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IDENTIFICATION AND ASSESSMENT OF TAIWANESE CHILDREN'S CONCEPTIONS OF LEARNING MATHEMATICS

Received: 30 April 2010; Accepted: 25 January 2011

ABSTRACT. The aim of the present study was to identify children's conceptions of learning mathematics and to assess the identified conceptions. Children's conceptions are identified by interviewing 73 grade 5 students in Taiwan. The interviews are analyzed using qualitative data analysis methods, which results in a structure of 5 major conceptions, each having 2 subconceptions: *constructivist* (*interest* and *understanding*), *interpretivist* (*liberty* and *innovation*), *objectivist* (*academic goal* and *perseverance*), *nativist* (*confidence* and *anxiety* (reverse)), and *pragmatist* (*vocational goal* and *application*). The conceptions are assessed with a self-developed questionnaire, titled "the Conception of Learning Mathematics Questionnaire" (CLMQ), which is administered to 513 grade 5 students in Taiwan and examined with a reliability measure, confirmatory factor analysis, and correlations with 2 criteria: mathematics achievement and approaches to learning mathematics. The results show that the CLMQ has desirable internal consistency reliability and construct validity. The conceptions are also sensibly in relation to the 2 criteria, suggesting that the CLMQ is a valid measure for evaluating the quality of children's learning mathematics in relation to teaching contexts.

KEY WORDS: approaches to learning, conceptions/beliefs of learning, mathematics learning

Introduction

The conceptions of learning are critical factors in relation to the quality of student learning environments, processes, and outcomes (Dart, Burnett, Purdie, Boulton-Lewis, Campbell & Smith, 2000; Liem & Bernardo, 2010; Marton, 1983; Trigwell & Ashwin, 2006). This has attracted educational and psychological researchers to delve into the nature of the conceptions of learning. This line of works, however, mostly focuses on general learning, reading, and science (e.g., Dahl, Bals & Turi, 2005; Eklund-Myrskog, 1998; Marton, 1981; Saljo, 1979; Tsai, 2004). Relatively few studies in the field of mathematics education use the term "conceptions" (Crawford, Gordon, Nicholas & Prosser, 1998), but mathematics-related "beliefs" refer to student constructed meanings in the subjective, affective, or epistemic aspect of learning mathematics (Chiu, 2009; Leder & Forgasz, 2002; McLeod, 1994; Op't Eynde, De Corte & Verschaffel, 2006; Schoenfeld, 1989).

International Journal of Science and Mathematics Education (2012) 10: 163–191

Furthermore, past studies normally focus on university students and some on secondary school students, but very few of these studies focus on primary school children (Klatter, Lodewijks & Aarnoutse, 2001). Since the conceptions of learning are culturally or contextually dependent (Marton, Watkins & Tang, 1997; Tsai, 2004), there appears to be a need for research to initially identify children's conceptions of learning mathematics, especially for children from a culture that is relatively rarely researched. The identified conceptions are best assessed for their quality in terms of theoretical constructs and practical use. The purpose of the present study, therefore, was to identify Taiwanese primary school children's conception of learning mathematics with a qualitative methodology and to assess the identified conceptions with a quantitative methodology. Both of these methodologies are formal ways to study the conceptions of learning, as shown by the following literature review.

Identification of the Conceptions of Learning

Qualitative methodologies are general ways of identifying student conceptions of learning. The history of research on the conceptions of learning can be tracked back to Marton & Saljo's (1976) study in Sweden; in a naturalistic experiment, students were asked to read an article and describe what they had learned and how they had gone about reading it. The interview data were later analyzed with phenomenography, and the results showed that students generally expressed one of the two major intentions and strategies: (1) reading for words and facts or (2) reading for meaning (Marton, 1981). Saljo's (1979) study, by interviewing teenagers and adults for their experiences of learning, generates five conceptions of learning: (1) an increase in knowledge; (2) memorizing; (3) an acquisition of facts or principles; (4) an abstraction of meaning; and (5) an interpretive process aimed at understanding reality, with (2) and (3) in relation to surface processing and (4) and (5) in relation to deep processing. Similar conceptions are identified using interview and qualitative data analysis methods (e.g., phenomenography and grounded theory) by later studies done on students from higher and secondary education and from diverse cultures focusing on diverse domains. For example, Marton, Dall'Alba & Beaty (1993) in Sweden, Eklund-Myrskog (1998) in Finland, and Marshall, Summer & Woolnough (1999) in the UK studied university students' conceptions of general learning, while Tsai (2004) in Taiwan studied high school students' conceptions of learning science. The conceptions of general learning and those of science learning appear to largely overlap (76%), as indicated by Tsai & Kuo (2008).

Research on mathematics and young children, however, reveals a different picture. Crawford et al. (1994) in Australia used a questionnaire to collect data from first year university students, analyzed the data with phenomenography, and found two sets of conceptions of learning mathematics: (1) fragmented conceptions (mathematics as numbers/rules/formula with applications to problems) and (2) cohesive conceptions (mathematics as thinking and understanding for complex problems and the world. Based on a comprehensive review of literature, Kloosterman & Stage (1992) developed the *Indiana Mathematics Belief Scales*, which included 36 items grouped into five factors: confidence in solving word problem, simple and step-by-step procedures for solving problems, importance of word problems, and effort for increasing ability. Op't Eynde, De Corte & Verschaffel (2002) posited a framework of student mathematics-related belief system which included three major sections: beliefs about (1) mathematics education (mathematics as a school subject, mathematical learning and problem solving, mathematics teaching in general); (2) self (self-efficacy, control, task value, and goal orientation); and (3) social context (teacher and student constructed social norms and socio-mathematical norms in class). Kloosterman (2002) developed nine categories of interview questions for understanding high school students' beliefs about mathematics and learning mathematics: feelings about school and about mathematics, effort in mathematics, non-school influences on motivation, self-confidence in mathematics, natural ability in mathematics, goal orientation and effort, study habits in mathematics, mathematics content, and students' expectations of teachers. Klatter et al. (2001) studied grade 6 children in the Netherlands and found a different structure of the conceptions of general learning, which included: purpose of school, learning orientation, regulation, learning demands, and mental activities. The present finding is likely to be somewhere between those of Crawford et al. and Klatter et al. as the present study places a special focus on mathematics and primary school children. In summary, the research of conceptions/beliefs in mathematics education and for young children has gradually extended to include a wide range of interactions between cognitive, affective, and contextual factors, while that in the fields of science and general learning and for older students tends to focus on cognitive factors.

