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Correlation Functions for Storage Data and Nonreturn-to-Zero-Inverted Modulated Data of A High-Density Magnetic Recording Channel with Partial Erasure Effect

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***Abstract*—As magnetic recording densities grow, the nonlinear distortions, known as transition shift and partial erasure, arise and limit the performance of the detector. Despite that the transition shift can be compensated by a write current technique, the effect of partial erasure complicates the evaluation of the correlation functions for the storage data and the nonreturn-to-zero-inverted modulated data. In this article, we observe the dependence between the partial erasure effect and storage data, and derive the correlation functions for storage data and nonreturn-to-zero-inverted modulated data in a high density magnetic recording channel. The closed-form of the correlation functions are obtained, these results are applied to verify the timing function of the Muller & Mueller's phase detector in a partial response class IV channel via computer simulations.**

***Index Terms*—partial erasure, phase detector, timing function, partial response class IV, nonreturn-to-zero-inverted modulated data**

I. Introduction

As magnetic recording densities grow, the nonlinear effects, known as transition shift and partial erasure, arise and limit the performance of the detector [1]-[3]. In designing a detector, a timing recovery loop or phase-locked loop is essential for retrieving the storage data and its task is to properly sample the readback signal for further signal processing such as equalization or channel decoding [4]-[9]. In addition, a phase detector is required to estimate the timing error; several methods [10]-[13] have been proposed for realizing the phase detector via using the sampling data. Among these, the Muller & Mueller's (M&M's) phase detector [14] is often adopted for its simple realization in a digital communication system, including the magnetic recording system. Although some primitive results have been discussed[15], [16], few researchers have directly investigated on the timing function of the M&M's phase detector in the presence of the nonlinearities for a magnetic recording channel, These nonlinear effects complicate the statistics of the storage data, which makes the derivation of the timing function non-trivial and difficult. Fortunately, several nonlinear channel models [17]-[21] have been proposed; the simple partial erasure channel model is applied for analyzing the timing function in this article. We first discuss the statistical properties of the storage data and the nonreturn-to-zero-inverted (NRZI) modulated data, and then focus on the derivation of the timing function in the presence of partial erasure with the assumption that the transition shift has been eliminated by the write current precompensation [22], [23]. These results are applied to verify the timing function of the Muller & Mueller'sphase detector in a partial response class IV channel via computer simulations. The results can also be applied in high order partial responses and are expected to be used in designing a minimum mean square error equalizer in the future.

II. Channel Model and Partial Responses

Several channel models have been proposed to describe the nonlinear effects of the magnetic recording channel; the simple partial erasure channel is adopted in this article. Assume that the nonlinear transition shift has been eliminated by the precompensation technique. In terms of the simple partial erasure model, the playback signal of a high-density recording channel can be expressed as

$$y(t) = \sum_k r_k b_k h(t - kT) \tag{1}$$

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where the effective transition-width ratio r_k is determined by the neighboring transitions and is given by

$$r_k = \begin{cases} 1, & b_{k-1} = b_{k+1} = 0 \\ \gamma_1, & \mid b_{k-1} \mid \neq \mid b_{k+1} \mid \\ \gamma_2, & \mid b_{k-1} \mid = \mid b_{k+1} \mid = 2 \end{cases} \quad (2)$$

The nonreturn-to-zero-inverted modulated input b_k with the period T may be of values $\{+2, 0, -2\}$. The term $h(t)$ represents the isolated transition response, which is often modeled as the Lorentzian function

$$h(t) = \frac{1}{1 + (\frac{2t}{\mathrm{PW}_{50}})^2} \quad (3)$$

where the normalized density, PW_{50}, quantifies the recorded bit density. The simple partial erasure model characterizes the channel nonlinearity by the reduction parameters γ_1 and γ_2; when $\gamma_1 = \gamma_2 = 1$, the model is reduced to a linear model. The simple partial erasure model commonly sets $\gamma_2 = \gamma_1^2$, and the nonlinear effect is determined by the reduction parameter γ_1.

For high-density magnetic storage, the model duration is usually long and this often increases the complexity for realizing the detector. Therefore, a linear transversal filter is first applied to partially equalize the channel response to a desired target response such that the effective channel length is reduced. In most applications, the magnetic channel is equalized to a partial response model [2] and then the maximum likelihood (ML) detector is designed to improve the performance in high-density magnetic storage [1]. For the partial response class IV (PR4) magnetic recording, the channel response $h(t)$ is equalized to a proper target response $s(t)$ given by

$$s(t) = q(t) + q(t - T) \quad (4)$$

where $q(t)$ represents a raised-cosine function. As recording densities increase, high order partial responses, such as extended partial response class IV (EPR4) and extended enhanced partial response class IV (EEPR4), are adopted and the corresponding target response $s(t)$ and output d_m are listed in table I.

