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Fuzzy Estimation Methods and their Application in Real Estate Evaluation

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ABSTRACT

The evaluation of real estate is complex, it involves many factors, such as peoples' utility (human subjective recognition) and the economic efficiency etc. In this paper we develop the traditional real estate appraisal approach the market comparison approach by integrating it with fuzzy estimation which reflects the imprecise and vague nature of human thinking. The primary main methods used are fuzzy statistical analysis, fuzzy-weighting and fuzzy ranking. This integrated procedure is aimed at yielding appropriate and reasonable rank and value of utility. We also give empirical examples to illustrate the techniques and how to evaluate the real estate. Result shows that fuzzy statistics with soft computing are more realistic and reasonable in the realistic evaluation. Finally certain comments are suggested for the further studies.

Keywords: Fuzzy estimation; Real estate valuation; Fuzzy statistics; Fuzzy weight

1. Introduction

Why do we think we have experienced such acceleration in prices of real estate in Shanghai, China? How are people recognizing the future values of a real estate? Should we depend on the banks to recognize the real value of the property we want to purchase?

Usually, brokers use licensed appraisers to estimate the value of a house. Moreover, the licensed appraisers use comparable sales and if there are not any, they have to improvise and use some comparable cases from similar areas. If the evaluation really gets skewed, what the licensed appraisers can do is use comparable cases from recent sales as close to the likeness of the property as possible. Because of these licensing appraisal practices the next properties sold could be over or under valued. How do they know the true value and how do they evaluate?

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There have been numerous literatures in the methodology of real estate valuation. Mackmin [10] showed some implications of DCF (Discounted Cash Flow) for the property valuation. Ekelid *et al.* [5] have focused on the structure and arguments in appraisal reports and how these have changed over time and on the treatment of different types of uncertainty. Janssen and Yang [9] used the regression approach to estimate the market value of a townhouse. Monte Carlo simulations were used to incorporate the uncertainty of valuation parameters by Hoesli *et al.* [7]. Chen and Khumpaisal [3] introduced a novel decision-making approach to risk assessment in commercial real estate development based on the ANP (Analytic Network Process) model. McAllister and Loizou [11] proposed an alternative approach that uses pricing signals from traded cash flows to appraise the various incomes and costs. Özsoy and Sahin [16] used the CART (Classification and Regression Tree) approach to analyze empirically major factors that affect housing price in Istanbul, Turkey.

All of these studies mentioned above are concerned with how to reduce the risk and uncertainty involved in the imprecise and vague information in the process of valuation. Risk and uncertainty are inherent parts of the valuation process as the price given is unable to be inclusive of all current and future influences on the value of the asset. In this paper, we are proposing an alternative method which uses fuzzy logic to improve the traditional way of valuation by more accurately reflecting the real way of human thinking.

In practice, in the real world, the price provided in the real estate valuation report supplied by the valuation agency is an exact number. However, the buyer and the seller are the different stakeholders in the commercial transaction. The more benefits the buyer obtains the less the seller can earn from his/her house. Different stakeholders have different expected prices and this price will not be the valuation price except in some rare situations. Actually in the real world, when we ask the holders how many the commercial houses worth, usually they would like to show an interval price. In order to work on the data, we need to provide an improved fuzzy-based methodology in the real estate valuation which involves in the attributes of impreciseness and vagueness.

There has been considerable and increasing attention paid to the idea of fuzzy logic since it was introduced by Zadeh as a modification of conventional mathematical Set Theory (Zadeh [20]). The basic aim is that vagueness and ambiguity can be described and distinguished mathematically. Fuzzy theory has been widely used a variety of authors. Verma [18] used fuzzy logic to construct offender profiles since the police officers received descriptions of suspects that were fuzzy in nature. Wu and Tseng [19] applied fuzzy regression models to business cycle analysis. Wu and Tseng [19] proposed new approaches on market research with fuzzy statistical analysis. Metaxiotis *et al.* [12] integrated fuzzy logic into a decision support system. Zhang *et al.* [21] proposed the fuzzy discrete-event simulation to model the uncertain activity duration. Ohdar and Ray [14] introduced the fuzzy-based approach to measure and evaluate the performance of suppliers in the supply

chain. Ordoobadi [15] used fuzzy logic to evaluate advanced technologies for decision makers and provided a model based on fuzzy logic for decision makers to help them with selection of appropriate suppliers in 2009.