Assessment of the Conceptions of Learning

The initially identified conceptions of learning can be quantified and made into questionnaires, which then can be used to assess their relationships with student characteristics, learning environments, and learning outcomes

(Crawford et al., 1998; Purdie & Hattie, 2002). Quantitative methodologies are relatively formal ways to assess the identified conceptions of learning, and the conceptions of learning are related to achievements and approaches to learning (Tsai, 2004). Therefore, the criterion validity of the conceptions of learning identified by a study can be assessed by relating the conceptions to achievement and approaches to learning.

Achievement. There are positive relationships between achievements and desirable conceptions/beliefs of learning (mathematics) (e.g., constructing meaning, understanding, confidence, interest, and usefulness) and negative relationships between achievements and undesirable conceptions of learning, such as rote learning and memorization (Cano & Cardelle-Elawar, 2004; Grootenboer & Hemmings, 2007; McLean, 2001).

Approaches to Learning. Research on approaches to learning bases its theories on Marton & Saljo's (1976) and Marton's (1981) experimental and qualitative studies and examines the theories with self-developed questionnaires and quantitative methodologies (Biggs, Kember & Leung, 2001; Boyle, Duffy & Dunleavy, 2003; Entwistle & Peterson, 2004; Entwistle, McCune & Walker, 2001; Leung, Ginns & Kember, 2008). According to Biggs (1993, 2001) and Kember, Wong & Leung (1999), approaches to learning include two distinct constructs: deep and surface approaches. While the deep approach is proper engagement in tasks with interest and meaning, the surface approach refers to keeping away from trouble, with a fear of failure by rote learning and narrowing targets. Kember, Biggs & Leung (2004) further indicated a more complicated factor structure of learning approaches: deep approaches including deep motives (comprising *intrinsic interest* and *commitment to work*) and deep strategies (comprising *relating ideas* and *understanding*) and surface approaches including surface motives (comprising *fear of failure* and *aim for qualification*) and surface strategies (comprising *minimizing scope of study* and *memorization*).

Researchers are interested in exploring the relationships between approaches to learning and the conceptions of learning. Given their common theoretical base of phenomenographic studies (Marton, 1981), there are generally positive relationships between deep approaches and deep processing/qualitative conceptions of learning (e.g., understanding, learning new things, and situated learning) and negative relationships between surface approaches and surface processing/quantitative ones (e.g., memorization; Burnett, Pillay & Dart, 2003; Duarte, 2007; Trigwell & Ashwin, 2006; Zhu, Valcke & Schellens, 2008.

Approaches to learning are significant predictors of achievement (Cano, 2005; Diseth & Martinsen, 2003). Deep approaches normally relate to achievement positively and surface approaches relate to achievement negatively (Booth, Luckett & Mladenovic, 1999; Burton, Taylor, Doswling & Lawrence, 2009; Saljo, 1981; Zeegers, 2001), with some exceptions (Kember, Jamieson, Pomfret & Wong, 1995; Newble & Hejka, 1991). These exceptions are attributed to educational environments such as fixed curriculum, examination procedure, and non-sufficient work.

In summary, a qualitative and quantitative mixed methodology (Onwuegbuzie & Johnson, 2006) is used to answer the following two research questions, respectively:

Research question 1: What are children's conceptions of learning mathematics?
Research question 2: What is the quality of children's conceptions of learning mathematics? In other words: What are the reliability, construct validity, and criterion validity of the Conceptions of Learning Mathematics Questionnaire (CLMQ) developed based on the answers to research question 1?

Study 1: Identifying Children's Conceptions of Learning Mathematics

Aims

The aim of study 1 was to identify children's conceptions of learning mathematics, in other words, to answer research question 1 by a qualitative methodology. A group of primary school children are interviewed for their experiences and perception of learning mathematics. All the interviews are analyzed using qualitative data analysis methods.

Method

Participants. The research participants were 73 grade 5 students (38 boys and 35 girls) from four classes at a primary school in Taiwan. They were chosen through balancing their classes, gender, and mathematics achievement.

Data Collection. The participating students were interviewed according to the following three-stage procedure. At stage 1, the interview began by asking the children to solve two or four mathematical problems taken from their textbooks, which were taught by their teachers in class. This was to remind children of their learning experiences in their mathematics classrooms and other settings. As such, we can increase the opportunity to obtain children's conceptions of learning that accurately and fully reflect the whole learning contexts. During stage 2, students were asked for their experiences of solving the problems in relation to their experiences of learning mathematics. At stage 3, students were asked the following questions about what they understood by learning mathematics:

Do you have anything that you wish to say about mathematics or learning mathematics?

What is mathematics? What does mathematics look like?

What do you mean by learning mathematics?

How do you feel about learning mathematics?

The students were asked the questions alternatively in order to make sure that all related issues had been raised by the students. The interviews lasted between 16 and 60 min. All the interviews were conducted by the same interviewer, audio-recorded, and transcribed.

Data Analysis

The verbatim transcripts of student interviews were analyzed by open coding, theme finding, constant comparison, and theory generation. These were a combination of methods suggested by the methodologies of general qualitative data analysis (Miles & Huberman, 1994), phenomenography (Marton, 1981), and grounded theory (Charmaz, 2000; Strauss & Corbin, 1990, 1998). For the present data, the researcher first read the transcription, identified keywords, and categorized similar keywords into themes. Then, the themes were identified as either motive- or strategy-related. Next, the themes were restructured into higher order categories based on the meaning of the themes and the juxtaposition of the themes in the interview transcription. Finally, all the themes and higher order categories were given "names" for the conceptions identified. A research assistant double-checked the coding and disagreements were resolved by discussion.

