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Emerald Book Chapter: Price errors from thin markets and their corrections: Studies based on Taiwan's political futures markets

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PRICE ERRORS FROM THIN MARKETS AND THEIR CORRECTIONS: STUDIES BASED ON TAIWAN'S POLITICAL FUTURES MARKETS☆

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ABSTRACT

While it has been claimed in many empirical studies that the political futures market can forecast better than the polls, it is unclear upon which our forecast should be based. Standard practice seems to suggest the use of the closing price of the market, as a reflection of the continuous process of information revealing and aggregation, but we are unsure that this practice applies to thin markets. In this chapter, we propose a number of reconstructions of the price series and use the closing price based on these reconstructed series as the forecast. We then test these ideas by comparing their forecasting performance with the closing price of the original series. It is found that forecasting accuracy can be gained if we use the closing price based on the smoothing series rather than the original

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series. However, there is no clear advantage by either using more sophisticated smoothing techniques, such as wavelets, or using external information, such as trading volume and duration time. The results show that the median, the simplest smoothing technique, performs rather well when compared with all complications.

MOTIVATION AND INTRODUCTION

The purpose of this chapter is to deal with the possible mispricing behavior when the underlying market is *thin*, and the information aggregation mechanism may malfunction. While this issue can arise quite generally from many real markets, what motivates us in this study are the familiar *prediction markets*, in particular, the *political futures markets*, which are mainly designed for the prediction of political events such as the elections of mayors, legislators, and the president (Wolfers & Zitzewitz, 2004; Rhode & Strumpf, 2004).

The performance of the prediction markets is usually based on their competition using polls, and the common practice is to take the *last* output (the last observation in the market price series) as the prediction made by the political futures market and to compare it with the latest available polls as the counterpart (Forsythe, Nelson, Neumann, & Wright, 1992; Berg, Forsythe, & Rietz, 1997; Berg, Forsythe, Nelson, & Rietz, 2000; Pagon, 2005). The reason for taking the last observation is mainly due to the typically decreasing sequence of the absolute forecast error with time. In other words, it indicates a process of *information revealing through time*. A good example is shown by Wolfers and Zitzewitz (2004, p. 111, Fig. 1). Doing so implicitly assumes that any single observation (market price) in time can be regarded as an outcome of a reasonable information aggregation of the market participants.

However, this assumption can crucially depend on whether the market is thick enough. When the market is thin, the theoretical support for using point estimation may become weak, and the ''asset'' can easily be mispriced due to either the manipulation from or extreme expectations of a small group of market participants. Tomek (1980), for example, remarked, ''a major concern about thin markets is that the number of transactions is so small that 'unwarranted' price behavior occurs'' (*ibid*., p. 434). Nelson and Turner (1995) expressed a similar concern, ''prices reported from a thin public market are not representative of those that would result from a large population of buyers and sellers, either because of sample selection, or price manipulation through collusive agreements among buyers" (*ibid.*, p. 149). This shows the possibility that the observed price may not be used directly as the true price. The question is then what the true price is when the market is thin. Alternatively put, how should we construct the true price from a series of observations generated by a thin market?

In this chapter, using empirical data, we shall first show that the point estimation (the last-minute price) is not an ideal representative of the behavior of the prediction market. Its mispricing behavior is consistently revealed when it is compared with other reconstructs. We basically consider two different kinds of constructs. The first kind is based on *the price series alone*, that is, the reconstruction uses only the information of price series alone, which is usually known as *smoothing* or *denoising* in econometrics. While there are many such techniques, only two are considered in this chapter. We first consider a very simple smoother, and then compare it with a more sophisticated one. For the former, we use the central tendency, such as the *median*, as the smoother; for the latter, we choose *wavelets* due to their greater popularity.

The denoising techniques above basically assume that the observed price series is filled with noises and, by appropriately weighted averaging the series, the true price can be revealed. These techniques use the information regarding *prices only*. However, the prediction market is continuous in time, and the price would not even exist unless there were at least one transaction happening at that specific moment. Therefore, the price that we generally have is not continuous and not even periodical but is discrete in time with irregular interruptions of different durations of market quietness. Furthermore, when the price is observed, it may result from a transaction involving different trading volumes. The standard smoothing techniques above, however, do not integrate this background information into their weighting schemes. In this chapter, we, therefore, propose the second kind of smoothing techniques, which can fully take the background information, such as trading volume and duration, into account. Hence, prices are smoothed by weights based on volume, time, or both. These newly constructed prices are then compared with the first kind of smoothers to see whether the use of this additional background information is worthwhile.

The rest of the chapter is organized as follows. The first section gives a brief general review of the idea of thin markets and prediction markets. The next section introduces specifically the two political futures markets considered in this study, one from Academia Sinica, and one from National

Chengchi University. The corresponding data structure and some general statistical properties of these data are reviewed. The section following discusses the two reconstruction schemes (denoising schemes): the single-variate and the multi-variate weighting schemes. The next section compares the prediction performances of the "true prices," which are constructed differently, based on the actual observed prices (vote shares). The final section concludes the chapter with the implications of our findings for thin markets and points to directions for further study.