Fuzzy logic was introduced in real estate appraisal by a great many of scholars. Fuzzy logic was introduced to deal with the risk and uncertainty which were poorly considered in real estate analysis by Byrne [1]. D'Amato [4] applied fuzzy theory to appraise property and made a comparison between this method and the most common statistic instruments. González and Formoso [6] improved mass appraisal techniques using FRBS (Fuzzy Rule-Based System) to model the real estate market and compared it with traditional hedonic regression model. Pagourtzi *et al.* [17] proposed a new methodology and discussed the architecture for a decision support system for real estate analysis based on GIS (Geographic Information System) integrated with fuzzy theory and spatial analysis. Hui *et al.* [8] integrated the housing indicator data such as RI (Rental Index) with fuzzy logic system to assist practitioners as well as investors on decision making in real estate investment.

In this paper we focus on the valuation approach in real estate. We take the city of Kunming in Yunnan, China, as an empirical study. How to get a reasonable and realistic price of real estate is the major requirement for buyers, sellers and valuation practitioners.

2. On traditional evaluation techniques with real estate appraisal

There are many traditional methods to estimate the value of real estate. These include the *Market Comparison Approach, Income Approach, Cost Approach, Hypothetical Development Method*, and so on which can be selected for different appraisal purposes and subject properties.

The Market Comparison Approach compares the subject property with a similar property whose trading date is close to the appraisal date. It then modifies some of influence factors in order to evaluate an objective and reasonable price or value of the real estate. The Income Approach evaluates the future incomes of the subject property and calculates the present value with the proper capitalization rate. It then takes the present value as the evaluated price. The Cost Approach uses the rebuilding price or reset price of the subject property and the deduction of the depreciation to compute objective and reasonable price or value of the subject property. The Hypothetical Development Method is on an assumption that the property is developed and then deducts the expected development costs, taxes and profits from the predicted value of the developed property.

All of these methods are applied widely in the valuation of real estate. One common feature of all of them is that the valuation result is a specific real number. However the trade price of the property is not the appraisal price. The appraisal price can be quite close to the final trade price but will not be the same except in some occasional situations.

Furthermore, from the seller's perspective, he maximizes his profit by raising the price of the property. Similarly the buyer maximizes his utility by reducing the price of the property simultaneously. Obviously, both of the expected prices of seller and buyer are highly unlikely to be the appraisal price which is a specific real number supplied be the appraisal report. There exists a range which includes the expected prices of the seller and buyer.

Therefore, we need to use fuzzy data to evaluate a reasonable range of the appraisal price which can satisfy both the seller and buyer.

Just like other business, real estate has various types of value. There is bank value, tax value, book value and asset value. In most cases this evaluation process is designed not only for financial purposes but also based on utility. Most people would be happy to pay the bank appraisal price because they know the real values of the property have not been considered and the property is worth more.

Note that in this paper we are not talking about residential raw housing real estate. We are talking about homes with acreage, lake and river frontage, elevated views as well as panoramic views. These properties offer something unique and beneficial to the owner or buyer and therefore command a premium price. Nowadays it is not as time consuming as before because there are several websites available on the net that really help to know which location is suitable for investment in real estate and why. Contact to the property agents of that particular location through mail and send their enquires whatever in your mind or call them and ask about different questions of property buying, selling and renting of that area. The information may give us enough knowledge about that particular area's real estate investment and we can start to get a good picture of the property business in that region. Before considering any location we need to know some things about that region. Such as: *Population, Tax and ownership law, Rents of property, Buyer and Sellers of that particular region, Future projects and Developments, Temperature and Environment, Traffic convenient, Management cost.*

3. Evaluation based on the fuzzy statistical analysis

Traditional statistics deals with a single answer or certain range of answer through sampling survey, but it has difficulty in reflecting people's incomplete and uncertain thought. In other words, these processes often ignore the intriguing, complicated and yet sometimes conflicting human logic and feeling. If people can use the membership function to express the degree of their feelings based on their own concept, the result will be closer to their real thought. For instance, when people process a pollution assessment, they classify the distraction into two categories: pollution and non-pollution. This kind of classification is not realistic, since pollution is a fuzzy concept (degree) and can hardly be justified by true-false logic. Therefore, computing the information based on fuzzy logic would be more reasonable.