Conceptions of Learning Mathematics Identified

The final structure consisted of five major conceptions of learning mathematics: *constructivist*, *interpretivist*, *nativist*, *objectivist*, and *pragmatist* conceptions, named partially with reference to Driscoll (2000) on the epistemological sources of learning theories. Each conception included two subconceptions, one as more *motive*-related and the other as more *strategy*-related. Table 1 shows the outline of the conceptions and their respective subconceptions.

Constructivist Conceptions

Students with constructivist conceptions engaged in one motive-related theme—*interest*—and one strategy-related theme—*understanding*. A girl stated in the interview:

> The ability to understand is endless if you 'use your heart' …If you really don't understand, just ask as quickly as possible …If you say 'I hate it. I don't want to do it,' sure you can't understand it …The feeling of understanding is so wonderful. I've experienced it …Failure is that you stay at the same stage, not stepping further …I can't predict the future.

This girl engaged in learning mathematics with the understanding of "here and now." This full engagement linked to interest in the search for meanings and challenges, as three students stated:

> Mathematics is great fun …For example, if I don't understand but later I understand it, it's great fun.

> Mathematics is interesting if there are harder or challenging problems …If similar problems are repeatedly taught, it's easy and not interesting.

> Mathematics is about understanding its meaning, not just calculating.

There was a clear link between the states of understanding and those of feeling interested. Something interesting implied that it was a thing that

TABLE 1

Children's conceptions of learning mathematics identified in study 1

Conceptions of learning mathematics	*Motive-related subconceptions of learning mathematics*	*Strategy-related subconceptions of learning mathematics*
Constructivist	Interest	Understanding
Interpretivist	Liberty	Innovation
Nativist	Confidence	Anxiety (reverse)
Objectivist	Academic goal	Perseverance
Pragmatist	Vocational goal	Application

triggered thinking, which finally achieved the states of understanding and satisfaction.

Interpretivist Conceptions

Interpretivist students emphasized the *liberty* provided by their learning environment, which gave them opportunities for *innovation*. They wished few constraints from their teachers. As two girls stated:

> About learning mathematics, if the teacher asks me to do like this or like that, I will dislike it …Teachers shouldn't ask too much, e.g., about the solution procedures. …My previous teacher asked us to calculate like that, as it's easier to get right answers. I'll especially hate it if teachers want us to follow their procedures and their procedures are complex.

> I feel that teachers give us very little space to think to the extent that we can't even speak out about how to solve a problem …The teacher only gave us 30 seconds, ten seconds or five seconds. The time is not enough …I hope teachers give us larger space.

Then, the two girls spent a significant amount of time stating the issue of restrictions from their teachers in the interview. They, however, did not say anything about what they would do if there were no restrictions. A boy stated his reaction to the restriction:

> Sometimes I like to find new answers …not necessarily to use the teachers'.

A liberal teacher and environment is likely to be a condition precedent to innovating behaviors. In other words, if there are "restrictions," there will be little space for students to perform creative behavior. Under the condition of restrictions, the first issue is to get rid of the "restrictions," and then it is likely to put children's desire to innovate and experiment into practice.

Nativist Conceptions

Students with *nativist* conceptions were motivated by their *confidence*, *self-concept*, or *perceived ability* in mathematics. If a failure occurred, the concern on ability often generated *anxiety* or *fear*, which was a negative psychological reaction, sometimes accompanying hurting or avoidance behavior. The following statements from five students show these conceptions:

> I like mathematics more …as I'm better at it (than the other school subjects).

> Mathematics is like sun and moon. Sometimes you feel you are smart. But when you fail or do the problems wrong, it'll be as cold as the moon …It means a terrible mood …If I do not get a good mark in school tests, I'll hit the wall of my room until my hands hurt … because I do not do it well myself.

The harder the mathematics problem is, the less possible that I'll be able to do it. If I can't do it, I'll become very nervous.

I feel it's hard sometime and don't want to learn it.

Horrible, mathematics is so horrible …sometimes it's a big monster, very difficult.

Students with *nativist* conceptions seemed to seriously monitor the degrees of problem difficulty and their achievement in relation to problem difficulty. They further perceived external achievement as a critical criterion of their inborn ability. In the interview, many students talked about the degrees of problem difficulty, but *nativist* students especially placed the issue at the center of their learning mathematics, while other students placed the issue in the background. For example, students with positive *constructivist* conceptions preferred hard and challenging problems and enjoyed the process of solving hard problems. Students with *interpretivist* conceptions did not state anything about problem difficulty. Students of positive *objectivist* conceptions, who will be introduced in the next section, worked really hard to overcome difficult problems in order to achieve their future aims. The students with *nativist* conceptions placed too much emphasis on external problem difficulty. They inferred their internal self-image or self-concept by seriously measuring the degrees of problem difficulty. As a result, *anxiety* became a response to low achievement partly in order to avoid negative self-concept.

Objectivist Conceptions

Students with *objectivist* conceptions set an objective, external goal, normally an *academic goal* for a desirable development at a later educational stage. They were sensitive to time lines and aimed at achieving the goal with *perseverance* or *effort*. As two boys stated:

Primary mathematics is the basis. You have to establish the basis at primary school. You have to understand more, to go to there, and to strive for it …If you are not able to do it well, you'll be miserable at junior high school.

Mathematics is like a clock …If you miss something at primary school, you won't be taught about it later. It's like time disappearing …So cherish the time.

The goal was clearly set and effort was taken, working hard along a clear time line. The children were looking into the future. A girl's statements depicted in detail the methods of effort-taking:

My thoughts about mathematics is that …I should try much harder for mathematics …because sometimes there're inevitably one or two problems that you are not able to solve …

(My methods are) to review more at home …read self-learning practice books, read some problems, do the problems, and ask my father to have a look at what I did …All these are related to what I learn at school.

The strategies for studying mathematics focused on the school curriculum and the students directly worked for it; none of the curriculum can be carelessly ignored. The *objectivist* students worked hard and overlearned in order to make sure that they could master every piece of knowledge that they had to learn.