THIN MARKETS AND PREDICTION MARKETS

Thin Markets

The term "thin market" was popularized by Gray (1960) in the context of futures markets, and since then, our studies of the thin-market phenomena have been largely associated with agricultural markets, futures markets, and capital markets. For example, the absence of thinness has been considered to be a property of efficient capital markets, which in turn are regarded as the major protection of investors (Stigler, 1964). The concept of the thin market is mainly concerned with *pricing problems*, such as weak price discovery, imprecise pricing, large pricing errors, and excessive price fluctuations, as reflected in the following quotations:

> Many feel that the prices discovered in these thin markets do not represent true market conditions and should not be used as guides in pricing direct sales. (Kohls & Uhl, 1990, p. 213)

> an efficient market is commonly expected to display the property of resilience (to use an unfamiliar word for a property whose absence is called "thinness"). Resilience is the ability to absorb *market* bid or ask orders (i.e., without a price limit) without an appreciable fluctuation in price. (Stigler, 1964, p. 127)

With different pricing problems in mind, researchers may define thin markets in different ways. Stigler (1964) defined thinness as follows: a market is *not thin* if "market buy and sell orders of a magnitude consistent with random tenders with an unchanged equilibrium price do not change transactions prices considerably" (*ibid.*, p. 127). In a similar vein, Silber (1975) defined thinness as "a market is commonly called thin if a large change in price is associated with a small change in supply or demand" (*ibid.*, p. 129). Hayenga, Gardner, Paul, and Houck (1979), however, laid more emphasis on the imprecise pricing, "markets with little trading volume and liquidity in which individual firms or offers to buy or sell can sometimes exert 'undue' influence on price or other terms of trade …" (*ibid*., p.7).

The definitions above indicate that market thinness is a relative (more-or-less) concept, rather than an absolute (yes-or-no) concept. In empirical studies, a number of proxy variables have been proposed for the degree of market thinness, such as bid-ask spreads, trading volumes, the number of market participants, and trading frequencies. Of course, the precise list of variables to use may vary from market to market. For example, in the case of securities markets, Silber (1975) also added the total assets of a listed company and the total value of the outstanding value of a security to the list. In agricultural markets, trading volume and trading frequencies are usually considered to be the important parameters (Tomek, 1980; Kohls & Uhl, 1990; Tomek & Robinson, 1990). Finally, in experimental markets with a controlled environment, the number of market participants has been frequently used as the only exogenous variable in a thin market (Nelson & Turner, 1995; Bossaerts & Plott, 2002).

We shall later return to examine some of these proxy variables in the two political futures markets studied in this chapter, following a brief review of the prediction markets.

Prediction Markets

Prediction markets, also known as information markets, are markets where participants trade in contracts whose payoff depends on unknown future events; accordingly, the contracts are also called *event futures*. These markets "are designed and conducted for the primary purpose of aggregating information so that market prices forecast future events. These markets differ from typical, naturally occurring markets in their primary role as a forecasting tool instead of a resource allocation mechanism" (Berg, Nelson, & Rietz, 2003, p. 1).

The idea of the prediction market is well understood, thanks to the famous Iowa experimental run in 1988, which was designed to forecast the outcome of the presidential election of the United States in that year. The experiment also coined the name of the Iowa Electronic Markets (IEMs), perhaps the best-known prediction markets. The idea of using prediction markets as a forecasting tool was soon popularized by the involvement of other research institutes and even private firms. The application domain was also quickly extended to encompass economic, entertaining, scientific, and policy-related events. Wolfers and Zitzewitz

(2004) provides an excellent glossary of different kinds of prediction markets.

The wide acceptance of the prediction market as a forecasting tool already indicated that this design may actually work well. As a matter of fact, there are already a number of empirical studies that show the forecasting superiority of these markets (many of these studies can be found in Wolfers & Zitzewitz, 2004, or Chen, Chu, & Mullen, 2006). The most essential ingredient that makes the prediction market work is the *information aggregation mechanism*. However, formal theoretical study of this mechanism is rare, and Chen et al. (2006) provide the only investigation known to us.

The prediction market is typically thin, and there is a concern that in this thin market, prices might be less informative due to the *partisan betting* behavior of biased traders (Hansen, Schmidt, & Strobel, 2004). Traders' preferences over parties or candidates tend to color their perceptions, creating a *wishful thinking* effect (Babad, 1997; Forsythe, Reitz, & Ross, 1999; Price, 2000). Nonetheless, it was found that the prediction markets were not much influenced by these biased individual traders (Forsythe et al., 1999; Hanson & Oprea, 2009). However, because of thinness, it was also noticed that the price data generated in the prediction market may not directly be used to make forecasts, as in the quotation we use from Kohls and Uhl (1990) in the first section of this chapter. In practice, simple smoothing techniques have been applied to preprocess these data (Chen & Plott, 2002; Huber & Hauser, 2005), although the convention is to make the forecast based on the closing price (Forsythe et al., 1992; Berg et al., 1997, 2000; Pagon, 2005). In this chapter, we shall conduct a systematic study to compare the forecasting performance based on smoothing prices with that based on the closing price. Our empirical examination uses the data from the two prediction markets, to which we now turn.

AI-ECON AND IOP POLITICAL FUTURES MARKETS

Thinness of the Markets

The IOP prediction market was established in 2003 at the Institute of Physics (IOP), Academia Sinica in Taipei, and since then, it has been run several times to predict many important political events, including both the

Table 1. AI-ECON and IOP Political Futures Markets.

AI-ECON	
Opening day	November 9, 12:00 am (midnight)
Closing day	December 9, 15:30 pm
Number of participants (registered)	532
Number of participants (active)	366
Trading volume (accumulated)	115,763
Outstanding volume	33,467
Transaction density	6.84% (Taipei)
	3.33% (Kaohsiung)
IOP	
Opening day	November 10, 12:00 am (midnight)
Closing day	December 9, 15:30 pm
Number of participants (registered)	645
Number of participants (active)	427
Trading volume (accumulated)	147,477
Outstanding volume	41,721
Transaction density	8.03% (Taipei)
	2.67% (Kaohsiung)

US and the Taiwan Presidential Election in 2004 (Wang, Liu, Yu, & Li, 2004; Wang, Li, Tai, & Chen, 2009; Wang, Tseng, Li, & Chen, 2006; also see http://socioecono.phys.sinica.edu.tw). The AI-ECON prediction market was established in 2006 at the AI-ECON Research Center, National Chengchi University in Taipei (see http://futures.nccu.edu.tw). It has been applied to predict the opening day of the High Speed Railway in Taiwan. In November, 2006, both elections were applied to predict the Taipei and Kaohsiung City mayoral elections, which provide the basis of the empirical study in this chapter.