Mode, mean and median are essential statistics in analyzing the sampling survey. Calvo and Mesiar [1] proposed the generalized median by discussion aggregation operators closely related to medians and to propose new types of aggregation operators appropriate, both for the cardinal and ordinal types of information. But they didn't give a realistic example. In this paper we present the definitions of fuzzy mode, fuzzy mean and related properties. We hope via these new techniques peoples' thought can be extracted and known in a more precise way.

If people can use the membership function to express the degree of their feelings based on their own choices, the result presented will be closer to real human thinking. Therefore, collecting information based on the fuzzy mode should be the first step to take. Since a lot of times, the information itself is embedded with uncertainty and ambiguity, it is natural for us to propose fuzzy statistics, such as fuzzy mode and fuzzy median, to fit the modern requirement. In this and next section we demonstrate the definitions for fuzzy mode and fuzzy mean generalized from traditional statistics.

3.1 How to get fuzzy number

Definition 3.1 Fuzzy number

Let U denote a universal set, $\{A_i\}_{i=1}^n$ be a subset of discussion factors on U, and $\Lambda(A_i)$ be a level set of A_i for $i=1,2,\ldots,n$. The fuzzy number of a statement or a term X over U is defined as:

$$\mu_U(X) = \sum_{i=1}^{n} \mu_i(X) I_{A_i}(X) \tag{3.1}$$

Where $\{\mu_i(X), 0 \le \mu_i(X) \le 1\}_{i=1}^n$ are set of membership functions for corresponding factor in $\{A_i\}_{i=1}^n$, and. $I_{A_i}(x) = 1$ if $x \in A_i$; $I_{A_i}(x) = 0$ if $x \notin A_i$ If the domain of the universal set is continuous, then the fuzzy number can be written as: $\mu_U(X) = \int_{A_i \subseteq U} \mu_i(X) I_{A_i}(X)$.

In the research of social science, the sampling survey is always used to evaluate and understand public opinion on certain issues. The traditional survey forces people to choose one answer from the survey, but it ignores the uncertainty of human thinking. For instance, when people needs to choose the answer from the survey which lists five choices including "Very satisfactory," "Satisfactory," "Normal," "Unsatisfactory," "Very unsatisfactory," traditional survey become quite exclusive.

The advantages of valuation with fuzzy numbers include: (i) The valuation process becomes robust and consistent by reducing the degree of subjectivity of the evaluator. (ii) Self-potentiality is highlighted by indicating individual distinctions. (iii) It provides the evaluators with an encouraging, stimulating and self-reliant guide that emphasizes individual characteristics. The drawback is that the calculation process will be a little more complex than the conventional one.

Example 3.1 *The use of fuzzy number in a sampling survey about favorite tourist attractions.*

Consider a fuzzy set of favorite tourist attractions for a person as shown in Table 3.1. Note that in the extreme cases when a degree is given as 1 or 0, that is "like" or "dislike", a standard "yes" and "no" are in a complementary relationship, as in binary logic. Let A_1 represents for "favorite tourist attractions", A_2 "dislike the tourist attractions".

Table 3.1 Comparing fuzzy number with integral number favorite tourist attractions .