Pragmatist Conceptions

Students with *pragmatist* conceptions focused on the long-term *vocational goal*. They also appreciated the efficacy of the *application* of mathematics to their lives and on the street:

If we can't do mathematics, it will be a big trouble when we do jobs in the outside world. For instance, you have to calculate how much stock is in a shop and how many products should be imported.

We are likely to use mathematics when we work …Mathematics is useful for life.

Mathematics is in fact quite easy. It is because if you don't learn mathematics well, you'll be unable to understand things on the street …If I don't even understand mathematics, I'll be cheated by others.

Sometimes you'll have to use mathematics. For example, divide things to some people. You don't have to count it one by one.

The students with *pragmatist* conceptions appeared to be "happy students" in learning mathematics. They acknowledged the value of mathematics in terms of the long-term and present utility. As a result, they did not avoid learning mathematics. Practical mathematics was their study focus. They were not obligated to study difficult and academic mathematics if their knowledge and skills were enough for earning a living or dealing with daily problems.

STUDY 2: ASSESSING CHILDREN'S CONCEPTIONS OF LEARNING MATHEMATICS

Aims

The aim of study 2 is to assess children's conceptions of learning mathematics identified in Study 1, i.e., to answer research question 2, by a quantitative methodology. A questionnaire, the CLMQ, is developed to model the conceptions, and the reliability, construct validity, and criterion validity of the questionnaire were assessed. The reliability of the questionnaire is examined using the internal consistency measure of Cronbach's alpha. The construct validity of the conceptions is assessed by confirmatory factor analysis (CFA; Onwuegbuzie & Johnson, 2006). We wish to see desirable reliability coefficients for each subscales of the conceptions and acceptable data fit to the factor structure of the conceptions (Table 1 and Figure 1). The criterion validity of the conceptions is assessed by examining the relationships between the conceptions and both mathematics achievement and approaches to learning (e.g., Biggs, et al., 2001). In summary, based on the meanings of the conceptions identified in study 1 and past research finding in relation to approaches to learning, we can propose three hypotheses:

Hypothesis 1: The reliability of The CLMQ is acceptable.
Hypothesis 2: The construct validity of the CLMQ is acceptable.
Hypothesis 3: The criterion validity of the CLMQ is acceptable.
Hypothesis 3-1: There are positive relationships between the five conceptions and mathematics achievement.
Hypothesis 3-2: There are positive relationships between the five conceptions and deep approaches to learning mathematics.
Hypothesis 3-3: There are negative relationships between the five conceptions and surface approaches to learning mathematics.

A further look at the differential meanings of the five conceptions and approaches to learning suggests that the relationships will be in varying degrees. Constructivist and interpretivist conceptions are more internally oriented and objectivist and pragmatist conceptions more externally oriented. Constructivist conceptions (interest and understanding) may be highly related to deep approaches (interest and maximizing meanings) because of the high overlap between their meanings. Nativist conceptions focus on confidence, which highly links to achievements.

Method

Participants. The research participants were 513 grade 5 students (254 girls and 259 boys) from 18 classes in three schools. The classes were normal ones in Taiwan and all had mixed ability. Each class had similar numbers of boys and girls. There are around 23–34 students in the classes (median = 28, mode = 31). The participating classes were normally distributed in student ability based on school achievement test results in the previous year.

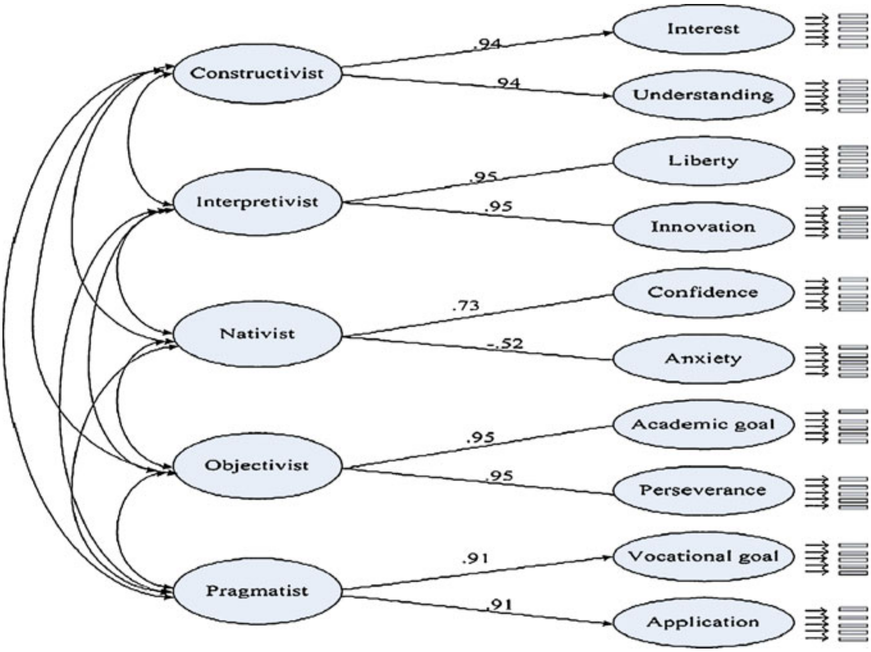

Figure 1. Factor structure of the Conceptions of Learning Mathematics Questionnaire. The items and factor loadings of the subconceptions leading to the items are presented in Table 2

Measure 1: Conceptions of Learning Mathematics Questionnaire. The CLMQ was developed based on the findings of study 1 (Tables 1 and 2). The questionnaire contained five major conceptions, each of which had one motive-related subconception and one strategy-related one. Each subconception consisted of five items (Table 1). Students were asked to response to each item using a five-point Likert scale ranging from 1 (strongly disagree) to 5 (strongly agree).