While both markets ended at 15:30 pm on December 9, 2006, the AI-ECON prediction market started one day earlier than the IOP market (Table 1). From beginning to end, there were 532 participants registered at the AI-ECON prediction market, and 645 participants registered at the IOP prediction market. However, many of them did not engage in a single ask or bid, not to mention engage in trade. If we exclude these idle participants, there were 366 active participants in the AI-ECON market, and 427 in the IOP market. Fig. 1 shows the accumulated number of participants over the entire trading period, whereas Fig. 2 depicts the accumulated trading volumes during this period. It can be seen that, in terms of both the accumulated number of participants and trading volumes, the AI-ECON market is *thinner* than the IOP market for almost the entire second half of

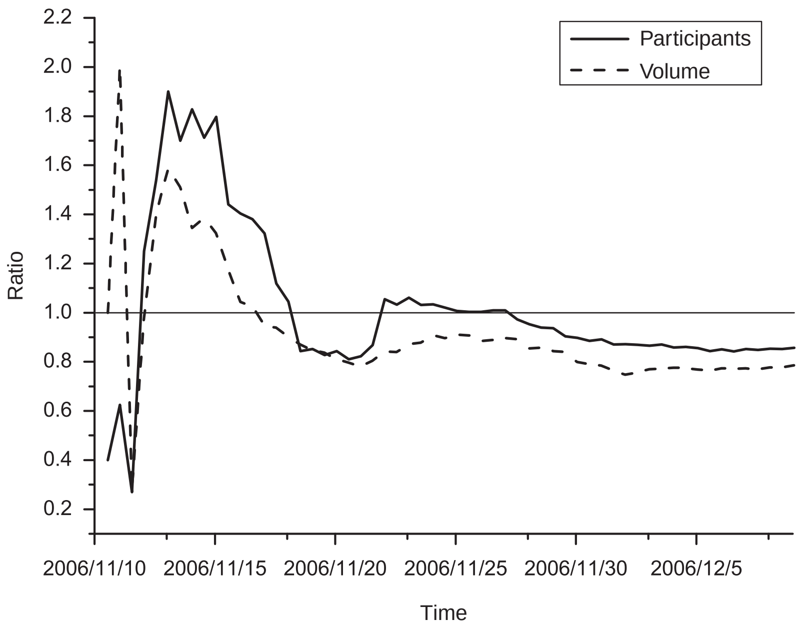

Fig. 1. Accumulated Number of Active Participants of the AI-ECON and IOP Prediction Markets.

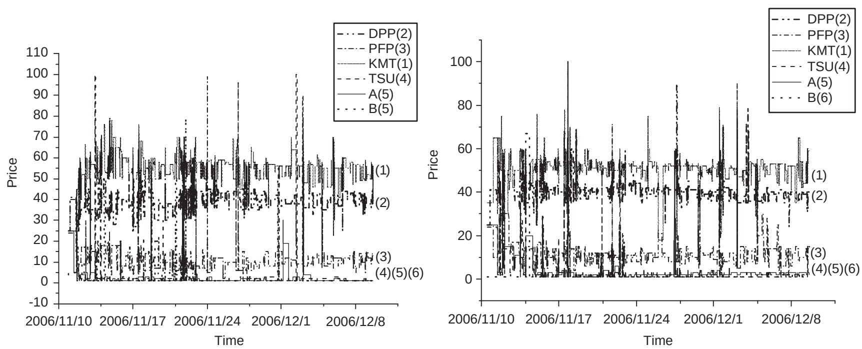

Fig. 2. Accumulated Trading Volume of the AI-ECON and IOP Prediction Markets.

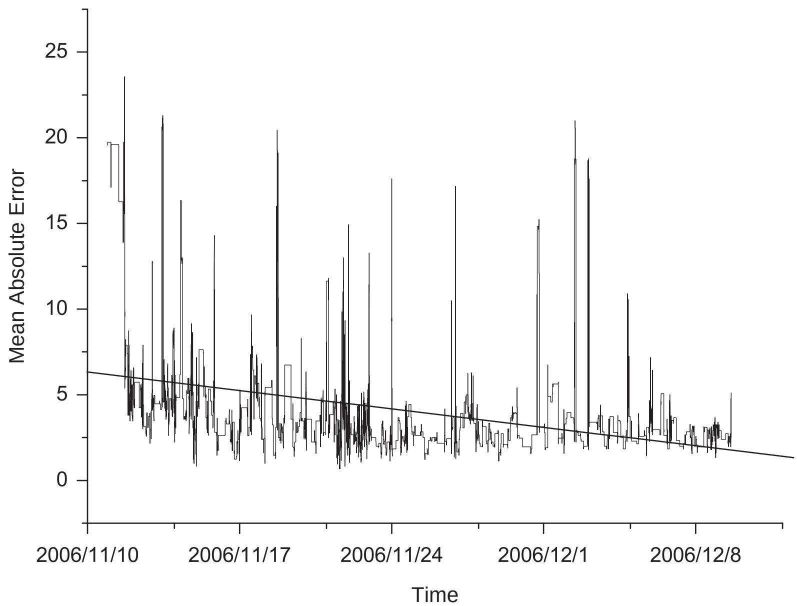

Fig. 3. Size Comparison between the AI-ECON Market and the IOP Market.

the trading period. To make the comparison even more transparent, Fig. 3 depicts the ratio of the size of AI-ECON market to the size of the IOP market, in terms of both the number of active participants and trading volumes. While the ratios can be above or below 1 (100%), after November 20, they are mostly below 1, which indicates that the size of the IOP market eventually surpasses that of the AI-ECON market.