Favorite Tourist Attractions	A_1	A_2	A_1	A_2
Degree of feelings	$\mu_{A_1}(X)$	$\mu_{A_2}(X)$	binary logic	
Jin Dian	1	0	X	
Xi Shan	0.4	0.6		X
Ye Ya hu	0.7	0.3	X	
Stone Forest	0.8	0.2	X	
National Museum	0.3	0.7		X

Based on the analysis of binary logic, we can find that he likes Jin Dian, Ye Ya hu and Stone Forest but dislikes Xi Shan and National Museum. On the other hand, the fuzzy statistical result can be represented as:

$$\mu_{A1}(X) = 1I_{JinDian}(X) + 0.4I_{XiShan}(X) + 0.7I_{YeYahu}(X) + 0.8I_{StoneForest}(X) + 0.3I_{NationalMuseum}(X)$$
$$\mu_{A2}(X) = 0I_{JinDian}(X) + 0.6I_{XiShan}(X) + 0.3I_{YeYahu}(X) + 0.2I_{StoneForest}(X) + 0.7I_{NationalMuseum}(X)$$

This means that the person likes Jin Dian 100%, YeYahu 70%, Stone Forest with 80%. He dislikes XiShan 60%, dislikes National Muesum 70%, and dislikes National Museum 70% of degree.

Therefore, based on the binary (like or dislike) logic, we can see only the superficial feeling about people's favorite tourist attractions. With the information of fuzzy response we will see a more detailed data representation.

3.2 How to compute fuzzy data

Definition 3.2 Fuzzy mode (data with interval values)

Let U be the universal set (a discussion domain), $L = \{L_1, L_2, \ldots, L_k\}$ a set of k-linguistic variables on U, and $\{FS_i = [a_i, b_i], a_i, b_i \in R, i = 1, 2, \ldots, n\}$ be a sequence of random fuzzy sample on U. For each sample FS_i, if there is an interval $[a, b]$ which is covered by certain samples, we call these samples as a cluster. Let MS be the set of clusters which contains the maximum number of sample, then the fuzzy mode FM is defined as

$$FM = [a,b] = \{\cap [a_i,b_i][a_i,b_i] \subset MS\}.$$

If $[a,b]$ *does not exist (i.e.,* $[a,b]$ *is an empty set), we say this fuzzy sample does not have fuzzy mode.*

Example 3.2 *Suppose we have the following sample: (2,5), (2,4), (2,3), (3,10), (2,3), (3,4), (3,5) Then by definition, the set of clusters which contains the maximum number of sample is {(2,5), (2,4), (3,10), (3,4), (3,5)}. Hence FM= (3,4)*

Definition 3.3 Fuzzy mean (data with interval values)

Let U be the universe set, and $FS_i=[a_i,b_i]$, i=1, 2, …, n be a sequence of random fuzzy sample on U. Then the fuzzy mean is defined as $E(X) = [\frac{1}{n}\sum_{i=1}^{n} a_i, \frac{1}{n}\sum_{i=1}^{n} b_i]$.

3.3 How to compute fuzzy weight

In the valuation process, people usually treat each factor with the equal weight. That is, we assume that the factors have the same contribution to the universe domain. However, in order to get a more accurate valuation, we had better use different weight, according to their contributions to the object, for different factor. Since then, the macro-performance valuation will reflect the real world situation.

To investigate the fuzzy weight of each factor, we may use the fuzzy set theory and sampling survey technique. Especially, using fuzzy memberships and multiple values assignment, we can get an appropriate fuzzy weight for the object. Hence, let's give a detailed process about how to get fuzzy weight.

Here, we propose the calculation process of entity fuzzy weight:

Step 1: *First, determine the effective factors* $A = \{A_1, A_2, \ldots, A_k\}$ *for the real estate appraisal*

Step 2: *Ask each interviewee i to give the importance of factors set with a membership* m_{ij} $\sum_{j=1}^{k} m_{ij} = 1$. *Let* m_{ij} *be the membership of importance of factor j for the i th interviewee*

Step 3: *Calculate the fuzzy weight* w_j *of* A_j *by* $w_j = \frac{\sum_{i=1}^{n} m_{ij}}{\sum_{j=1}^{k}\sum_{i=1}^{n} m_{ij}}$.

Example 3.4 Suppose there are five experts rank the influence factors of real estate valuation, see Table 3.4.

Table 3.4 Fuzzy weights of the influence factors of real estate valuation.

