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Using an Optimization Approach to Design an Insole for Lowering Plantar Fascia Stress—A Finite Element Study

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Abstract—Plantar heel pain is a commonly encountered orthopedic problem and is most often caused by plantar fasciitis. In recent years, different shapes of insole have been used to treat plantar fasciitis. However, little research has been focused on the junction stress between the plantar fascia and the calcaneus when wearing different shapes of insole. Therefore, this study aimed to employ a finite element (FE) method to investigate the relationship between different shapes of insole and the junction stress, and accordingly design an optimal insole to lower fascia stress.

A detailed 3D foot FE model was created using ANSYS 9.0 software. The FE model calculation was compared to the Pedar device measurements to validate the FE model. After the FE model validation, this study conducted parametric analysis of six different insoles and used optimization analysis to determine the optimal insole which minimized the junction stress between plantar fascia and calcaneus.

This FE analysis found that the plantar fascia stress and peak pressure when using the optimal insole were lower by 14% and 38.9%, respectively, than those when using the flat insole. In addition, the stress variation in plantar fascia was associated with the different shapes of insole.

Keywords—Plantar fascia, Finite element method, Insole, Optimization, Biomechanics.

INTRODUCTION

Heel pain is a commonly encountered orthopedic problem which can cause weight-bearing difficulty owing to significant discomfort.[13] Plantar fasciitis is a serious disorder which can induce plantar heel pain.[7,13] Over a million people in the United States suffer from chronic plantar heel pain every year.[20] The etiology of heel pain can be divided into multiple factors including increasing age and increasing body weight. Intensive exercise, prolonged standing or walking are also major risk factors. Plantar fascia plays an important role in stabilizing the foot arch during walking. Excessive or repetitive traction of the fascia may cause microtrauma and result in plantar heel pain. Pes planus or excessive subtalar pronation will overstretch the fascia and may cause plantar fasciitis.[13]

Foot orthoses are widely used to treat foot problems. Recent studies reported that foot orthoses can reduce heel pain by relieving strain on the plantar fascia,[16,17,18] and reduce pronation of the foot and collapse of the foot arch.[15] Heel pads can redistribute foot plantar pressure,[11] as well as reduce the symptoms of plantar fasciitis.[21] Total contact insoles can lower plantar pressure and transfer pressure from the rearfoot to the midfoot region with a view to moderating plantar heel pain.[3] Therefore, many different brands of total contact insoles have been developed for the patient with plantar heel pain. However, little information is available on the relationship between different total contact insoles and plantar fascia stresses since it is difficult to measure the load sharing of plantar fascia *in vivo*. Therefore, it is worth investigating how plantar fascia stress is associated with the insole shape.

The finite element (FE) method has been used widely to analyze foot biomechanics because the FE method is able to control different parameters to model load sharing in the foot.[10] The 2D foot FE model was implemented to undergo stress analysis of standing foot following surgical plantar fascia release,[8] and estimate plantar soft tissue loading in the standing diabetic foot.[9] The 2D FE analyses addressed that plantar fasciotomy caused von Mises stress increases in the bones and plantar ligaments.[23] After the 3D foot FE model was developed, influence of different plantar fascia stiffness on stability of the ankle-foot complex,[4] and variation of plantar fascia tension under different Achilles tendon loadings[5] were further investigated.

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These studies reported that tension of plantar fascia increased with either increase of the Achilles tendon tension or vertical compressive force on foot. However, decreasing the stiffness of plantar fascia would increase the strains of the long and short plantar and spring ligaments significantly.

In addition, the FE model combined with optimization analysis is an effective simulation tool for assisting with the design of medical devices or implants, such as spinal cages,[24] thumbspica splints,[12] and spinal braces.[19] In FE simulation, using design optimization can rapidly determine the relationship between insole shape and plantar fascia stress. Thus, this study aimed to establish a 3D foot FE model and to implement an optimization tool to design an insole, as well as to investigate the relationship between insole shape and plantar fascia stress.