Measure 2: Approaches to Learning Mathematics Questionnaire. This measure was also used to examine the criterion validity of the CLMQ by computing its correlations with the five CLMQ conceptions. The Approaches to Learning Mathematics Questionnaire (ALMQ) was adapted from the questionnaire developed by Whitebread & Chiu (2004), which focuses on primary school children's mathematical problem-solving skill and was developed based on the questionnaire of Biggs et al. (2001) for examining college students' learning approaches. There appears to be no other questionnaires to date that focus on children's approaches to learning in relation to mathematics. The ALMQ contained two subscales: deep and surfaces approaches. Each subscale had five items,

TABLE 2

Cronbach's α and factor loadings for the items of the conceptions of learning mathematics questionnaire

Subconceptions/subscales and items	*α*	*Factor loadings*
Interest	0.90	
I feel that mathematics is interesting.		0.82
Learning mathematics is great fun.		0.84
I enjoy my time studying mathematics.		0.76
I look forward to my mathematics lessons.		0.74
Solving mathematics problems is interesting.		0.85
Understanding	0.86	
To understand mathematics thoroughly is my goal.		0.76
My aim is to have a deep understanding of mathematics.		0.73
My goal is to understand mathematics in detail.		0.75
Understanding every concept in mathematics is my goal.		0.79
Understanding why a method works is my aim of learning mathematics.		0.73
Liberty	0.86	
I solve mathematics problems with my own methods freely.		0.78
I freely solve mathematics problems according to what I think.		0.71
I use mathematics concepts to solve mathematics problems liberally.		0.76
I can liberally choose diverse methods to solve mathematics problems.		0.78
I can freely explain mathematics concepts that I learn.		0.71
Innovation	0.84	
I always use new ideas to do mathematics.		0.69
I always find new methods to solve mathematics problems.		0.69
I can think of many ways to solve one mathematics problem.		0.75
I can always change the way of problem solving and create a better one.		0.74
I can use ways to solve mathematics problems that others can't think of.		0.69
Confidence	0.91	
My mathematics ability is not bad.		0.85
I have a good ability in mathematics.		0.90
I get good marks in mathematics.		0.90
I do well in mathematics.		0.85
Mathematics is one of the subjects I am good at.		0.86
Anxiety	0.79	
I feel dreadful about doing mathematics.		0.73
I am afraid of doing mathematics.		0.79
Taking mathematics tests scares me.		0.78
It scares me to think that I will be taking harder or more advanced mathematics.		0.54

My mind goes blank, unable to think clearly, when doing mathematics.		0.37
Academic goal	0.82	
I need to learn mathematics well in order to get into the school I dream of.		0.59
I have to be good at mathematics so that I can study in a good school.		0.52
I need mathematics, which can help with my later learning.		0.74
The better I learn mathematics, the better chance I can get into a good school.		0.71
Learning mathematics is worthwhile because it will help me get in a good school.		0.75
Perseverance	0.82	
Taking effort can improve my mathematics ability.		0.71
Being hard-working can increase my mathematics ability.		0.67
The only way to improve my mathematics ability is to make an effort.		0.66
I make an effort in mathematics because this can increase my mathematics ability.		0.69
I try hard in mathematics in order to enhance my mathematics capacity.		0.70
Vocational goal	0.89	
I need to be good at mathematics so that I can do my ideal job.		0.80
I need to do well in mathematics in order to get the job I want.		0.86
Learning mathematics will help me with career prospects.		0.68
Things I learn in Mathematics can help me get a job.		0.81
Doing mathematics well can give me more chances to find a job.		0.73
Application	0.78	
Learning mathematics can benefit me in solving daily-life problems.		0.68
Mathematics can help me deal with things in my daily life.		0.73
Mathematics is very useful in daily life.		0.54
Mathematics is a worthwhile and necessary subject for my daily life.		0.68
Mathematics can be used to solve daily life problems.		0.77

The factor loadings of the items are based on the CFA structure of Figure 1

rather than ten items as used by Biggs et al., in order to save the children time of filling in the CLMQ and ALMQ at the same time.

Measure 3: Mathematics Achievement. This measure was used to examine the criterion-related validity of the CLMQ by computing its correlations with the five CLMQ conceptions. The children's mathematics achievements were collected from their two school examinations, one before and one after the administration of the two questionnaires (the CLMQ and ALMQ). As such, the achievement data may best represent the children's mathematics ability near the time of data collection. In Taiwan, school examinations were normally administered three times in each semester and at the same time for classes in the same school, and at similar times for different schools. The examination contents were the same for students in the same school.

Statistical Methods. The major statistical method used here was structural equation modeling using the software of LISREL 8.72 (Du Toit & Du Toit, 2001; Joreskog & Sorbom, 2001, 2005). CFA was performed to validate the two-layer factor structure of the CLMQ identified in study 1 and to validate the two-factor structure of the ALMQ, i.e., to examine the construct validity of the CLMQ and ALMQ. The indices of model fit used here included χ^2, comparative fit index (CFI), non-normed fit index (NNFI), and root mean square error of approximation (RMSEA). A non-significant χ^2 value indicates a good fit. The χ^2 value, however, may become significant just because of a large sample size, and so χ^2 may not be a suitable index when the sample size is large (Bollen & Long, 1993; Browne & Cudeck, 1993). The values of CFI and NNFI larger than 0.90 and that of RMSEA smaller than 0.05 indicate a good fit, a RMSEA value between 0.05 and 0.08 indicates a reasonable fit, and factor loadings above 0.50 and below 1.00 are desirable (Hair, Anderson, Tatham & Black, 1998; Hair, Black, Babin, Anderson & Tatham, 2006; Schumacker & Lomax, 1996).

Internal consistency reliability for each of the subscales of the CLMQ and ALMQ were examined by obtaining the values of Cronbach's α. According to Murphy & Davidshofer (2005), reliability estimates larger than 0.80 are high and those below 0.60 are low; measures of personality and attitudes have a reliability of 0.64, on average.

The children's mathematics achievement scores were standardized into z scores for each class, each school, and then all the students. This procedure can reduce the confounding effect of differential examination contents and scoring standards between different classes and schools.

Reliability and Construct Validity of the CLMQ

The coefficients of internal consistency (Cronbach's α) for the five student conceptions of learning mathematics were 0.93 (constructivist), 0.91 (interpretivist), 0.87 (nativist), 0.89 (objectivist), and 0.90 (pragmatist). The internal consistency coefficients of the ten subconceptions ranged from

0.78 to 0.91 (Table 2). Most of the coefficients of internal consistency were high, which supported hypothesis 1.