The sampled output y_m at $t = mT + \tau_m T$ is thus given by

$$y_m = \sum_{k=-L_1}^{L_2} r_{m-k} b_{m-k} s(kT + \tau_{m-k} T) \quad (5)$$

where τ_m is the timing error; the parameters L_1 and L_2 are chosen such that the samples of $s(t)$ for $k < -L_1$ and $k > L_2$ are small enough to be neglected. The sampled output is passed through a designed detector including timing recovery and equalization such that the storage data can be retrieved.

III. Correlation Functions for the Simple Partial Erasure Model

The statistics for the channel model are often required to design a detector in most applications. In the past decade, the recording density is moderately low and the channel model is linear, the statistics for the channel mode are trivial, and given in the following. The data sequence a_m is often assumed independently and identically distributed (IID), i.e.,

$$\begin{cases} \mathrm{E}[a_{m-i} a_{m-j}] = 0 & \text{, if } i \neq j \\ \mathrm{E}[a_{m-i} a_{m-j}] = 1 & \text{, if } i = j \end{cases} \quad (6)$$

where $\mathrm{E}[\cdot]$ represents the expectation operation. Accordingly, the statistics for the NRZI encoded data b_m are given by

$$\begin{cases} \mathrm{E}[b_{m-i} b_{m-j}] = 2 & \text{, if } i = j \\ \mathrm{E}[b_{m-i} b_{m-j}] = -1 & \text{, if } |i-j| = 1 \\ \mathrm{E}[b_{m-i} b_{m-j}] = 0 & \text{, else} \end{cases} \quad (7)$$

However, the above correlation function (7) is no longer valid for a high-density recording channel. In the presence of the partial erasure, the transition-width ratio r_m is determined by b_{m-1} and b_{m+1} or equivalently a_{m-2}, a_{m-1}, a_m and a_{m+1}, and the dependence between r_m and a_m complicates the evaluation of the statistics. The dependence for the data a_m, b_m and r_m are observed and listed in table II. Consider the all possible combinations of the data sequence, and we can obtain the corresponding correlation functions via computer simulations.

a_{m-2}	a_{m-1}	a_m	a_{m+1}	b_{m-1}	b_m	b_{m+1}	r_m
-1	-1	-1	-1	0	0	0	1
1	-1	-1	-1	-2	0	0	γ_1
-1	1	-1	-1	2	-2	0	γ_1
1	1	-1	-1	0	-2	0	1
-1	-1	1	-1	0	2	-2	γ_1
1	-1	1	-1	-2	2	-2	γ_2
-1	1	1	-1	2	0	-2	γ_2
1	1	1	-1	0	0	-2	γ_1
-1	-1	-1	1	0	0	2	γ_1
1	-1	-1	1	-2	0	2	γ_2
-1	1	-1	1	2	-2	2	γ_2
1	1	-1	1	0	-2	2	γ_1
-1	-1	1	1	0	2	0	1
1	-1	1	1	-2	2	0	γ_1
-1	1	1	1	2	0	0	γ_1
1	1	1	1	0	0	0	1

TABLE II
THE DEPENDENCE FOR THE DATA a_m, b_m AND r_m

A. Correlation Function for Initial Data Sequence

The correlation functions for the initial data sequence a_m can be given by

$$\begin{cases} \mathrm{E}[r_{m-l} a_{m-i} a_{m-j}] = \sigma(\gamma_1, \gamma_2), \\ \qquad \text{if } i = j \;\forall\; l \\ \mathrm{E}[r_{m-l} a_{m-i} a_{m-j}] = \rho(\gamma_1), \\ \qquad \text{if } l = i \text{ and } i = j + 1 \\ \mathrm{E}[r_{m-l} a_{m-i} a_{m-j}] = \rho(\gamma_1), \\ \qquad \text{if } l = i - 2 \text{ and } i = j + 1 \\ \mathrm{E}[r_{m-l} a_{m-i} a_{m-j}] = 0, \\ \qquad \text{else} \end{cases} \quad (8)$$

where

$$\sigma(\gamma_1, \gamma_2) = \frac{1}{4} + \frac{1}{2}\gamma_1 + \frac{1}{4}\gamma_2 \quad (9)$$

class of partial response	target response $s(t)$	output d_m
PR4	$q(t)+q(t-T)$	$a_m - a_{m-2}$
EPR4	$q(t)+2q(t-T)+q(t-2T)$	$a_m + a_{m-1} - a_{m-2} - a_{m-3}$
EEPR4	$q(t+3q(t-T)+3q(t-2T)+q(t-3T)$	$a_m + 2a_{m-1} - 2a_{m-3} - a_{m-4}$

