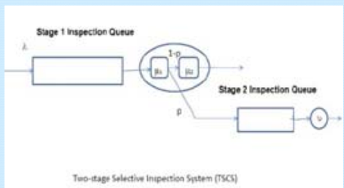
- Accuracy of Approximation

p	$E(W_1^q)$	$E(W_2^q)_{approx}$	$E(W_2^q)_{simu}$	95% CI of $E(W_2^q)_{approx}$	error of $E(W_2^q)_{approx}$	$E(W)$	$E(T)$
0.20	0.6094	0.0315	0.0311	(0.0303, 0.0320)	1.34%	0.6157	0.7420
0.25	0.4722	0.0419	0.0410	(0.0382, 0.0439)	2.09%	0.4827	0.6114
0.30	0.3787	0.0536	0.0525	(0.0493, 0.0558)	2.09%	0.3947	0.5259
0.35	0.3108	0.0669	0.0632	(0.0598, 0.0670)	5.89%	0.3342	0.4677
0.40	0.2592	0.0823	0.0783	(0.0709, 0.0856)	5.12%	0.2921	0.4281
0.45	0.2188	0.1002	0.0949	(0.0862, 0.1036)	5.62%	0.2639	0.4023
0.50	0.1862	0.1214	0.1135	(0.1051, 0.1220)	6.96%	0.2469	0.3877
0.55	0.1594	0.1468	0.1407	(0.1303, 0.1510)	4.33%	0.2401	0.3833
0.60	0.1369	0.1779	0.1685	(0.1566, 0.1800)	6.89%	0.2436	0.3893
0.65	0.1178	0.2169	0.2031	(0.1874, 0.2189)	6.79%	0.2588	0.4069
0.70	0.1014	0.2674	0.2544	(0.2352, 0.2736)	5.13%	0.2887	0.4391
0.75	0.0872	0.3358	0.3190	(0.2876, 0.3504)	5.27%	0.3390	0.4919
0.80	0.0747	0.4339	0.4218	(0.3769, 0.4667)	2.87%	0.4218	0.5771

Table 1. Performance of a Single Server TSCS with $\lambda = 8.5, \mu_1 = 20, \mu_2 = 15, \nu = 8.7$.



- v.s



Unique Advantage of TSCS

p	$E(D_l)$	$E(D_h)_{approx}$	ρ_1	ρ_2	$E(T)_{OSCS}$	ρ_{OSCS}
0.12	0.3513	0.4550	0.9514	0.4229	0.0871	0.7576
0.13	0.2974	0.4093	0.9426	0.4581	0.0979	0.7767
0.14	0.2578	0.3792	0.9338	0.4933	0.1107	0.7958
0.15	0.2275	0.3597	0.9250	0.5286	0.1262	0.8149
0.16	0.2036	0.3483	0.9162	0.5638	0.1452	0.8340
0.17	0.1842	0.3437	0.9074	0.5990	0.1691	0.8531
0.18	0.1682	0.3453	0.8986	0.6343	0.2000	0.8721
0.19	0.1548	0.3532	0.8898	0.6695	0.2418	0.8912
0.20	0.1433	0.3682	0.8810	0.7048	0.3012	0.9103
0.21	0.1335	0.3919	0.8721	0.7400	0.3926	0.9294
0.22	0.1249	0.4274	0.8633	0.7752	0.5515	0.9485
0.23	0.1173	0.4803	0.8545	0.8105	0.8950	0.9676
0.24	0.1106	0.5617	0.8457	0.8457	1.8935	0.9867
0.25	0.1046	0.6960	0.8369	0.8810	infinity	1.0058
0.26	0.0993	0.9488	0.8281	0.9162	infinity	1.0248
0.27	0.0944	1.5766	0.8193	0.9514	infinity	1.0439
0.28	0.0900	5.5666	0.8105	0.9867	infinity	1.0630

Table 3. *Performance Differentiation Between Low-risk and High-risk Customers in a Two Stage Security Check Single Server System* $\lambda = 52.8571, \mu_1 = 300, \mu_2 = 60, \nu = 15, h_1 = 3, h_2 = 2.$

