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# To choose or not? That is the question of memberships

## Fuzzy statistical analysis as a new analytical approach in child language research

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**Abstract**

**Purpose** – The purpose of this paper is to propose a new analytical approach in human thought computation with an application to the child language acquisition research.

**Design/methodology/approach** – Certain fuzzy statistical concepts, such as fuzzy samples, fuzzy mode, fuzzy category test, and their relevant properties, were presented. Empirical data sets on children's language and cognitive development were discussed. Finally, the paper makes a comparison result between the traditional statistical analysis and fuzzy statistical analysis.

**Findings** – The results show that the new method is better able to capture the intricacies and complexities of the nature and processes in acquiring languages than the traditional methods.

**Originality/value** – The paper presents fuzzy statistical analysis as a new analytical approach in child language research.

**Keywords** Fuzzy logic, Statistical analysis, Children (age groups), Language

**Paper type** Research paper

### 1. Introduction

Statistics such as the mean (the average), median (the value of the middle item) and mode (data value with the greatest frequency) are useful measures in reflecting characteristics of sample distributions. These statistics can be conveniently computed from sets of data and are widely employed in many research areas. Though mostly illustrating central tendencies of the samples, each statistics has its particular functions and often serves different purposes in different applications. For example, an investigation of people's opinions on a particular issue would usually use the method of opinion polls for data collection. The analysis of such data are obviously concerned more with the mode than with the mean.

These measures in traditional statistics, in some cases, can be inadequate or even fail to capture the richness and complexities of human behaviors if the values from which they are derived are discrete and mutually exclusive categories or levels. Take opinion polls for an example, often times the choices the respondents are facing are discrete, such as "satisfactory" or "not satisfactory"; "in favor" or "oppose"; "excellent," "ok," "poor" and so forth.

These choices tend to be built on a binary or categorical logic. In the real world, as we all know, an individual's opinions or views are far from being binary or discrete.

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In most cases, people's opinions or views are complex and "fuzzy,", e.g. they may be satisfactory, but not quite so; they may feel it is "ok," but they are leaning toward "poor." The true opinion may fall somewhere on a continuum, moving or "spilling over" in between two mutually overlapping categories.

In view of the inadequacy of the traditional categories and analyses, many researchers have been experimenting a promising analytical approach based on fuzzy logic – fuzzy statistical analysis. This powerful research tool was presented first by Zadeh (1965) as "fuzzy set theory." Since then the applications of fuzzy statistical analyses are extended to traditional statistical inferences and methods in a variety of areas such as education, psychology, economics, public administration, and so forth. Lowen (1990), Ruspini (1991), Dubois and Prade (1991), and Tseng and Klein (1992), in a series of studies, have demonstrated the approximate reasoning econometric methods using fuzzy statistics. Wu and Hsu (2004) and Wu and Sun (1996, 2001) developed fuzzy time series model to overcome the bias of stock market systems which are frequently considered "unreasonable." Hwang and Wu (1995) proposed fuzzy statistical testing method to discuss the stationarity of Taiwan's short-term money demand function. Guariso *et al.* (1992) identified the model construction through qualitative simulation. Wu and Yang (1998) demonstrated the concepts of fuzzy statistics and applied it to social surveys; Wu and Tseng (2002) used fuzzy regression method of coefficient estimation to analyze Taiwan's monitoring index of economics.

This paper is a demonstration of how this new analytical method can be used in research on children's language and cognitive development; how fuzzy statistical analysis could capture better the intricacies and complexities of human cognitive functioning in language learning than the traditional analyses. Fuzzy categories and degree of membership were introduced first. The definitions of fuzzy numbers, fuzzy mode (FM), fuzzy median, and their relevant properties were then given. Empirical data on children's language and cognitive development were reanalyzed using the fuzzy analyses. The results were presented in comparison with those by the traditional analytical method to demonstrate the effectiveness and advantages of the new method.