Both markets are open 24 hours a day; in other words, they are continuously run with no break, and trade can take place at any point in time. However, the market cannot continuously successfully match bids and asks, and there is a large proportion of time when the market has no trade at all. If we define the *transaction density* of the market as the time with trades divided by the total market time, then we can see from Table 1 that both markets are rather quiet. More than 90% of the time, the markets remain quiet.

Silber (1975) proposes a number of variables that can contribute to the thinness of the market. Some of them fit our descriptions, including trading volume, market participants, trading intensity, and outstanding volumes (the only one which we do not have here is the total assets of firms). Based on these criteria, the AI-ECON market can be considered to be thinner than the IOP market, while both are thin from the viewpoint of the respective real markets.

Futures and Prices

In the Taipei City mayoral election, there were six candidates. Four of them were nominated by their affiliated political parties, which were in fact the four major political parties in Taiwan, namely, the Democratic Progressive Party (DPP), the Kuomintang (KMT), the People First Party (PFP), and the Taiwan Solidarity Union (TSU). In addition to these party nominees, there were two independent candidates, which shall be denoted as candidates "A" and "B" in this chapter. Then, six futures contracts are made with these six candidates accordingly. For simplicity, we shall denote the six futures contracts by the respective parties: DPP, KMT, PFP, TSU, A, and B.

While the prices of the six futures contracts are determined by the secondary market, which is run as a usual order-driven market, the initial prices are given by the primary market. In fact, the six futures contracts together as a *bundle* are available from the primary market at a cost of 100 tokens (fictitious money). Any registered participant can obtain the *bundles* of futures contracts directly from the primary market at the initial price and then trade each of the contacts separately in the secondary market by using either the market order or the limit order. Fig. 4 is the time series of the

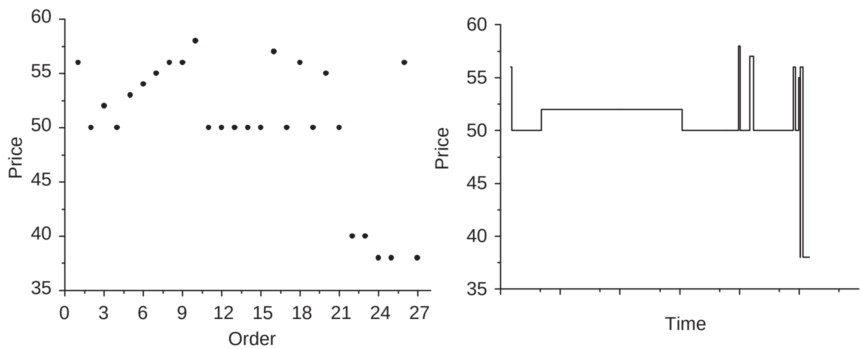

Fig. 4. The Minute-by-Minute Prices of the AI-ECON and IOP Political Futures Markets for the Taipei Mayoral Election. *Note:* The Left Panel Corresponds to the AI-ECON Market, Whereas the Right Panel Corresponds to the IOP Market.

Table 2. Vote Shares and the Closing Prices (Taipei City Mayoral Election).

	Forecasts					
	DPP	KMT	PFP	TSU	A	B
AI-ECON	45	38	13	1	1	1
IOP	40	59	15	3	2	1
Vote share	40.89	53.81	4.14	0.26	0.61	0.29
Errors (absolute)						
AI-ECON	4.11	15.81	8.86	0.47	0.39	0.71
IOP	0.89	5.19	10.86	2.74	1.39	0.71

Notes: The upper panel shows the closing prices of the six futures in both the AI-ECON and the IOP prediction markets. The closing price is the price of the last minute of the market time, that is, the price at 2:29 pm on December 9. These prices are then used to forecast the "true price," which is the vote share of each of the candidates. The lower panel then depicts the absolute forecasting errors.

minute-by-minute prices of the six futures in both the AI-ECON and the IOP markets.

The exercise price at the expiration data is determined by the vote share won by each candidate. Designed in this way, it is hoped that the price can predict the vote share. For example, one usually uses the closing price to forecast the vote share of each candidate. Table 2 gives the closing prices of the Taipei futures in both the AI-ECON and the IOP market. The upper half of the table gives the vote share won by each candidate, and the lower half gives the absolute forecasting errors for each candidate.

We have the identical structure for the Kaohsiung City mayoral election, except the number of futures offered for this election is only four. In addition to the candidates nominated by the DPP, KMT, and TSU, there are two other candidates, who played rather negligible roles. Therefore, we combine the vote shares of the two candidates and offer only one single futures for this combination.

RECONSTRUCTION OF THE PRICE

As we mentioned earlier, in the literature on prediction markets, it is quite common to use the closing price as the main forecast. Doing so may go well with a continuous information aggregation and information revealing

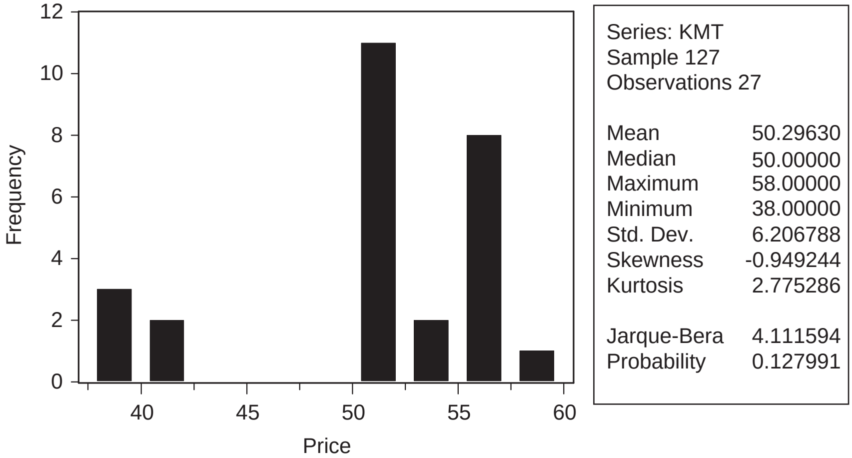

Fig. 5. Mean Absolute Forecast Error of Taipei Futures (Vote Share %): AI-ECON. *Note:* The Series Shown Above is the Time Series of the Absolute Forecast Error in Terms of Vote Share, Averaged over Six Futures (Candidates) in the AI-ECON Prediction Market. The Straight Line is the Regression Line Fitted to the Series.