The result of a CFA test revealed a reasonable fit to the a priori two-layer factor structure of the CLMQ (Tables 1 and 2 and Figure 1): There were five major conceptions of learning mathematics, each conception containing two subconceptions ($\chi^2(1{,}161) = 3{,}095.43$, $p < 0.05$; CFI = 0.98; NNFI = 0.98; RMSEA = 0.057). The factor loadings leading from the subconceptions to their respective items were all above 0.50 and below 1.00, except one item in anxiety (Table 2). The parameter estimates and fit indices indicated that the a priori structure of the CLMQ identified in study 1 was generally proper. The result supported hypothesis 2.

Reliability and Construct Validity of the ALMQ

The coefficients of internal consistency (Cronbach's α) for the two approaches to learning mathematics were 0.81 (deep approaches) and 0.69 (surface approaches). The result of a CFA test revealed a reasonable fit to the a priori two-factor structure of the ALMQ ($\chi^2(34) = 115.22$, $p < 0.05$; CFI = 0.96; NNFI = 0.95; RMSEA = 0.068). The factor loadings leading from the subscale to their respective items were all above 0.50 and below 1.00, except two items in surface approaches (Table 3). The factor correlation between the deep and surface approaches was −0.58, which was consistent with the theoretical notions of Biggs et al. (2001). The parameter estimates and fit indices indicated that the a priori structure of the ALMQ was generally proper, especially given the few items for each subscale.

Criterion Validity of the CLMQ

The five conceptions of learning mathematics and deep approaches to learning mathematics were positive concepts and their means were all above the average 3.00 (score range = 1–5); the mean of surface approaches, with a meaning in the negative direction, was below 3.00 (Table 4). The result showed that Taiwanese children had desirable conceptions of and approaches to learning mathematics. The results of one-sample t tests on whether the means of the five conceptions and two approaches are significantly different from 3.00 revealed that all the means were significant from the average 3.00, except for the two non-significant means of the constructivist and interpretivist conceptions. The result implied that children in Taiwan were not keen to take constructivist and interpretivist conceptions in learning mathematics compared with other conceptions.

TABLE 3

Cronbach's α and factor loadings for the items of the approaches to learning mathematics questionnaire

Sub-approaches/subscales and items	α	*Factor loadings*
Deep approaches	0.81	
I have to do enough work on mathematics until I feel satisfied.		0.75
I test myself on mathematics problems until I understand it completely.		0.70
Studying mathematics can be highly interesting once I get into it.		0.55
Studying mathematics can at times be as exciting as a novel or movie.		0.70
I work hard at studying mathematics because it is interesting.		0.74
Surface approaches	0.69	
Students shouldn't spend a significant amount of time studying mathematics problem if it won't be tested.		0.55
I do not find mathematics very interesting so I keep my work to the minimum.		0.66
Studying mathematics in depth will create confusion, when all we need is a passing acquaintance with it.		0.47
I study only the mathematics material that will be in the examination.		0.40
Mathematics is a boring subject, so I don't need to spend much time on it.		0.66

Pearsons' correlation coefficients between the variables of the five conceptions, achievement, and two approaches showed sensible outcomes. Correlations between the five conceptions showed medium to slightly high relationships, with the highest relationships between the objectivist and pragmatist conceptions (0.84) and between the constructivist and interpretivist ones (0.78) and the lowest relationships between the nativist conception and both the pragmatist conception (0.41) and objectivist conception (0.47).

There were medium-positive relationships between the five conceptions and mathematics achievement (0.33 – 0.49). Mathematics achievement had the highest correlation with the nativist conception (0.49), the next highest correlation with the objective conception (0.38), and smaller correlations with the other three conceptions (0.33 or 0.34).

All the five conceptions had positive relations with deep approaches (0.62–0.88); in addition, the correlation between constructivist conceptions and deep approaches was the largest (0.88). All the five conceptions

TABLE 4

Means, standard deviations, one-sample *t* test results, and correlations for the five conceptions of learning mathematics, mathematics achievement, and two approaches to learning mathematics

			Correlation						
	Mean	*SD*	*1*	*2*	*3*	*4*	*5*	*6*	*7*
Constructivist	3.07	1.01							
Interpretivist	3.04	0.93	0.78						
Nativist	3.22[a]	0.89	0.66	0.64					
Objectivist	3.60[a]	0.88	0.75	0.64	0.47				
Pragmatist	3.55[a]	0.99	0.74	0.60	0.41	0.84			
Achievement	0.00	0.95	0.34	0.34	0.49	0.38	0.33		
Deep approaches	3.15[a]	0.93	0.88	0.74	0.62	0.72	0.70	0.33	
Surface approaches	2.36[a]	0.84	−0.39	−0.30	−0.44	−0.32	−0.31	−0.19	−0.39

All correlations are significant at the 0.01 level (two-tailed tests). The score range of the five conceptions and two approaches is 1–5. The achievement scores are standardized

[a]$p<0.05$, in one sample *t* test on whether the means of the five conceptions and the two approaches are significantly different from 3.00 (the average)

had negative relations with surface approaches (−0.30 to −0.44). Deep and surface approaches had a negative relation (−0.39). Achievement had a positive relation with deep approaches (0.33) and a negative relation with surface approaches (−0.19). These results supported hypothesis 3.

Discussion

Consistency Between the Conceptions of Learning Mathematics Identified in the Present Study and Those in the Literature of Mathematics Education

The five conceptions identified by the qualitative methodology were distinct notions with rich meanings, reflecting children's epistemological beliefs in relation to their experiences of learning mathematics as a domain in the context of mathematics teaching (Österholm, 2009). These conceptions are highly consistent with the beliefs system posited by researchers in mathematics education. Constructivist and interpretivist conceptions reflect children's beliefs about their experiences of mathematical problem solving and learning in relation to teaching contexts. Nativist, objectivist, and pragmatist conceptions reveal children's beliefs about self as learners of mathematics in the affective aspects and as responses to class and social norms in society (Op't Eynde et al., 2002; Kloosterman, 2002).