TABLE I
TARGET RESPONSE $s(t)$ AND THE CORRESPONDING OUTPUT d_m

and

$$\rho(\gamma_1) = \frac{1}{4} - \frac{1}{4}\gamma_1^2 \tag{10}$$

B. Correlation Function for NRZI modulated Data Sequence

In some applications, the correlation functions for the NRZI modulated data sequence b_m may be required. Hence, we also obtain the correlation functions for the NRZI modulated data sequence. They are given in the following equations (11) and (12).

$$\begin{cases} \mathrm{E}[r_{m-i}b_{m-i}b_{m-j}] = \frac{1}{2} + \gamma_1 + \frac{1}{2}\gamma_2, \\ \qquad \text{if } |i-j| = 0 \ \forall\ i \text{ and } j \\ \mathrm{E}[r_{m-i}b_{m-i}b_{m-j}] = -\frac{1}{2}\gamma_1 - \frac{1}{2}\gamma_2, \\ \qquad \text{if } |i-j| = 1 \ \forall\ i \text{ and } j \\ \mathrm{E}[r_{m-i}b_{m-i}b_{m-j}] = -\frac{1}{4} + \frac{1}{4}\gamma_2, \\ \qquad \text{if } |i-j| = 2 \ \forall\ i \text{ and } j \\ \mathrm{E}[r_{m-i}b_{m-i}b_{m-j}] = 0, \qquad \text{else} \end{cases} \tag{11}$$

and

$$\begin{cases} \mathrm{E}[r_{m-i}b_{m-i}r_{m-j}b_{m-j}] = \frac{1}{2} + \gamma_1^2 + \frac{1}{2}\gamma_2^2, \\ \qquad \text{if } |i-j| = 0 \ \forall\ i \text{ and } j \\ \mathrm{E}[r_{m-i}b_{m-i}r_{m-j}b_{m-j}] = -\frac{1}{4}\gamma_1^2 - \frac{1}{2}\gamma_2\gamma_1 - \frac{1}{4}\gamma_2^2, \\ \qquad \text{if } |i-j| = 1 \ \forall\ i \text{ and } j \\ \mathrm{E}[r_{m-i}b_{m-i}r_{m-j}b_{m-j}] = -\frac{1}{8} + \frac{1}{4}\gamma_2\gamma_1 - \frac{1}{4}\gamma_1 + \frac{1}{8}\gamma_2^2, \\ \qquad \text{if } |i-j| = 2 \ \forall\ i \text{ and } j \\ \mathrm{E}[r_{m-i}b_{m-i}r_{m-j}b_{m-j}] = -\frac{1}{16} + \frac{1}{8}\gamma_2 - \frac{1}{16}\gamma_2^2, \\ \qquad \text{if } |i-j| = 3 \ \forall\ i \text{ and } j \\ \mathrm{E}[r_{m-i}b_{m-i}r_{m-j}b_{m-j}] = 0, \qquad \text{else} \end{cases} \tag{12}$$

IV. Applying the Correlation Function to Derive the Timing Function

As well-known, the timing error τ_m for the Muller & Mueller's phase detector is realized by

$$\tau_m = y_m\hat{d}_{m-1} - y_{m-1}\hat{d}_m \tag{13}$$

where $\hat{d}_m$ is an estimate of the output for the partially equalized channel. Assume the timing error τ_m varies slowly, and the timing function is defined as the mean of the timing error τ_m that is denoted by

$$\begin{aligned} \mathrm{E}[\tau_m] &= \mathrm{E}[y_m\hat{d}_{m-1} - y_{m-1}\hat{d}_m] \\ &= \mathrm{E}[A_m - B_m] \end{aligned} \tag{14}$$

where $\mathrm{E}[\cdot]$ represents the expectation operation, and the two additional terms A_m and B_m are given by

$$\begin{cases} A_m = \left(\sum_{k=-L_1}^{L_2} r_{m-k}b_{m-k}s(kT+\tau_{m-k}T)\right)\hat{d}_{m-1} \\ B_m = \left(\sum_{k=-L_1}^{L_2} r_{m-k-1}b_{m-k-1}s(kT+\tau_{m-k}T)\right)\hat{d}_m \end{cases} \tag{15}$$

The timing function is derived as follows with the assumption that the estimate $\hat{d}_m$ is correct. For a PR4-equalized channel, the partial response output $d_m = a_m - a_{m-2}$, and the target response is given by $s(t) = q(t) + q(t-T)$. In addition, the correlation functions of (6) and (8) can be applied for deriving the timing function; $\mathrm{E}[A_m]$ and $\mathrm{E}[B_m]$ can be obtained in the following equations (16) and (17).