METHODS

FE Models

The foot geometry was obtained from computed tomography (CT) images with 1 mm slices from a 24-year-old male (height: 179 cm; mass: 79 kg) subject's foot in the unloaded neutral position. A 3D FE model consisting of plantar fascia, bones, ligaments, and skin was reconstructed using the FE software ANSYS 9.0 (Swanson Analysis System, Inc., Houston, TX). All phalanges, cartilages, bones, and skin were simulated by SOLID 45 elements. The SOLID 45 element used for the 3D modeling of solid structure is defined by eight nodes having three degrees of freedom at each node: translations in the nodal x, y, and z directions. The plantar fascia and major ligamentous structures such as the deltoid ligament, the lateral collateral ligament, the short plantar ligament, the long plantar ligament, and the spring ligament were created using tension-only LINK 10 elements. The LINK 10 element has three degrees of freedom at each node: translations in the nodal x, y, and z directions. The foot FE model comprised 34251 nodes and 38908 elements as shown in Fig. 1. All materials used in this FE model are listed in Table 1.[2,4–6,8]

Boundary and Loading Conditions

In this study, a balanced, symmetric standing condition was considered for the FE analysis. The force of extrinsic and intrinsic muscle was ignored. Since the

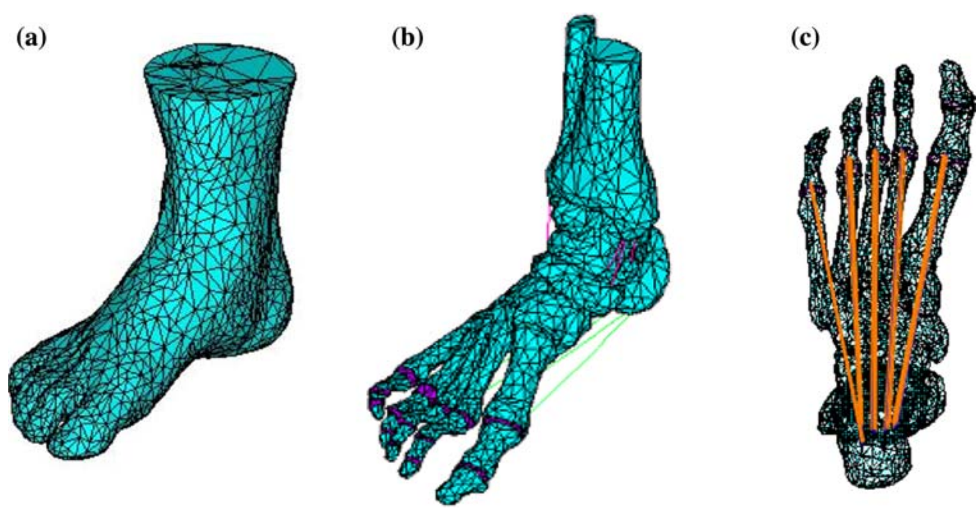

FIGURE 1. (a) Complete FE model. (b) Bony structure and cartilage. (c) Plantar fascia.

TABLE 1. Material properties and element types used in the FE model.

Component	Element type	No. of element	Cross-sectional area (mm^2)	Modulus of elasticity (MPa)	Poisson's ratio (v)
Bone	Solid 45	14,133	–	7,300	0.3
Cartilage	Solid 45	856	–	1	0.4
Soft tissue	Solid 45	23,862	–	0.15	0.45
Ligament	Link 10	42	18.4	260	–
Fascia	Link 10	15	58.6	350	–
Insole	Solid 45	17,324	–	0.4	0.2
Ground	Solid 45	2,922	–	1,000,000	0.1