The subconceptions identified (Table 1) are also consistent with the results of research on beliefs in mathematics education. "Confidence" is a highly emphasized concept in past studies on beliefs in mathematics education since mathematics is a salient school subject in which performance is attributed to innate ability (Burton, 2004; Wheeler & Montgomery, 2009; Whitebread & Chiu, 2004). The other subconceptions, i.e., interest, understanding, liberty, innovation, goals (application), perseverance, and anxiety, are also variables widely included in studies researching the belief systems of student learning mathematics in relation to mathematics teaching (Malmivuori, 2006; Schommer-Aikins, Duell, & Hutter, 2005; Sullivan, Tobias & McDonough, 2006). In general, the result is also consistent with the notion that conceptions or epistemological beliefs of learning are domain specificity, contextual sensibility, and cultural dependence (Op't Eynde et al., 2006; Presmeg, 2002).

Consistency Between the Conceptions of Learning Mathematics Identified and Those in the Literature on General Learning and Science Education

The finding that the conceptions include both motive-related and strategy-related subconceptions is consistent with the notions of approaches to learning (e.g., Biggs, 2001) and the findings of Klatter et al.'s (2001) study for primary school children, although past studies for secondary and university students placed more emphasis on the cognitive aspect (e.g., Marton, 1981; Tsai, 2004).

The contents of the conceptions/subconceptions are generally consistent with those identified in studies on the conceptions of learning in the domains of science (e.g., Tsai, 2004, pp. 1739–1741) and on approaches to learning (e.g., Kember et al., 2004, pp. 278–279) in the aspects of constructivist conceptions (including interest and understanding), anxiety or fear of failure, and pragmatist conceptions (including vocational goal and application). The conceptions that share similarity in the meanings include (1) "seeing in a new way" in Tsai, "relating ideas" in Kember et al. vs. "intepretivist conceptions" (including liberty and innovation) in the present study; (2) "testing" in Tsai vs. "academic goal" in the present study; and (3) "commitment to work" in Kember et al. vs. "perseverance" in the present study. The conceptions that past studies indicate but the present study lacks are memorization, increase of knowledge, minimizing scope of study, and calculation; the conception that is missing in past studies is confidence. No children in the present study stated that memorization was a way to learn mathematics. Neither did any children suggest an "increase of knowledge" nor "minimizing scope of study" in the interview. The reason for this may be that mathematics is a special domain which emphasizes procedural knowledge, and there appears to be almost no "conceptual knowledge" particular in need to take effort to memorize. In addition, mathematics teaching based on constructivism has been formally introduced into Taiwanese classrooms since 1993 (Ministry of Education in Taiwan) and rote learning is discouraged by teachers and governments especially in primary education (Chiu & Whitebread, 2011). Calculation appears to be a general practice in learning mathematics. Children took "calculation" for granted partly because hand calculation without using calculators or computers is the general practice in Taiwanese mathematics classrooms. As such, "calculation" is not a distinctly meaningful conception in learning mathematics for these children.

Acceptable Reliability and Construct Validity of the CLMQ

The conceptions of learning mathematics (Table 1) identified by study 1, based on a qualitative methodology, were assessed by study 2, based on a quantitative methodology with the CLMQ (Table 2). The results supported hypotheses 1 and 2. The CLMQ has desirable internal consistency reliability (indicated by Cronbach's α) for each subscale and construct validity (indicated by the result of CFA). The medium to slightly high correlations between the five conceptions suggest that the five conceptions are distinctly differential concepts, although some are more similar to others. Constructivist and interpretivist conceptions are similar partly because both are internally oriented in addressing the issue of personal development and desires for learning mathematics. Objectivist and pragmatist conceptions are more similar because both are externally oriented, striving for some external goals, e.g., study in a good school and application to daily life. The result is consistent with the notion of approaches to learning (e.g., Biggs, 2001), with constructivist and interpretivist conceptions relating to intrinsic motivation and objectivist and pragmatist conceptions relating to extrinsic motivation.

Acceptable Criterion Validity of the CLMQ

The correlations between the five conceptions of learning mathematics in the CLMQ, mathematics achievements, and the two approaches to learning mathematics in the ALMQ (Table 3) supported hypothesis 3 (Table 4). Students' approaches to learning can serve as "products" to detect the quality of teaching, as indicated by Biggs (2001), Kember Charlesworth, Davies, McKay & Stott (1997), and Newble & Hejka

(1991). Following this logic, achievement and approaches to learning here can serve as the "products" to assess the characteristics of children's conceptions of learning mathematics, which in turn may be used to suspect features of the quality of mathematics teaching.

Mathematics Achievement. All of the five conceptions of learning mathematics identified are significantly correlated with mathematics achievements in a medium degree (correlation coefficients from 0.33 to 0.49 in Table 4). Mathematics achievement has relatively slightly higher correlations with nativist conceptions (including confidence and anxiety (reverse), 0.49) and objectivist conceptions (including academic goal and perseverance, 0.38). Past studies have consistently indicated that ability-related beliefs (e.g., confidence, self-concept, and self-efficacy) have a reliable relationship with achievement (Juter, 2005; Lokan & Greenwood, 2000; Marsh & Hau, 2004; Meyer & Koehler, 1990; Pietsch, Walker & Chapman, 2003; Pintrich & De Groot, 1990; Seegers & Boekaerts, 1996; Zimmerman, 1995). Mathematics anxiety was generally negatively related to achievement (Clute, 1984; Ho et al., 2000) and confidence (Turner, Thorpe, & Meyer, 1998; Meece, Wigfield & Eccles, 1990). The present nativist conception combined ability-related beliefs with anxiety and therefore can highly predict achievement. As for the aspect of objectivist conception, the study of Seegers, van Putten & de Brabander's (2002) shows that children's "task orientation" has positive impacts on effort investment; in addition, the perception of poor achievement will lead to low willingness to invest effort. There are also medium relationships between mathematics achievement and the constructivist, interpretivist, and pragmatist conceptions (0.33–0.34). This result is consistent with the results of past researches on beliefs in mathematics education using a quantitative methodology, which indicate that student views of useful and positive mathematics are positively correlated with mathematics achievement (Grootenboer & Hemmings, 2007; Schommer-Aikins et al., 2005). An intervention study conducted by Mason & Scrivani (2004) shows partial support to the positive impact of innovative mathematics teaching on student achievement. The results suggest that taking conceptions of learning mathematics at a positive direction is desirable in terms of achievements.