## 2. Somewhere in between

In traditional sampling surveys, what statistical analysis is dealing with are binary and mutually exclusive categories or levels. Such categories or levels often illustrate poorly ambiguities and uncertainties inherent in the nature of human thoughts and decisions. In other words, these categories and analyses fail to capture the intriguing, complicated, and sometimes conflicting human perceptions and reasoning under investigation. For instance, a dichotomous response to a survey question on environmental pollutions as "polluted" or "not polluted" would be terribly misleading and imprecise. Such an all-or-none logic obviously cannot do justice to people's opinions of environmental pollutions. The answer, obviously, has to be represented by degrees of perception about pollutions. A different notion of categorization is thus needed for the analysis to represent the varying degrees and nuances of the spectrums of matters and issues.

### *2.1 Fuzzy categories and degree of membership*

The notion of "fuzzy categories" and "degree of membership" (Zadeh, 1965; Nguyen and Walker, 2000) has been developed in statistics to serve the demand for a new way of analysis. Classical view of categorization based on an all-or-none membership, i.e. one either belongs or does not belong to a certain category, has been seriously challenged by research in recent years in philosophy, linguistics, and psychology on the nature and structure of categories (Armstrong *et al.*, 1983; Brown, 1986; Rosch, 1973; Rosch *et al.*, 1976; Taylor, 1995). The classical or Aristotelian theory of categorization is inadequate since various members of a natural category, such as the word "games" as proposed by Wittgenstein (1953), do not all share a single set of common properties based on which to distinguish "games" from "non-games." In other words, there is no single distinctive attribute that is essential for distinguishing one game from the other. What there is, is what Brown (1986, p. 472) described as "some set of attributes, none true of all instances, but each true of some, a collection of overlapping short-range similarities."

The category are not bounded by clear-cut boundaries or dividing lines. The boundary of the category is thus "fuzzy," and the membership of a category is "by degrees." The classical theory of categorization contains mostly a binary or two degrees of membership, i.e. one either belongs or does not belong to the category; while fuzzy categories contain varying degrees of membership.

The best example of a category was extensively and systematically researched by Rosch (1973) and Rosch *et al.* (1976) in the early 1970s. Rosch (1973) asked people to rate instances of a category for "goodness of category membership" or what she called "prototypicality" on a scale. Together with her studies on color categorization, Rosch (1973, p. 31) came to the conclusion that "some reds are redder than others. The same is true for other kinds of categories,", i.e. just as some colors are the best examples of the categories, so are prototypical members in other categories in the world. Further, the degree of prototypicality of the member in the category reflects the degree of category membership. Such membership has been proven through her empirical studies to be psychologically real, i.e. people really deal with categorization that way in every day world.

In summary, recent research on categorization has greatly changed our view of the nature and structure of natural categories. Contrary to the classical theory of a binary and mutually exclusive construct of categories, the boundary of a category is fuzzy and the membership of a category is by degree of its prototypicality, which is not a set of defining features shared by all the members, but a criss-crossing network of similarities called "family resemblances." Such an understanding of categorization is employed in research methods and in a way "quantified" in fuzzy statistics.

### *2.2 Fuzzy number and degree of membership*

The quantification of fuzzy categories and degrees of membership in fuzzy statistics is represented by the "fuzzy property," which means that information in each category is by nature ambiguous, fuzzy, and tends to "spill over" to the bordering categories. Therefore, we need to put in the categories with "fuzzy numbers" instead of "real numbers." An explanation of fuzzy numbers may be formulated as below:

*Definition 2.1.* Let $U$ denote a universal set, $\{A_i\}_{i=1}^{n}$ be a subset of discussion factors on $U$, and $\Lambda(A_i)$ be a level set of $A_i$ for $i = 1,2,\ldots,n$. The fuzzy number of a statement or a term $X$ over $U$ is defined as:

$$\mu_U(X) = \sum_{i=1}^{n} \mu_i(X) I_{A_i}(X), \tag{2.1}$$

where $\{\mu_i(X), \quad 0 \leq \mu_i(X) \leq 1\}_{i=1}^{n}$ are set of membership functions for corresponding factor in $\{A_i\}_{i=1}^{n}$, and $I_{A_i}(x) = 1 \quad \text{if } x \in A_i; I_{A_i}(x) = 0 \quad \text{if } x \notin A$. If the domain of the universal set is continuous, then the fuzzy number can be written as: $\mu_U(X) = \int_{A_i \subseteq A} \mu_i(X) I_{A_i}(X)$.

*Example 2.1.* Use of fuzzy numbers in the sampling survey about favorite games.

Consider a fuzzy set of favorite games for a person as shown in Table I. Note that in the extreme cases when a degree is given 1 or 0, that is "like" or "dislike," a standard "yes" and "no" are in complement relationship, as in binary logic. Let $A_1$ represents the semantics of "favorite game," $A_2$ is "dislike the game."

Based on the analysis of binary logic, we can find that he likes the baseball and football game, dislikes basketball, golf and tennis. On the other hand, the fuzzy statistical result can be represented as:

$$\mu_{A_1}(X) = 1I_{\text{baseball}}(X) + 0.3I_{\text{basketball}}(X) + 0.8I_{\text{football}}(X) + 0.4I_{\text{golf}}(X) + 0.2I_{\text{tennis}}(X),$$

$$\mu_{A_2}(X) = 0I_{\text{baseball}}(X) + 0.7I_{\text{basketball}}(X) + 0.2I_{\text{football}}(X) + 0.6.I_{\text{golf}}(X) + 0.8I_{\text{tennis}}(\text{X}).$$

Which means that the person likes the baseball game with 100 percent of degree, likes the basketball game with 30 percent of degree and likes the football with 80 percent of degree? While, he dislikes the basketball game with 70 percent of degree, dislikes the football game with 20 percent of degree, dislikes the golf game with 60 percent of degree, and dislikes the tennis game with 80 percent of degree.

Therefore, based on the binary (like or dislike) logic, we can only see the superficial feeling about people's favorite game (the rightest columns of Table I under "binary logic"). While via the information of fuzzy response we will see a more detailed data representation.

## 3. Fuzzy statistics and related properties

### *3.1 Fuzzy mode*

Fuzzy statistical analysis requires a different practice in data collection. It is recommended that more flexibility in levels of codings be provided to the respondents during data collection, if such a practice is feasible. If not, the uncertainty and ambiguity embedded in the data can be dealt with in the analysis using the "fuzzy mode." In the following section, we will demonstrate how they differ from the traditional statistics.

*Definition 3.1.* FM (data with multiple values).

Let $U$ be the universal set (a discussion domain), $L = \{L_1, L_2, \ldots, L_k\}$ be a set of $k$-linguistic variables on $U$, and $\{\text{FS}_i, \quad i = 1, 2, \ldots, n\}$ be a sequence of random fuzzy sample on $U$. For each sample $\text{FS}_i$, assign a linguistic variable $L_j$ a normalized

**Table I.** Comparing fuzzy numbers with real (integral) numbers for favorite games

| Favorite game | $A_1$ | $A_2$ | $A_1$ | $A_3$ |
|---|---|---|---|---|
| Degree of feelings | $\mu_{A_1}(X)$ | $\mu_{A_2}(X)$ | Binary logic | |
| Baseball | 1 | 0 | √ | |
| Basketball | 0.3 | 0.7 | | √ |
| Football | 0.8 | 0.2 | √ | |
| Golf | 0,4 | 0.6 | | √ |
| Tennis | 0.2 | 0.8 | | √ |

membership $m_{ij}\left(\sum_{j=1}^{k} m_{ij}=1\right)$, let $S_j=\sum_{i=1}^{n} m_{ij}$, j $=1,2,\ldots,k$. Then, the maximum value of $S_j$ (with respect to $L_j$) is called the FM of this sample. That is FM $=\{L_j|S_j=\max_{1\le i\le k} S_i\}$.