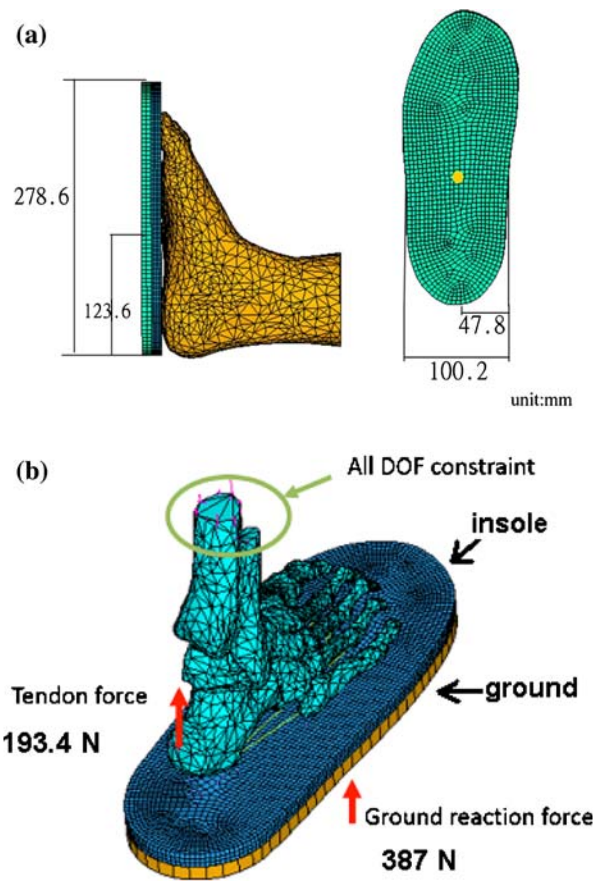

FIGURE 2. Loading and boundary condition (a) location of ground reaction force measured from the Pedar system (b) tendon force, ground reaction force, and boundary constraint.

subject weighed 774 N, half that weight, namely about 387 N, was regarded as the reaction force on a single foot. The location of the loads was determined by in-shoe plantar pressure measurement using the Pedar system (Pedar-X; Novel, Inc., Germany) during balanced, symmetrical standing. Besides the ground reaction force, a tensile force acting on the Achilles tendon was considered in this study. The force generated by the Achilles tendon is about 50% of the foot reaction force during balanced standing, based on a previous study,[22] so a vertically directed upward force of 193 N acting on the Achilles tendon was used in the FE model. In terms of the boundary conditions, the superior surfaces of the tibia, the fibula, and the soft tissue were fixed completely as shown in Fig. 2. The mechanical interaction between the foot and insole was simulated by 5480 node-to-surface contact elements, and the interface between the insole and the ground was simulated by 5792 surface-to-surface contact elements. Additionally, the ground material was simulated as a rigid horizontal plate with greater Young's modulus. Therefore, this ground material only represented a rigid surface acted to foot to mimic ground–foot contact behavior.[4]

FE Model Validation

One subject who supplied the CT images wore a flat insole with a thickness of 10 mm. This insole was one kind of thermal plastic elastomer mixed trans-polyisoprene and ethylene-vinyl-acetate, named SM-1R (Vers, Co., USA). Thus the material properties of this insole was measured at the Poisson ratio of 0.2 and the Young's modulus of 0.4 MPa based on the test procedures of ASTM E132-97 and ASTM D575-91 by using an HT-9102 computer-operated Servo Control Materials Testing System (Hung Ta Instrument Co. Ltd., Taipei, Taiwan). These material properties were used for the insole FE model. The foot pressure in a standing posture was measured by the in-shoe Pedar system and calculated by the FE analysis. In order to validate the foot FE model, the foot was divided into three regions consisting of the forefoot, the midfoot, and the rearfoot region for comparing the differences between the Pedar measurement and the FE analysis. The forefoot region consisted of the phalanges and the metatarsal bones; the midfoot region included the cuneiform, navicular, and cuboid bones; and the rearfoot region was made up of the calcaneus and the talus. The contact area, as well as the peak and mean plantar pressure from the FE analysis and the Pedar measurements, were compared.

Optimization Analysis

According to a previous study,[15,21] an insole with arch support and heel cup can reduce heel pain. Thus, the design domain covering heel cup and arch support was mainly determined from a parametric analysis of the insole as shown in Fig. 3a. The values of the 15 points on the insole were variable and were thus able to be used to determine the new insole shape. In order to simplify the design process, the conforming index (CI), as defined below, was used.

$$\mathrm{CI} = \frac{\sum_{i=1}^{15} \frac{X_i}{P_i}}{15} \times 100\%$$

where X_i is the distance from the ground to the new insole surface and P_i is the distance from the ground to the foot plantar surface.

The CI is an index representing contact status between foot and insole, e.g., the CI is 100% when representing a total contact insole, while 0% represents a flat insole.