$E(D_l) = E(T_1)$

$E(D_h) \approx E(D_h)_{\text{approx}} = E(W_1^q) + 1/\mu_1 + E(W_2^q)_{\text{approx}} + 1/\nu$

Real-Dataset

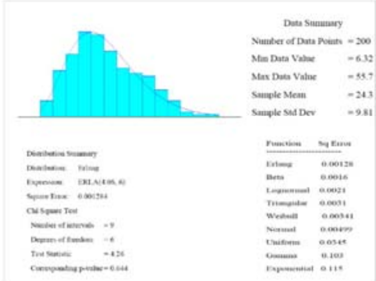
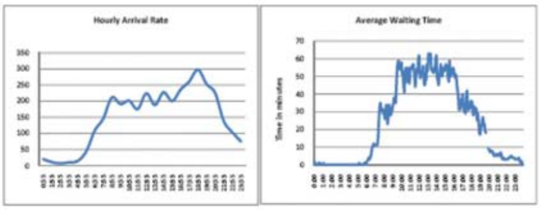


Figure 7. Fit All Options to a Dataset of Phase 1 (Initial Screening) Durations at a Border-Crossing Station.

IV Practical Value of the Study



(http://www.wcog.org/Data.aspx).

Figure 9. Pacific Highway Crossing Station -Southbound Traffic Data on July 9 2010.

Figure 10 presents the performance curves for these two periods computed from the TSCS model. With the security dataset used in Section 2.2, we obtain the minimum $p_0 = 0.109$ for achieving $P(FC) \leq 0.001$. In practice, a $0.11 \leq p_0 \leq 0.12$ was used and our TSCS model in Figure 10 shows the average waiting times at these practical p_0's are between 88 and 50 minutes for the period of 12:00 to 15:00 and between 48 and 33 minutes for the period 17:00 to 20:00, respectively. The average waiting times are consistent with the actual recorded average waiting times of 54 minut for 12:00 to 15:00 and 34 minutes for 17:00 to 20:00 in the right graph of Figure 9.

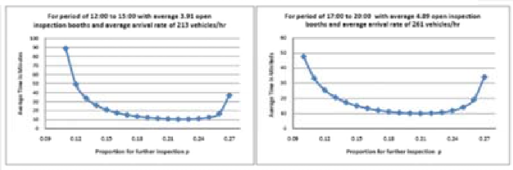


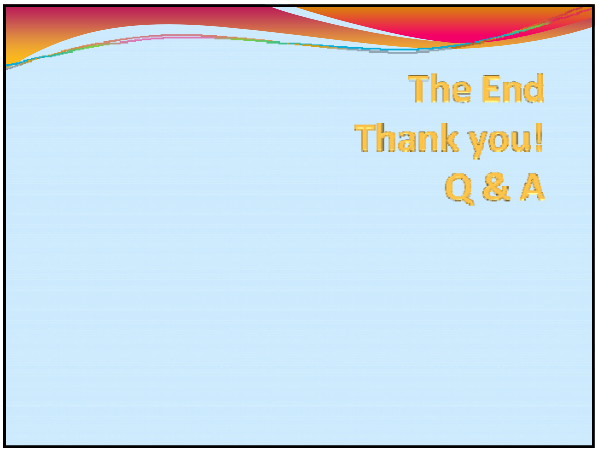
Figure 10. Average Waiting Time as a Function of p for Two Heavy Traffic Periods on July 9, 2010 at Pacific Highway South Crossings with Service Rates of 50 Vehicles per Hour for Stage 1 and 15 Vehicles per Hour for Stage 2.

Information from our model

The optimal p^* obtained from the TSCS model are 0.21 and 0.22 for the two periods of interest Both optimal proportions for further inspection significantly reduce the average waiting times t about 10 minutes for these two periods as shown in Figure 10. Note that the performance char acteristics of the TSCS model have been verified by extensive simulations. In addition, using th optimal p^* can also improve the security screening level as $P(FC)$ is reduced from 0.000998 a $p_0 = 0.11$ to 0.000867 at $p^* = 0.21$. Another implication is that the optimal p^* is relatively insensi tive to the arrival rate change (almost the same p^* for the two periods) due to the congestion-base staffing (adjusting the number of open booths according to the queue length - see Zhang 2009)

V Conclusion

- Addressed the issue of trade-off between security level and customer service quality
- Combined – analytical model, approximations, simulations, and empirical studies
- Generated valuable information and insights for practitioners
- Stimulate more future research topics – e.g. we are now working on the multi-class customer security-check system for airport checkpoints.