process. In fact, the statistics generally show that the forecast error tends to decline monotonically with time (see, e.g., Wolfers & Zitzewitz, 2004, p. 112, Fig. 1). As shown in Fig. 5, our time series for the absolute forecasting error can also be approximated by a downward-slopping linear regression line. However, the deviation from and fluctuation around the line, which one may anticipate from a thin market, is too wide to support the closing price as the best forecast. In this section, we, therefore, propose a number of reconstructions of the original series.

Reconstruction Using the Price Series Alone

The most straightforward idea is to *smooth* the originally very fluctuating price, which is also what data miners usually do in their data processing.

Median

The simplest way to do smoothing is to just take the *mean* or *median* of a sample of prices. Nelson and Turner (1995) indicated that prices obtained from thin markets exhibit no apparent bias, but the price variability may be greater than in markets with greater trade volume. This implies that long-term price averages calculated from thin markets may be reflective of supply and demand conditions. In prediction markets, instead of the closing price, Huber and Hauser (2005) used the average of the last two market days as the forecast.

Consider the transaction price over a fixed interval. Denote this series by $\{p_i^H\}$, where i is the ith transaction observed at the interval within a time horizon of H. The unit of H is a *day;* therefore, $H = 1$ refers to a horizon of one single day. Fig. 6, as an illustration, gives the price series of the KMT candidate for Taipei Mayor on the last day. The right panel shows the price second by second; in other words, it is the usual time series depicted

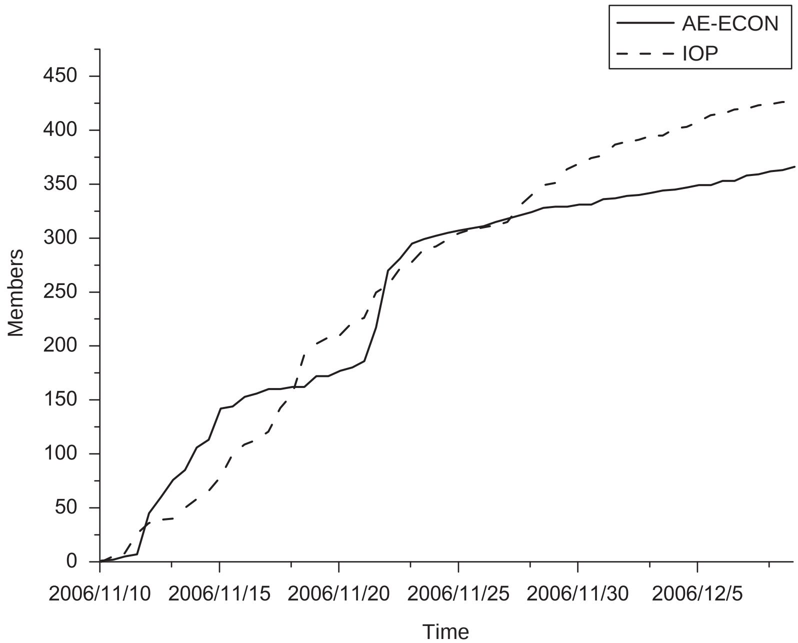

Fig. 6. The Transaction Price and the Second-by-Second Price of the KMT Taipei Futures: AI-ECON Market. *Note:* The Left Panel Above is the Transaction Price of the Last Market Day. They are Presented in the Transaction Order. The X-Axis, Therefore, Corresponds the Transaction Order and Not Time. The Right Panel Gives the Second-By-Second Market Price, and the X-Axis Corresponds to Time.

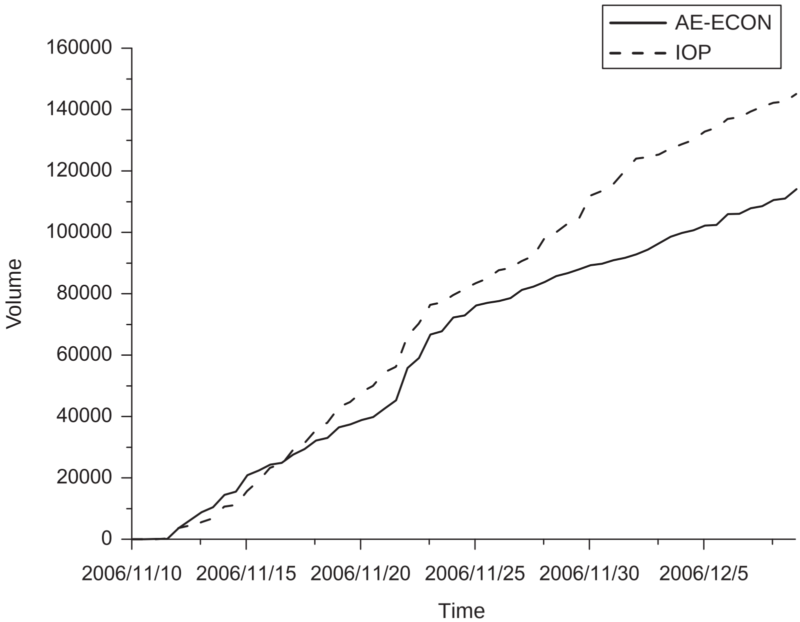

Fig. 7. The Histogram of the Transaction Price of the KMT Futures, AI-ECON Market: December 9, 2006.

with time as the X-axis, or simply, $\{p_t\}$ ($t \in [T-1, \ T]$). Nonetheless, the left panel is the transaction price in sequel over the time interval [T−1, T], whose X-axis is not time but transaction order. The $\{p_i^{H=1}\}$ introduced above refers to the left panel, rather than the right one.