Design Optimization

In order to achieve an optimal insole shape minimizing the tensile stress on the plantar fascia, this study implemented a design optimization approach to predict

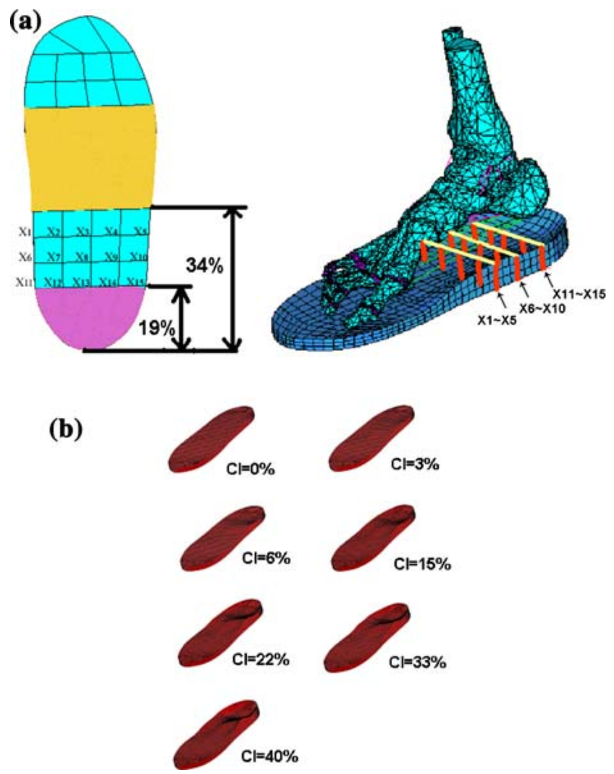

FIGURE 3. (a) A total of 15 design variables were measured from ground to foot plantar surface to define the CI value. (b) Seven insole FE models with different CI values.

the insole shape. Design optimization usually employs two types of variables to execute the design process: design variables, and the objective function.[1] In order to minimize the junction stress between the plantar fascia and the calcaneus, the von Mises stress between the plantar fascia and the calcaneus was defined as the objective function. The 15 points from X_1 to X_{15} were used to form a new insole and consequently were defined as the 15 design variables as shown in Fig. 3a.

Before running the optimization approach, it was necessary to give some initial values to perform the optimization calculation. Generally, the optimization analysis took a long time to determine the optimization values. Therefore, in order to speed up the optimization calculation, this study created seven insole FE models with CI values of 0, 3, 6, 15, 22, 33, and 40% beforehand (Fig. 3b) to undergo parametric analysis, and predicted a reasonable range of initial values, as well as investigated the relationship between insole shape and plantar fascia stress. After acquiring a group of initial values from the parametric analysis, this study performed an optimization analysis.

This study performed an optimization analysis using the subproblem approximation method.[1] This method of optimization can be described as an advanced, zero-order method. The objective function is first approximated by a fully quadratic representation with cross terms by means of least-squares fitting, and the design variables are handled in a similar manner except that only a quadratic fit is used. Then, the constrained minimization problem is converted to an unconstrained problem using penalty functions. The search for a minimum of the approximated function is carried out by applying a sequential unconstrained minimization technique at each iteration. To achieve optimization solution, the iteration of 20 times and a convergence tolerance of 0.01 were given in the program.

RESULTS

FE Model Validation

In terms of the peak pressure, the measured value from the Pedar system was greater than the calculated value from the FE model as shown in Fig. 4, especially in the rearfoot region. However, the measured mean pressure was close to the calculated mean pressure. For both the FE analysis and the Pedar measurement, the mean plantar pressure in the rearfoot region was greater than that in the midfoot and forefoot regions. Also, the measured contact area of 11,176 mm^2 was close to the 11,502 mm^2 as calculated from the FE model.

Parametric and Optimization Analyses

In the parametric analysis, the plantar fascia stress using the insole with a CI value of 40% was less by about 12% than that using the flat insole as shown in Fig. 5. The plantar fascia stress was inversely proportional to the CI value of the insole. In addition, the 15 design variables for the insole with a CI value of 40% were regarded as initial values for the optimization analysis, as listed in Table 2.