The End
Thank you!
Q & A

國科會補助計畫衍生研發成果推廣資料表

日期:2012/09/28

<table>
<tr><td rowspan="3">國科會補助計畫</td><td colspan="2">計畫名稱: 以群聚計算法求解叉狀型等候系統之穩態機率(I)</td></tr>
<tr><td colspan="2">計畫主持人: 陸行</td></tr>
<tr><td>計畫編號: 100-2221-E-004-003-</td><td>學門領域: 作業研究</td></tr>
<tr><td colspan="3">無研發成果推廣資料</td></tr>
</table>

100 年度專題研究計畫研究成果彙整表

計畫主持人：陸行　計畫編號：100-2221-E-004-003-

計畫名稱：以群聚計算法求解叉狀型等候系統之穩態機率(I)

成果項目			量化			單位	備註（質化說明：如數個計畫共同成果、成果列為該期刊之封面故事...等）
			實際已達成數（被接受或已發表）	預期總達成數(含實際已達成數)	本計畫實際貢獻百分比		
國內	論文著作	期刊論文	0	0	100%	篇	
		研究報告/技術報告	0	0	100%		
		研討會論文	0	0	100%		
		專書	0	0	100%		
	專利	申請中件數	0	0	100%	件	
		已獲得件數	0	0	100%		
	技術移轉	件數	0	0	100%	件	
		權利金	0	0	100%	千元	
	參與計畫人力（本國籍）	碩士生	0	0	100%	人次	
		博士生	0	0	100%		
		博士後研究員	0	0	100%		
		專任助理	0	0	100%		
國外	論文著作	期刊論文	0	0	100%	篇	
		研究報告/技術報告	0	0	100%		
		研討會論文	0	0	100%		
		專書	0	0	100%	章/本	
	專利	申請中件數	0	0	100%	件	
		已獲得件數	0	0	100%		
	技術移轉	件數	0	0	100%	件	
		權利金	0	0	100%	千元	
	參與計畫人力（外國籍）	碩士生	0	0	100%	人次	
		博士生	0	0	100%		
		博士後研究員	0	0	100%		
		專任助理	0	0	100%		

其他成果 (無法以量化表達之成果如辦理學術活動、獲得獎項、重要國際合作、研究成果國際影響力及其他協助產業技術發展之具體效益事項等，請以文字敘述填列。)	目前仍在撰寫投稿論文。

	成果項目	量化	名稱或內容性質簡述
科教處計畫加填項目	測驗工具(含質性與量性)	0	
	課程/模組	0	
	電腦及網路系統或工具	0	
	教材	0	
	舉辦之活動/競賽	0	
	研討會/工作坊	0	
	電子報、網站	0	
	計畫成果推廣之參與（閱聽）人數	0	

國科會補助專題研究計畫成果報告自評表

請就研究內容與原計畫相符程度、達成預期目標情況、研究成果之學術或應用價值（簡要敘述成果所代表之意義、價值、影響或進一步發展之可能性）、是否適合在學術期刊發表或申請專利、主要發現或其他有關價值等，作一綜合評估。

1. 請就研究內容與原計畫相符程度、達成預期目標情況作一綜合評估

■達成目標

□未達成目標（請說明，以 100 字為限）

□實驗失敗

□因故實驗中斷

□其他原因

說明：

2. 研究成果在學術期刊發表或申請專利等情形：

論文：□已發表 ■未發表之文稿 □撰寫中 □無

專利：□已獲得 □申請中 ■無

技轉：□已技轉 □洽談中 ■無

其他：（以 100 字為限）

3. 請依學術成就、技術創新、社會影響等方面，評估研究成果之學術或應用價值（簡要敘述成果所代表之意義、價值、影響或進一步發展之可能性）（以 500 字為限）

具有技術開發的價值。