Experts \ Factors	Location	House Type	Community	Quality
1	0.25	0.25	0.25	0.25
2	0.3	0.25	0.3	0.15
3	0.25	0.3	0.2	0.25
4	0.4	0.1	0.35	0.15
5	0.3	0.1	0.4	0.2
Total Rank	1.5	1	1.5	1
Weight	0.3	0.2	0.3	0.2

It is easy to compute that $w_1 = \frac{\sum_{i=1}^{5} m_{i1}}{\sum_{k=1}^{4}\sum_{i=1}^{5} m_{ij}} = \frac{1.5}{5} = 0.3,\ldots,\ w_4 = \frac{\sum_{i=1}^{5} m_{i4}}{\sum_{k=1}^{4}\sum_{i=1}^{5} m_{ij}} = \frac{1}{5} = 0.2.$

4. An integrated fuzzy valuation process

Since human thought and behavior are typically vague and uncertain, analysis by traditional methods usually involves the following weaknesses: (a) The use of arithmetic in traditional questionnaires is often an over-explanation. (b) Experimental data are often overused just to cater to the need for apparent numerical accuracy. (c) For the sake of simplifying the evolutional model, the relationship between the actual condition and dynamic characteristics are neglected. That is why we will prefer to apply fuzzy theory to handle the questions that involve human opinion.

4.1 Determination about price fuzzy distribution

It is appropriate to apply the membership function, a more precise mathematical technique, in analyzing the fuzzy information. The value of the membership function, between 0 and 1, is derived from the characteristic function, to express the membership grade of each element in a set. There are many types of membership functions, such as Z-type, Λ-type, Π-type, S-type, etc, see Nguyen and Wu [13]. In this research we use Λ-type membership functions. It assesses the fuzzy interval of various valuations, and then calculates the fuzzy value of an enterprise according to appraiser's fuzzy weighting.

We also use the Λ-type to reflect the value for a commercial house price distribution. That is, we will give the price of commercial houses into different linguistic terms, such as, high-level, intermediate and unfurnished. Each term will correspond to a real value, which will be determined by the sampling survey and fuzzy statistical analysis.

4.2 The highly correlated property for the discussion factors

After detailed discussion from the above sections, an integrated process of fuzzy valuation is started by fixing the crucial affection factors of the commercial house. We use the weighted arithmetic average instead of the geometric average. The reason is that the factors are highly correlated; any extreme value of a certain factor will influence the real price of the commercial house. Take evaluating the price of commodity house as an example.

Suppose the factors of the valuation price are {*location, house type, community, quality*}. If the factor location is valueless, no matter the *community* is high and/or the other factors are high too, the integrated valuation will be low.

Finally we make weighted arithmetic average to get a more appropriate valuation. That is, suppose the factor sets is $A = \{A_1, A_2, \ldots, A_l\}$ and which corresponding to the with a weight set $w = \{w_1, w_2, \ldots, w_l\}$, then the integrated Valuation Price will be

$$\text{Valuation Price} = \prod_{i=1}^{l} A_i^{w_i}$$

4.3 The Mean Absolute Error for interval difference

Since an interval X_t can be represented by $X = (c;r)$, where c is the center of the interval and r is the radius of the interval. To consider the error of two intervals, we need to consider the difference of center as well as the difference of radius simultaneously. We propose the following definitions which are useful in determining the interval difference.

Definition 4.1 (Mean Absolute Error of Interval Location, MAEL)

Let $\{\hat{X}_t = (\hat{c}_t, \hat{r}_t); t = 1,2,\ldots n\}$ *be a sequence of predicted interval for* $\{X_t = (c_t, r_t); t = 1,2,\ldots n\}$. *The mean absolute error of interval location (MAEL) is given by* $MAEL = \frac{1}{n}\sum_{t=1}^{s} | c_t - \hat{c}_t |$.