In terms of the optimal insole, the 15 design variables did not exceed the range between the upper and lower bounds as listed in Table 2. The optimal insole was determined for a CI value of 54%, as shown in Fig. 5. The plantar fascia stress using the optimal

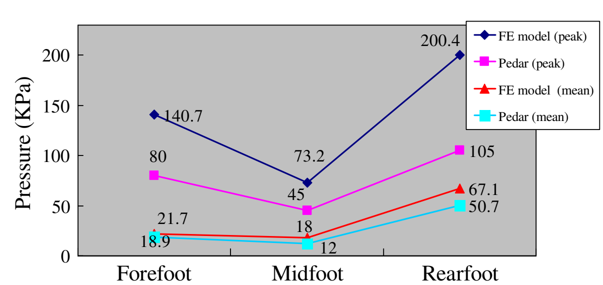

FIGURE 4. Comparison of peak and mean pressure between the Pedar measurement and FE analysis in forefoot, midfoot, and rearfoot region.

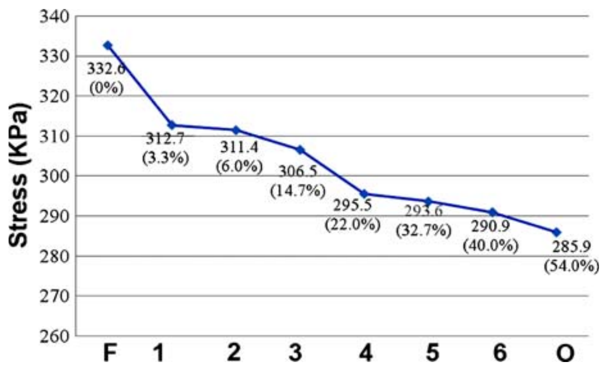

FIGURE 5. Comparison of junction stress between plantar fascia and calcaneus in the flat insole, the optimal insole, and the six insole shapes with different CI values. ***Note*****: F: flat insole, O: optimal insole, ( ): parenthesis indicates the CI value.**

TABLE 2. Fifteen design variables in the initial gauss values from the CI of 40%, the upper bond, the lower bond, and the optimal solution (Unit: mm).

	Upper bond	Initial gauss value	Optimal solution	Lower bond
X_1	25	28	28.4	31
X_2	17.8	20.8	22.3	23.8
X_3	13.3	16.3	16.8	19.3
X_4	9.1	12.1	11.7	15.1
X_5	13	16	18.4	19
X_6	16.6	19.6	21.3	22.6
X_7	11.8	14.8	17.4	17.8
X_8	9.4	12.4	12.8	15.4
X_9	9.4	12.4	11.5	15.4
X_{10}	13	16	18.3	19
X_{11}	16	19	19.8	22
X_{12}	5.6	8.6	6.8	11.6
X_{13}	5.5	8.5	8.8	11.5
X_{14}	5.5	8.5	9.6	11.5
X_{15}	14.8	17.8	19.7	20.8

TABLE 3. Comparison of the peak and mean pressure between the optimal insole and the flat insole (Unit: kPa).

	Peak pressure		Mean pressure	
	Optimal insole	Flat insole	Optimal insole	Flat insole
Forefoot	112.1	140.7	18.2	18.9
Midfoot	100.9	73.2	31.8	12.1
Rearfoot	122.4	200.4	32.2	50.7

insole was lower by about 14% than that using the flat insole.

In the rearfoot region, the peak and mean pressure when wearing the optimal insole were lower by 38.9% and 36.4%, respectively, than those when wearing the flat insole, as listed in Table 3. When compared to the flat insole, there was a significant pressure shift from the rearfoot to the midfoot region when wearing the optimal insole, as shown in Fig. 6. The peak plantar pressure in the rearfoot region was also dramatically decreased in the foot with the optimal insole.

The maximum strain of the plantar fascia decreased when wearing different insoles with different CI values as shown in Fig. 7. The maximum fascia strain of wearing the optimal insole was lower about 31.8% than that of wearing the flat insole.

DISCUSSION

A 3D foot FE model using actual geometry, including bony and soft tissue structures, was developed. This FE model was used to calculate fascia stress and to alter the parameters in order to predict the shape of the optimal insole. In the FE model validation, the contact area measured by the Pedar system was similar to that estimated by the FE model. However, the peak plantar pressure predicted by the FE model was higher than that measured by the Pedar system. The deviation may have been caused by the different resolutions between the Pedar system and the FE model. The Pedar system consisted of 99 sensors to measure the foot pressure, while the FE model used 554 nodes to calculate the foot pressure. As a result, a greater local stress was estimated by the FE analysis because of higher resolution. In order to account for the difference in resolution, the mean plantar pressure was used to compare the Pedar measurement and the FE analysis. When using this comparison, the FE analytic result was consistent with the Pedar measurement. Therefore, the foot FE model was considered accurate enough to investigate the effect of different insole shapes and was used to calculate the optimal insole.