Given a time horizon H, we can reconstruct the original time series $\{p_t\}$ by first taking all transaction prices observed at the time interval [t−H, t], and taking its median, say $\tilde{p}_t^H$. Doing this for each t, we can then have a new price series $\{\tilde{p}_t^H\}$ after smooth an illustration, Fig. 7 is the histogram of the series $\{p_i\}$ over the time interval $[T-1,T]$, and its median, denoted by $\tilde{p}_T^1$ is exactly 50.00.

Wavelets

To isolate signal discontinuities, we would like to have some very *short* basis functions. At the same time, to perform detailed frequency analysis, we would also like to have some very *long* basis functions. The best way to achieve this is to have *short high-frequency basis* functions and *long low-frequency ones*. This is exactly what wavelet transforms can do as will be shown below.

In the wavelet transforms, a function $f(t)$ can be expressed as an additive combination of the wavelet coefficients at different resolution levels. More precisely,

$$f(t) = \sum_j \sum_k \beta_k^j \psi_k^j(t) = \sum_k \alpha_k^{jo} \phi_k^{jo}(t) + \sum_{j \geq jo} \sum_k \beta_k^j \psi_k^j(t) \tag{1}$$

where

$$\alpha_k^{jo} = \int f \phi_k^{jo} dt, \quad \beta_k^j = \int f \psi_k^j dt \tag{2}$$

$$\phi_k^j(t) = 2^{j/2} \phi(2^j t - k), \ \ \psi_k^i(t) = 2^{j/2} \psi(2^j t - k) \tag{3}$$

α_k^{j0} represents smooth coefficients at the coarsest resolution level j_0, and β_k^j represents *detailed coefficients* at the finest resolution level j, where j indicates the frequency information, and k denotes the time information. The function ϕ is also denoted as the *father wavelet* or the *scaling* function. The linear combination of $\{\phi_k^j\}$ produces the *mother wavelet* function ψ. There are different wavelet bases. Different wavelet families make different trade-offs between how compactly the basis functions are *localized in space* and how *smooth* they are. Wavelet families proposed in the literature include the Harr wavelet, the Morlet wavelet, the Coiflet wavelet, the Meyer wavelet, and the Daubechies wavelet. The Haar wavelet is the simplest one, and it is often used for educational purposes. The Daubechies wavelet transform is perhaps the most elegant one, and hence has become the cornerstone of wavelet applications today.

There are no set rules for the choice of the mother wavelet used in wavelet analysis. The choice depends on the properties of the mother wavelet, and the properties of the time series to be examined. By eyeball-inspecting the price series of our futures, and by using the experiences gained from the literature, two families of mother wavelets have been considered in this chapter, namely, *Daubechies wavelets* and *biorthogonal wavelets*. These two families of wavelets are applied to smooth all price series. Nonetheless, based on the pattern of each series, we apply different Daubechies wavelets for different series. As the literature suggests, a Daubechies wavelet of order 4 (db4) is chosen for a more irregular series, and a Daubechies wavelet of order 1 (db1), also known as the Harr wavelet, is chosen for the more simple series. Similarly, for the family of biorthogonal wavelets, bior3.3 is applied to the more irregular series and bior1.3 to the more simple series. All wavelets are applied with six decomposition levels.

Finally, wavelet denoising modifies a set of wavelet coefficients in reference to a threshold value. In this study, we have used the *universal threshold* with the *soft* thresholding rule.

Reconstruction Using Volume and Time

The reconstruction based on the price series alone assumes that price errors can be eliminated by weighted averaging. While the weights assigned to each observation may depend on the specific smoothing techniques used, they are not explicitly determined by the background information underlying each price, such as transaction volumes and duration time between two consecutive transactions. In this section, we consider this *external smoothing* as an alternative to the usual internal smoothing.

Volume Weighted Average Price

Maybe the most intuitive way to do external smoothing is to weight each of the observed prices by their associated transaction volumes, which is known as the *volume weighted average price* (VWAP). The VWAP is used to calculate the average price of an asset or a good weighted by its trading volume. This measure has been extensively applied to financial markets and has also been applied to prediction markets (Berg et al., 2000; Chen & Plott, 2002).

Let N_t^H denote the number of transactions happening at the time interval $[t-H,t]$, and let p_i:$[t-H,H]$ be the price corresponding to the ith transaction, $i = 1, 2, \ldots, N_t^H$. In addition, let υ_i be the corresponding trading volume of the ith transaction. Then the VWAP $p_t^{v,H}$ is defined as follows.

$$p_t^{v,H} = \frac{v_i}{\sum_{i=1}^{N_t^H} v_i} p_i, \quad i \in [t-H, t] \tag{4}$$

Notice that H in Eq. (4) is a parameter, which is to be determined by the user. In our later empirical experiments, we shall explore different values of H, ranging from a window from one day to six days.

Time Weighted Average Price

Despite its simplicity and popularity, VWAP has its limitation. In particular, in a thin market, a *wishful-thinking* participant may bid or ask a price out of a reasonable range. When that happens, the offer may trigger a large trading volume, which, in turn, can cause this deviating price to be heavily weighted. However, this deviation may not last long, given the limited capital owned by each participant; therefore, the prices of the later transactions may soon get closer to normal. This implies that the duration time associated with this abnormal price is short, and, if we weight it by its duration time rather than trading volume, its biased influence can be radically controlled.