Definition 4.2 (Mean Absolute Error of Interval Scale, MAES)

Let $\{\hat{X}_t = (\hat{c}_t, \hat{r}_t); t = 1,2,\ldots n\}$ *be a sequence of predicted interval for* $\{X_t = (c_t, r_t); t = 1,2,\ldots n\}$. *The mean absolute error of interval scale (MAES) is given by* $MAES = \frac{1}{n}\sum_{t=1}^{n} | r_t - \hat{r}_t |$.

Consider the interval X= [4,7] = (5.5;1.5), and the forecast intervals $\hat{X}_1$ = [1,8] = (4.5;3.5) and $\hat{X}_2$ = [6,8] = (7;1) obtained by two different forecasting methods. The $MAES$ of $\hat{X}_1$, denoted $MAES_1$, is 2. The $MAES$ of $\hat{X}_2$, denoted $MAES_2$, is 0.5. Then $\hat{X}_2$ is good forecast interval than $\hat{X}_1$ from comparing $MAES_1$ an $MAES_2$. However, since the radius of $\hat{X}_1$ is larger than that of $\hat{X}_2$, the central point of $\hat{X}_1$ is closer to the central point of X. Since the range of $\hat{X}_1$ covers the range of the actual interval X is more than the range of $\hat{X}_2$ does. As a result, we still regard $\hat{X}_1$ as the better forecast interval. Consequently, while considering the efficiency of the interval forecasting, both the two indexes should approach the real values. Hence, by combining the two factors of the center and the radius of interval, we need to integrate those above definitions.

Definition 4.3 (Mean Absolute Error of Interval, MAEI)

Let $\{\hat{X}_t = (\hat{c}_t, \hat{r}_t); t = 1,2,...n\}$ *be a sequence of predicted interval for* $\{X_t = (c_t, r_t); t = 1,2,...n\}$. *The mean squared error of interval (MSEI) is given by*

$$MAEI = \frac{1}{n}\sum_{t=1}^{n} |c_t - \hat{c}_t| + \frac{1}{n}\sum_{t=1}^{n} |r_t - \hat{r}_t| = MAEL + MAES$$

Note that for some special purposes, we may use the Weighted Mean Absolute Error of interval, $WMAEI = w_1 \frac{1}{n}\sum_{t=1}^{n} |c_t - \hat{c}_t| + w_2 \frac{1}{n}\sum_{t=1}^{n} |r_t - \hat{r}_t| = MAEL + MAES, w_1 + w_2 = 2.$ instead of the $MAEI$.

Example 4.2 Let X = [4,7] = (5.5;1.5) be the real interval, $\hat{Y}$ = [1,8] = (4.5;3.5) and $\hat{Z}$ = [6,8] = (7;1) be the forecasting intervals obtained by two different forecasting methods. It is easy to calculate

$MAEL$ of $\hat{Y}$ = 1, $MRES$ of $\hat{Y}$ = 2, $MAEI$ of $\hat{Y}$ = 3, and
$MREL$ of $\hat{Z}$ = 1.5, $MRES$ of $\hat{Z}$ = 1.5, $MAEI$ of $\hat{Z}$ = 3

That is $\hat{Y}$ has more efficient forecasting location than $\hat{Z}$. While $\hat{Z}$ has more efficient forecasting interval scale than $\hat{Y}$. However they have the same $MAEI$.

Example 4.3 Let the interval time series be X_1 = [4,6] = (5;1), X_2 = [5,8] = (6.5;1.5), the predicted intervals are $\hat{X}_1$ = [2.8,5.4] = (4.1;1.3) and $\hat{X}_2$ = [3.8,7.8] = (5.8;2). Then

$MAEL$ = (| 5 – 4.1 | + | 6.5 – 5.8 |) / 2 = 0.8.
$MRES$ = (| 1 – 1.3 | + | 1.5 – 2 |) / 2 = 0.4
$MAEI$ = 0.8 + 0.4 = 1.2

5. A case study on the commercial house in Kunming (China)

In this section we present a case which shows how to valuate the price of real estate in Kunming by fuzzy estimation method. The fuzzy valuation process is as follows:

(1) Decide the influence factors which are of importance for the valuation price of commercial house.

(2) Apply the fuzzy ordering method to calculate the weights $\{w_1, w_2, \ldots, w_k\}$.