The present FE model predicted that the fascia strain was about 0.32% under a vertical force of 350 N. The strain data of our FE calculation was consistent with that of the previous study[5] reported the strain of ranging from 0.4% to 0.5% in same loading. This strain value was within the physiological region in normal standing and much lower than the failure strain of 10%.[14] Additionally, the fascia stress in present FE calculation approximately estimated 0.33 MPa in balanced standing, but this stress value was less than the 0.63 MPa reported from the previous 2D FE study.[23] This discrepancy may be attributed to the usage of nonlinear material in cartilage in previous 2D FE analysis. In our FE simulation, material property of the cartilage was assumed linear behavior, and consequently resulted in that our foot FE model was more rigid and less stress passed to plantar fascia.

A previous study reported that a lower junction stretch between the plantar fascia and the calcaneus may ease plantar heel pain.[15] Based on this finding, this

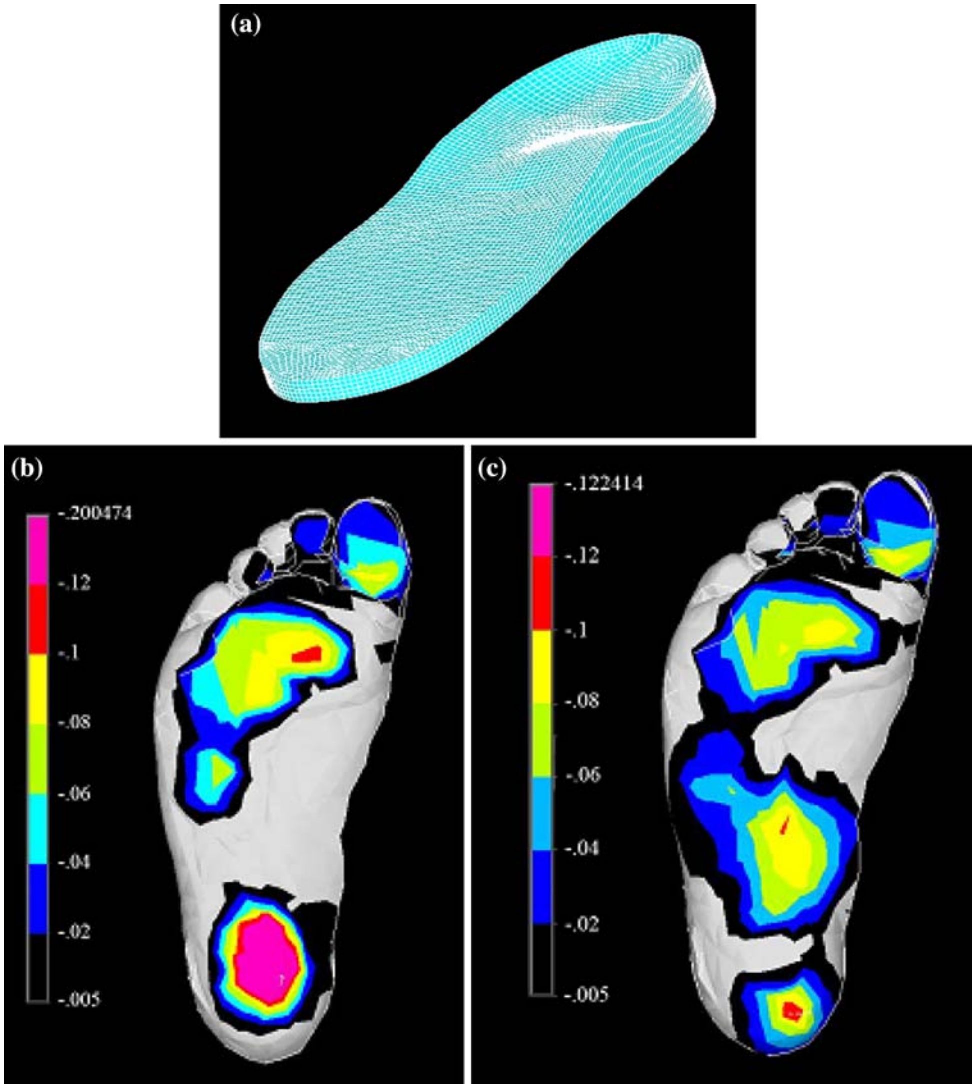

FIGURE 6. (a) Appearance of the optimal insole. (b) The plantar pressure distribution in the foot with the flat insole (MPa). (c) The plantar pressure distribution in the foot with the optimal insole (MPa).