Hence, an alternative reconstruction based on duration time is proposed. The time weighted average price (TWAP) involves calculating the average price weighted by the duration time. Specifically, let d_i be the duration time between the ith transaction and the $i+1$th transaction, that is,

$$d_i = t_{i+1} - t_i \tag{5}$$

Then the TWAP $p_t^{d,H}$ is defined as follows:

$$p_t^{d,H} = \frac{d_i}{\sum_{i=1}^{N_t^H} d_i} p_i, \quad i \in [t-H, t] \tag{6}$$

Volume and Time Weighted Average Price

Nonetheless, TWAP also has its problems. In a thin market, that the duration time for a price can be sustained for a long time may not be because it is reasonable, but simply because there is no one on-line in that period. In this case, TWAP may overweight some transactions taking place during the rather silent period. Since both VWAP and TWAP have their limitations, it is natural to consider some weighted average of the two. A simple linear weighted average is given as follows:

$$p_t^{vd,H} = \lambda p_t^{v,H} + (1-\lambda) p_t^{d,H}, \quad i \in [t-H, t] \tag{7}$$

The parameter λ in Eq. (7) is to be further explored by the user. By setting λ in the [0,1] interval, we actually consider a reconstruction $p_t^{vd,H}$, which lies between $p_t^{v,H}$ and $p_t^{d,H}$.

EMPIRICAL RESULTS

Table 3 summarizes the mean absolute forecasting error (MAE) over the 10 futures by using the closing price of either the original series or the reconstructed series as described in the previous section. The benchmark is the one using the closing price of the original series as the forecast. The rest all use the closing price of the reconstructed series. As indicated in Eqs. (4), (6), and (7), we consider different window lengths for the VWAP, the TWAP, and the volume and time weighted average price (VTWAP). H

Table 3. The Mean Absolute Forecasting Error: Overall.

Forecast/H	0	1	2	3	4	5	6
AI-ECON							
Benchmark	3.60						
Median		2.04	1.67	1.52	1.48	1.66	1.68
Wavelet-d	3.40	1.98	1.69	1.42	1.52	1.53	1.55
Wavelet-b	3.27	1.98	1.69	1.42	1.52	1.53	1.55
VWAP		2.00	1.70	1.54	1.52	1.44	1.60
TWAP		1.96	1.71	1.49	1.45	1.51	1.59
VTWAP		1.98	1.71	1.52	1.42	1.36	1.48
IOP							
Benchmark	2.78						
Median		2.17	2.27	2.372	2.37	2.42	2.32
Wavelet-d	2.80	2.53	2.49	2.47	2.62	2.74	2.71
Wavelet-b	2.84	2.54	2.49	2.47	2.63	2.74	2.71
VWAP		2.48	2.52	2.33	2.55	2.71	2.88
TWAP		2.66	2.59	2.45	2.62	2.72	2.72
VTWAP		2.57	2.56	2.39	2.59	2.72	2.80

Notes: The first column refers to the seven different forecasts made on either the original series or the reconstructed series. The benchmark is the one based on the closing price of the *original series* ($H = 0$). Wavelet-d and wavelet-b refer to the use of Daubechies wavelets and biorthogonal wavelets. VWAP, TWAP, and VTWAP correspond to the volume weighted average price, time weighted average price, and volume and time weighted average price, respectively. H, from 0 to 6, indicates the length of the window by which the median or the (weighted) average is taken.

originally is not a parameter for wavelets, but since the series after wavelet-denoising are still very irregular, we therefore decide to smooth them again by using the simple average with different window lengths (H).

General Performance

We hope that these empirical results can shed light on the following two questions.

- Would the forecast based on the reconstructed series perform better than the forecast based on the original series?
- Would the forecast based on the reconstruction using volume and time perform better than the forecast based on the one using price only?

From Table 3, the answer to the first question is affirmative. The closing price based on the original series (the benchmark) performs almost the worst. In the AI-ECON market, the MAE is 3.60, which is the worst, and in the IOP market, it is 2.78, which is ranked at the bottom, if not the worst. The series after taking wavelet transforms does not help much. In the case of the IOP market, they perform even worse. The result clearly suggests that the closing price of the original series may not be the best forecast that one can have from the prediction market, and this lesson is even more appealing when the market is thin or under a larger exposure of manipulation (there is reason to suspect that the AI-ECON market, which is located at a social science-oriented university, may suffer from more manipulations from participants than the IOP market, which is located at an academy of sciences).

What is particularly interesting is that the reconstructed series help the forecast of the AI-ECON market more than the forecast of the IOP market. Notice that, by using original series to forecast, the IOP market has a lower MAE than the AI-ECON market. However, after reconstruction, the results are reversed: AI-ECON performs better than the IOP market in all the reconstruction series with all time horizons (H). This reversal pattern provides a counterevidence on the positive relationship between market size and forecasting accuracy. As we noted in the third section, the IOP market has a larger market size than the AI-ECON market, and it does forecast better when using the original series; nonetheless, after reconstructing the series, the AI-ECON market outperforms the IOP market.

The reversal pattern also lends support to a recent result of Hanson and Oprea (2009), who gave a theoretical justification for the popular view that, instead of decreasing it, market manipulation can increase price accuracy. The background and the behavioral pattern of the two political futures markets, as discussed in the third section, motivate the conjecture that the AI-ECON market is under a larger exposure of manipulation than the IOP market. If this is indeed so, then we have shown that reasonable effort made to reconstruct the price would make this market forecast even better.

The second question to address is whether the smoothing using additional information can perform better than that using only price information. From Table 3, we fail to see significant benefits of using extra information. There is no clear indication that those weighted averages using either volume (VWAP) or duration time (TWAP) or both (VTWAP) can uniformly beat the smoothing using only price series. In particular, in the IOP market, the median always performs the best with respect to all time horizons applied. The AI-ECON market, however, does suggest there is evidence in support of using the volume or time in the reconstruction.