(3) Collect information of the similar property case and then use the Marketing Comparison Approach to modify the value of location, house type, community and quality of the cases.

(4) Investigate the transaction price of commercial house which located in different district. In generally, five cases are possible.

(5) Compute the fuzzy mode and fuzzy mean in order to get the valuation price of our subject property.

As is mentioned before, the most important effective factors of the commercial house valuation price are location, house type, community and quality.

The weights of location, house type, community and quality have been calculated in the section of 3.3.

Table 5.1 is the result of the surveys for weights with respect to the effective factors

Table 5.1 The weights of commercial house.

Factors	Location	House Type	Community	Quality
w	0.3	0.2	0.3	0.2

We use the Marketing Comparison Approach to choose some of the similar cases which depend on the most important effective factors which we will show later. Then the fuzzy estimation method will be applied to obtain the predicted value of our subject property. In this paper we suppose the appraisal purpose is to get the current market price of our subject property for the trader.

Generally, the effective factors of estimated value of a commercial home include: location, house type, community and quality. Take the commercial house in Panlong District as our subject property, so the value of location, house type, community and quality of the subject property can be standardized as 1. By using the Marketing

Comparison Approach, we need to choose other similar cases as the reference object to compare. Here we take the district of Kaiyue Times, ShiGuangJunYuan, PropertyCenter, HeTangYueSe and JiangDongWorld as our comparable market cases, which are quite similar in the effective factors.

Take the district of Kaiyue Times for an example then modify the location effective factor. The standard value of location of our subject property is 1 and the evaluating rule of the location is that the less far from the center of the city the more valuable it is. Since the location of the district of Kaiyue Times is much a little far away from the center of the city than our subject property, so we can modify the location effective factor by 0.9.

Modify the house type. Our subject property in Panlong District is with one living room, two bed rooms, two bath rooms, one study room, one kitchen and two balconies which is a quite normal type whose area is 120.5 square meters. The case in the district of Kaiyue Times is also designed similar as our subject property, but be decorated much more reasonable, easier living and with more bright sun shines. The area is 135 square meters. So we can modify the house type effective factor by 1.3.

Modify the community. The community of our subject property in Panlong District is like the other excellent districts in Kunming. It is covered with green grass and trees in the forest belt. There are some body building equipments and entertainment facilities for the residents.

The nearest bus station is about one kilometer away and it is convenient to go to the urban centre. There are many restaurants nearby that can supply lots of delicious dishes. However, the community in the case of the district of Kaiyue Times has more advantages. Besides, there is a modern shopping mall surrounding it and a hospital not far away. So we can modify the community effective factor by 1.1.

Modify the quality. The quality of our subject property in Panlong District is a common level and has 7 years history. However, the quality of the case in the district of Kaiyue Times has higher quality since the building only have been finished 2 years. So we can modify the quality effective factor by 1.2.

In order to get a more appropriate price we evaluate the price of our subject property by fuzzy estimation method.

The detailed valuation steps are as follows. The valuation price of commercial house is:

Valuation Price: {price, location, house type, community, quality}

The price initially used to get the valuation price is the transaction price of the cases. In this paper, we investigated the transaction price under the help of one real estate business agency which located in Kunming. We get the transaction price in the first market case of Kaiyue Times district is form 5151 to 6255 yuan per square meters. As is known to all, some of the commercial houses are decorated in a high grade level, some of them are of common level and some even are blank housing. Therefore, three typical cases in the district of Kaiyue Times are selected by the rank of decoration, and the interval is from 5151 to 6522 yuan per square meter.

Let

$P_{t+1} = P_t \cdot L_t^{\alpha} \cdot H_t^{\beta} \cdot C_t^{\gamma} \cdot Q_t^{\lambda}$

Where P_{t+1} = *the valuation price at time t+1,* P_t = *the price at time t.* L_t^{α} = *location affection,* H_t^{β} = *house type affection,* C_t^{γ} = *community affection,* Q_t^{λ} = *quality affection,* $\alpha, \beta, \gamma, \lambda$, *stand for the multiplicative weight.*

The Table 5.2 showed how we get the fuzzy valuation price of our subject property.