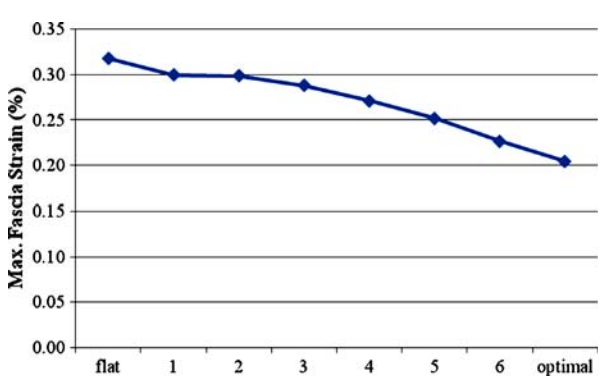

FIGURE 7. Comparison of the maximum strain of the plantar fascia among wearing different insoles during balanced standing.

study employed a numerical method to determine the optimal insole by minimizing the junction stress between the plantar fascia and the calcaneus. In this study, the optimal insole reduced the junction stress on the plantar fascia by about 14% compared to wearing the flat insole. In addition, the peak plantar pressure in the rearfoot region using the optimal insole was less by about 38.9% compared to that in the same region when using the flat insole. The analytic result from the optimal insole was consistent with that of the total contact insole[3] or custom-molded insole,[6] namely, increasing the contact area near the arch can effectively redistribute the greater plantar pressure, as well as transfer pressure to other foot regions to ease heel pain. However, a small difference was found between the optimal insole and the total contact insole, namely, that the optimal insole did not make complete contact

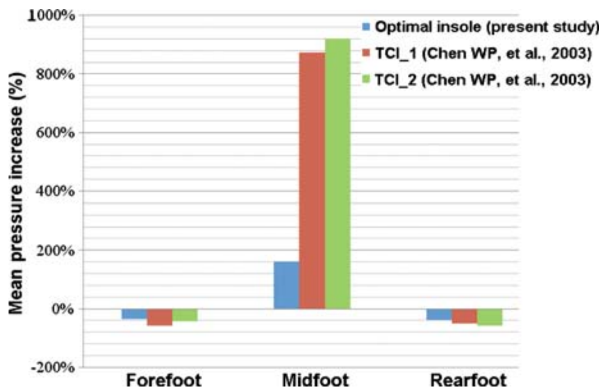

FIGURE 8. Mean pressure increase between the optimal insole and the total contact insole was respectively compared in the forefoot, the midfoot, and the rearfoot region. Mean pressure increase is defined that the optimal insole or the total contact insoles normalize to the flat insole. *Note*: TCI_1: First total contact insole; TCI_2: Second total contact insole.

with the foot plantar surface. As a result, in the midfoot region the mean pressure increase in the optimal insole was much lower than that in the total contact insole, as shown in Fig. 8. Generally, a common insole seldom has complete contact with the foot arch. Although a total contact insole can moderate heel pain, the extra volume completely touching the midfoot region sometimes induces an uncomfortable feeling near the arch during walking. Therefore, if the predicted optimal insole can ease heel pain without affecting midfoot pressure too much, this type of insole will be more easily accepted by patients than a total contact insole.

The FE model used to predict an optimal insole in this study made use of certain assumptions. All materials were idealized as homogeneous, linear, and elastic. The plantar fascia normally exhibits hyperelastic and viscoelastic characteristics, but this study simplified this nonlinear material behavior in order to converge to the optimal insole shape rapidly. The only intrinsic muscle force included was the Achilles' tendon force, but other muscle forces were ignored. This study simulated only the standing condition; the dynamic response while wearing the optimal insole was not considered.

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