Deep Approaches. Deep approaches are highly correlated with constructivist conceptions (including interest and understanding, 0.88 in Table 4). This result shows that constructivist conceptions are similar to the original meaning of deep approaches. The deep approach has similarly high correlations with the interpretivist, objectivist, and pragmatist conceptions (0.74, 0.72, and 0.70, respectively) and a slightly lower correlation with the nativist conception (0.62). This implies their meaning in an order from closer-to to farther-from the meaning of deep approaches. Deep approaches refer to active and engaging actions during learning by searching for meaning and based on interest, which therefore strongly link to constructivist conceptions (interest and understanding). Deep approaches may partially link to liberal, innovative, useful, perseverant, and goal-oriented subconceptions, which suggest effective pedagogies to advance students' deep approaches to learning mathematics. Research has indicated that confidence and anxiety (the subconceptions of the nativist conception) are highly correlated with achievements, which may link to the belief that mathematics belongs to smart students (Wheeler & Montgomery, 2009; Whitebread & Chiu, 2004). Performance or innate ability orientations appear to be far away from the central value of deep approaches (searching for meaning and based on interest) and may slightly hinder students from taking deep approaches to learning, although the correlation is still high and significant (0.62). These four conceptions are rarely investigated with deep approaches by the researchers of approaches to learning. Future research can study this topic further.

Surface Approaches. Surface approaches to learning mathematics are negatively correlated with all the five conceptions of learning mathematics. Surface approaches are best predicted by nativist conceptions, including confidence and anxiety (reverse) in a negative direction (−0.44 in Table 4). This result was consistent with the meaning of surface approaches defined by Biggs (2001) in the motive aspect: fear of failure. Next, surface approaches show a negative relationship with constructivist conception (−0.39), which is consistent with the notion that the strategy aspect of surface approaches includes narrow target and rote learning, an opposite strategy to understanding. The relationships between surface approaches and the other three conceptions are similar (−0.32 for objectivist, −0.31 for pragmatist, and −0.30 for interpretivist conceptions).

All of the correlations between the five conceptions and both deep and surface approaches are significant and in positive and negative directions, respectively. The result suggests that the five conceptions have a meaning in a positive direction with deep approaches and a negative direction with surface approaches. In addition, the correlations between deep approaches and the five conceptions are slightly stronger (0.62–0.88) than those between surface approaches and the five conceptions (−0.30 to −0.44). The result is consistent with past research on relationship between deep and surface approaches, which is negative in a medium degree (e.g., −0.23 in Biggs et al. 2001 and −0.39 in Table 4 of the present study).

Comparison Between the Results Obtained by the Two Methodologies

The qualitative methodology has successfully identified the dominating themes of a person in a specific time, but may ignore the juxtaposition of the conceptions and their relationships with mathematics achievement and other learning approaches of a larger population. In the interview, most children focused on a single conception, some children indicated two conceptions, some children failed to state any meanings about learning mathematics, and some children could not explore in depth the meanings that they posed. They might be confined by their ability of language in addressing the issue of "meaning." Survey, a quantitative methodology, is therefore used to assess the conceptions identified in study 1 further.

On the other hand, the correlations between the five conceptions of learning mathematics are all significant (Table 4). The correlation coefficients between the five conceptions and both achievement and surface approaches are medium and those between the five conceptions and deep approaches are high. In other words, the correlations between the conception types and criteria are slightly not distinguishable within a single criterion, but they are between criteria. The reason for this may be that qualitative and quantitative methodologies appear to retrieve different patterns of responses from research participants. Major concerns are likely to be retrieved in interviews, while surveys can remind children of the different patterns of conceptions of learning that may occur in their learning mathematics. Self-reports during filling in questionnaires, however, may suffer from response sets (Winkler, Kanouse, & Ware, 1982) in which participants show single tendency (tending to say "yes" or "no") to response to all items and fail to distinguish the meanings of different conception types.

Conclusion

The children's conceptions of learning mathematics are identified by means of a qualitative methodology. The conceptions (Table 1) identified in the present study include both cognitive/strategic/epistemological and affective/motivational/emotional conceptions. Furthermore, the identified structure of children's conceptions of learning mathematics is assessed with a questionnaire titled the CLMQ. The CLMQ has acceptable reliability, construct validity, and criterion validity, which implies that the children's conceptions of learning mathematics have a reliable and construct-valid structure. In addition, the conceptions are sensible in relation to two criteria—mathematics achievements and learning approaches—which suggests that the CLMQ is eligible for a valid measure for evaluating the quality of children's learning mathematics. The quality of children's learning may also suggest features of the quality of teachers' teaching mathematics in both the aspects of pedagogies, problem types, and classroom cultures (Lester, 2002; Liu, 2010; Yackel & Rasmussen, 2002). Educators may pay attentions to how a learning environment can be managed to facilitate the constructive and interpretivist conceptions (Mason & Scrivani, 2004). Future in-depth research on teaching practice can find critical teaching factors in relation to the CLMQ.

A mixed methodology is used in the present study, which provides an in-depth look at the issue of conceptions of learning. Past research on conceptions of and approaches to learning normally focus on college or secondary students and on the domain of general learning, reading, or science in the Western culture. The present study has successfully extended the research participants to primary school children and the domain of mathematics in an Eastern culture. The structure of the conceptions of learning mathematics identified and assessed in the present study should be further examined and elaborated for its theoretical characteristics and practical use and be further extended to other research participants and other cultures.

Acknowledgments

Part of this research was supported by National Science Council, Taiwan (NSC-95-2522-S-004-001).

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