Table 5.2 The valuation price of the commercial house in pan long district, Unit = RMB / m^2.

Factors / Cases	$Price_t$	Location	House Type	Community	Quality	$Price_{t+1}$
Kaiyue Times	[5151,6522]	0.9	1.3	1.1	1.2	[5613,7107]
ShiGuangJunYuan	[4103,7338]	1.1	0.9	0.8	0.9	[3785,6771]
PropertyCenter	[6111,7792]	0.8	1.1	0.9	0.8	[5398,6883]
HeTangYueSe	[5250,6535]	0.6	0.8	1.2	1.1	[4637,5773]
JiangDongWorld	[5206,6263]	0.7	0.9	1.1	1.2	[4888,5880]
Fuzzy Mode			[5398,5773]			
Fuzzy Mean			[4864,6483]			

From Table 5.2 we can find that the fuzzy mode and fuzzy mean of the valuation price. Take the first case as an example, from Definition 3.2, we can find that the [5151, 6522] is the transaction price of the market case of Kaiyue Times district. Finally computed with the weight and the modified value of location, house type, community and quality, we can get the fuzzy mean of the subject property, which is

$$[5151 \cdot 0.9^{0.3} \cdot 1.3^{0.2} \cdot 1.1^{0.3} \cdot 12^{0.2}, 6522 \cdot 0.9^{0.3} \cdot 1.3^{0.2} \cdot 1.1^{0.3} \cdot 1.2^{0.2}] = [5613,7107]$$

As is shown in the Table 5.2 we get the fuzzy mode is [5398, 5773] and the fuzzy mean is [4864, 6483]. It is not difficult to find that the interval of fuzzy mode is short than the fuzzy mean. It implies that the probabilities of the valuation price of our subject property are more likely in the fuzzy mode interval. However, the fuzzy mean interval implies the lowest price which the seller can accept and the highest price which the buyer can supply.

According to section 4.3 we can calculate the MAEL, MAES and MAEI. The Table 5.3 shows them.

Table 5.3 The MAEL, MAES and MAEI of the valuation price, Unit = RMB / m^2.

Cases \ Factors	$Price_{t+1}$ (c; r)	Fuzzy Mode (5585; 187)			Fuzzy Mean (5673; 809)		
		MAEL	MAES	MAEI	MAEL	MAES	MAEI
Kaiyue Times	(6360; 747)	775	560	1335	687	62	749
ShiGuangJunYuan	(5278; 1493)	307	1306	1613	395	684	1079
PropertyCenter	(6140; 742)	555	555	1110	467	67	534
HeTangYueSe	(5205; 568)	380	381	761	468	241	709
JiangDongWorld	(5384; 496)	201	309	510	289	313	602
Result		444	622	1066	461	273	735

It is easy for us seeing the result from table 5.3 that the fuzzy mode has more efficient forecasting location than fuzzy mean, since 444 is smaller than 461. While fuzzy mean has more efficient forecasting interval scale than fuzzy mode, since 273 is smaller than 622.

6. Conclusion

Estimating the value of real estate is a wide-ranging and complex area and its evaluation involves much dispute. The advantage of the fuzzy statistical analyzing techniques proposed in this article lies in the way it handles human thought and recognition, improving on vague measurement. The presented integrated procedure differs from the traditional assessment method, and establishes the membership grade of evaluator's weight to better capture real values. Moreover, suppose we are surveying real estate. No matter how carefully we read the measuring process, we can never be certain of the exact value, but we can answer with more confidence that the appropriate area lies within certain bounds. Though interval analysis and fuzzy set theory are areas of active research in mathematics, numerical analysis and computer science began in the late 1950s and early 1960s. The application to statistical evaluations in real estate is just beginning.

Using fuzzy statistical analysis we can get fuzzy data which can be applied in different areas. The methodology which integrates the traditional valuation approach with fuzzy logic shows us how the appraisals can valuate a commercial house in the form of a fuzzy interval which satisfies different components of real transactions.

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