Substituting (16) and (17) into (14), we can obtain the resulting timing function as

$$\mathrm{E}[\tau_m] = \sigma(\gamma_1,\gamma_2)\cdot f(\tau) + \rho(\gamma_1)\cdot g(\tau) \tag{18}$$

where the function $f(\tau)$ and $g(\tau)$ are in the following equations (19) and (20).

$$\begin{aligned} f(\tau) &= 2s(T-\tau) - 2s(-\tau) - s(3T-\tau) \\ &+ s(2T-\tau) - s(-T-\tau) + s(-2T-\tau) \end{aligned} \tag{19}$$

$$\begin{aligned} g(\tau) &= 2s(2T-\tau) - 2s(-T-\tau) - s(4T-\tau) \\ &+ s(T-\tau) - s(-\tau) + s(-3T-\tau) \end{aligned} \tag{20}$$

V. Computer Simulations

Computer simulations are given to verify the proposed equalizer in this section. The channel output is generated by using equation (5) with parameter settings $L_1 = L_2 = 7$, $\gamma_2 = \gamma_1^2$, and $\gamma_1 = 0.6$. Assume the decision $\hat{d}_m$ is correct, the timing functions generated by using equation (13) and (18) are shown in Fig. 1. The realized timing function (13) is plotted in symbol 'o' and the derived timing function (18) is shown in symbol '*'. The result indicates the derived timing function closely matches the realized one, which verifies the proposed results.

VI. Conclusion

We have derived the correlation functions for the storage data and the NRZI modulated data of a magnetic recording channel in the presence of the partial erasure. The closed-form of the correlation functions are obtained, these results are applied to verify the timing function of the Muller & Mueller's phase detector in a partial response class IV channel via computer simulations. The results can be applied in high order partial responses and are expected to be used in designing a minimum mean square error equalizer in the future.

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$$\begin{aligned} \mathrm{E}[A_m] &= \sigma(\gamma_1,\gamma_2)\cdot[s(T-\tau)-s(-\tau)-s(3T-\tau)+s(2T-\tau)] \\ &\quad +\rho(\gamma_1)\cdot[s(2T-\tau)-s(-T-\tau)-s(4T-\tau)+s(T-\tau)] \end{aligned} \tag{16}$$

$$\begin{aligned} \mathrm{E}[B_m] &= \sigma(\gamma_1,\gamma_2)\cdot[s(-T-\tau)-s(-2T-\tau)-s(T-\tau)+s(-\tau)] \\ &\quad +\rho(\gamma_1)\cdot[s(-\tau)-s(-3T-\tau)-s(2T-\tau)+s(-T-\tau)] \end{aligned} \tag{17}$$

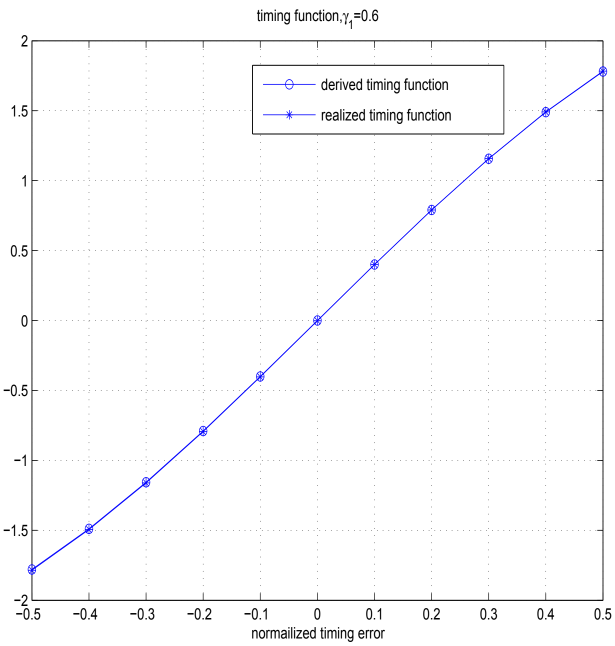

Fig. 1. Derived and realized timing functions for a PR4-equalized channel with $\gamma_1 = 0.6$

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