Individual Performance

We also give a detailed look at the forecasting performance using these reconstructed series. The futures related to the mayoral elections in Taipei and Kaohsiung are separated into Tables 4 and 5. The two tables are in sharp contrast. Generally speaking, the performance of the prediction markets for the Taipei mayoral election is much worse than the counterpart in Kaohsiung. This is true regardless of whatever measure we use. For example, in Taipei, the MAEs of the benchmark at AI-ECON and IOP are 5.10 and 3.63, respectively, while in Kaohsiung they are 1.34 and 1.50. This result is consistent with what we learn from the prediction market literature, namely, the forecasting accuracy of the prediction market can be adversely affected by the number of contracts (Berg et al., 1997). In our case, there are six futures in Taipei, but only four in Kaohsiung; therefore, this difference in the number of contracts may partially contribute to the difference in the forecasting performance between the two elections.

The performance of the benchmark model of the IOP market is superior to that of the AI-ECON market in forecasting the Taipei election (a MAE of 3.63 vs. 5.10), whereas this dominance is reversed in forecasting the Kaohsiung election (a MAE of 1.50 vs. 1.34). While this pattern seems to fit the market size intuition well, that is, the larger the market size, the better

Table 4. The Mean Absolute Forecasting Error (MAE): Taipei.

Forecast/H	0	1	2	3	4	5	6
AI-ECON							
Benchmark	5.10						
Median		2.27	2.07	1.73	1.67	1.96	2.00
Wavelet-d	4.80	2.60	2.12	1.67	1.85	1.89	1.98
Wavelet-b	4.51	2.59	2.13	1.67	1.85	1.89	1.98
VWAP		2.15	1.97	1.74	1.74	1.65	1.90
TWAP		2.56	2.10	1.74	1.78	1.84	1.95
VTWAP		2.36	2.04	1.74	1.64	1.55	1.73
IOP							
Benchmark	3.63						
Median		2.73	2.73	2.23	2.82	2.82	2.73
Wavelet-d	3.63	3.18	2.91	2.70	2.82	3.13	3.11
Wavelet-b	3.62	3.18	2.91	2.70	2.82	3.12	3.11
VWAP		3.23	3.13	2.98	3.11	3.35	3.60
TWAP		3.38	3.06	2.77	2.83	3.14	3.12
VTWAP		3.30	3.09	2.87	2.97	3.24	3.36

Table 5. The Mean Absolute Forecasting Error: Kaohsiung.

Forecast/H	0	1	2	3	4	5	6
AI-ECON							
Benchmark	1.34						
Median		1.71	1.08	1.21	1.21	1.21	1.21
Wavelet-d	1.31	1.05	1.03	1.06	1.02	1.00	0.91
Wavelet-b	1.41	1.05	1.03	1.06	1.02	1.00	0.91
VWAP		1.78	1.29	1.24	1.20	1.13	1.15
TWAP		1.05	1.14	1.12	0.95	1.02	1.05
VTWAP		1.41	1.21	1.18	1.08	1.08	1.10
IOP							
Benchmark	1.50						
Median		1.33	1.58	1.46	1.71	1.83	1.71
Wavelet-d	1.55	1.56	1.85	2.13	2.34	2.16	2.09
Wavelet-b	1.68	1.56	1.85	2.13	2.34	2.16	2.09
VWAP		1.36	1.36	1.36	1.72	1.76	1.79
TWAP		1.59	1.89	1.98	2.32	2.10	2.11
VTWAP		1.47	1.75	1.67	2.02	1.93	1.95

the forecasting accuracy, our refinement result with the reconstructed series shows that it is not necessarily so. As what we have seen in Table 3, after reconstruction, the AI-ECON market can forecast consistently better than the IOP market, even though the former has few participants engaging in the transaction of the Taipei political futures.

As to which reconstruction is the best, Tables 4 and 5 do give mixed results. For the IOP market, it does not appear that any complicated reconstruction can do better than just taking the median. In both the Taipei and the Kaohsiung elections, the median seems to perform the best regardless of the time horizon applied. On the contrary, for the AI-ECON market, the reconstruction based on wavelets performs the best in forecasting the Kaohsiung election, whereas the reconstruction using both volume and duration time performs a rather handsome job in forecasting the Taipei election.

To conclude, there is no strong evidence to show that the reconstruction using more information than prices will always provide us with more forecasting power.

CONCLUDING REMARKS

While the pricing problem caused by the thinness of the market has long been noticed in agriculture, futures, and capital markets, it has not drawn enough attention in the prediction markets. As a result, there is no clear indication on how the data from the prediction market should be used to make the best forecast. On the one hand, the gradual process of information aggregation and revelation seems to support the common practice, that is, to take the closing price from the original series as the forecast. On the other hand, the thinness of the market also prompts researchers to look for better forecasts by using smoothed series. There is, however, a lack of a systematic study to show how much difference we can make, and from what smoothing techniques. This chapter attempts to bring a better understanding of this issue.

Based on the data from recent experiments on two political futures markets in Taiwan, IOP and AI-ECON, we first show that some forecasting accuracy can be gained if we use the closing price based on the smoothed series rather than the original series. This basically confirms the suggestions made by some very early studies on thin markets. Second, we, however, also find that there is no clear advantage to be gained by either using more sophisticated smoothing techniques, such as wavelets, or using more external information, such as trading volume and duration time. The median, the simplest smoothing technique, performs surprisingly well as opposed to all complications.

By the same token, it does come as a surprise to us that using external information to smooth the data does not help much. Maybe this is because the behavioral foundations that we use to support the weights assigned to each price and the weights assigned to integrate the volume and time are not appropriate. More rigorous study of the underlying market microstructure, by possibly involving other sources of information, such as the spread, is a direction for further study.

Of course, due to the very limited explorations and very limited dataset involved, this chapter by no means attempts to answer any optimality-like issue, such as what the optimal smoothing is or what the optimal window length is. We are not sure about whether the answer can be easily found in a robust manner. The chapter, as an initial effort, attempts only to formally acknowledge the measurement error and the forecasting error that we may anticipate in a thin prediction market, in particular, when no reconstruction work is performed.

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