*Note.* A significant level $\alpha$ for FM can be defined as follows: let $U$ be the universe set (a discussion domain), $L=\{L_1,L_2,\ldots,L_k\}$ be a set of $k$-linguistic variables on $U$, and $\{\mathrm{FS}_i,\quad i=1,2,\ldots,n\}$ be a sequence of random fuzzy sample on $U$. For each sample, $\mathrm{FS}_i$ assign a linguistic variable $L_j$ a normalized membership $m_{ij}\left(\sum_{j=1}^{k} m_{ij}=1\right)$, let $S_j=\sum_{i=1}^{n} I_{ij}, j=1,2,\ldots,k, I_{ij}=1$ if $m_{ij}\ge\alpha, I_{ij}=0$ if $m_{ij}<\alpha$, $\alpha$ is the significant level. Then, the maximum value of $S_j$ (with respect to $L_j$) is called the FM of this sample. That is FM $=\{L_j|S_j=\max_{1\le i\le k} S_i\}$. If there are more than two sets of $L_j$ that reach the conditions, we call that the fuzzy sample has multiple common agreement.

*Definition 3.2.* FM (data with interval values).

Let $U$ be the universe set (a discussion domain), $L=\{L_1,L_2,\ldots,L_k\}$ be a set of $k$-linguistic variables on $U$, and $\{\mathrm{FS}_i=[a_i,b_i],\ a_i,b_i\in R,\ i=1,2,\ldots,n\}$ be a sequence of random fuzzy sample on $U$. For each sample $\mathrm{FS}_i$, if there is an interval $[c,d]$ which is covered by certain samples, we denote these samples as a clustering. Let MS is the set of clustering which contains the maximum number of samples, then the FM is defined as:

$$\mathrm{FM}=[a,b]=\{\cap[a_i,b_i]|[a_i,b_i]\subset\mathrm{MS}\}.$$

If $[a,b]$ does not exist (i.e. $[a,b]$ is an empty set), we say this fuzzy sample does not have FM.

*Example 3.1.* Suppose eight voters are asked to vote for a chairperson from four candidates. Table II is the result from two different types of voting: traditional versus fuzzy.

From the traditional voting, three vote for B. Hence, the mode of the votes is B. However, from the fuzzy voting, B only gets a total membership of 2.1, while C gets 3.4. Based on the traditional voting, B is voted for the chairperson; while based on the fuzzy voting or membership voting, C is the chairperson. The voters' preference is reflected more accurately in the fuzzy voting than is the traditional voting. In other words, C deserves to be the chairperson more than B does based on the fuzzy voting.

| | Candidate | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | Traditional voting | | | | Fuzzy voting | | | |
| Voter no. | A | B | C | D | A | B | C | D |
| 1 | | √ | | | | 0.7 | 0.3 | |
| 2 | √ | | | | 0.5 | | 0.4 | 0.1 |
| 3 | | | | √ | | | 0.3 | 0.7 |
| 4 | | | √ | | 0.4 | | 0.6 | |
| 5 | | √ | | | | 0.6 | 0.4 | |
| 6 | | | | √ | | | 0.4 | 0.6 |
| 7 | | √ | | | | 0.8 | 0.2 | |
| 8 | | | √ | | | | 0.8 | 0.2 |
| Total | 1 | 3 | 2 | 2 | 0.9 | 2.1 | 3.4 | 1.6 |

**Table II.** A voting comparison of eight voters

*3.2 Heuristic statistical properties related to the FM*

Though features of a population can be illustrated by certain statistical parameters, there we still many characteristics which are left out, such as expectation, medium, and mode. Especially, when we are going to investigate the public opinions in the social science research, traditional parameters do not seem to be enough for application. In order to depict the whole picture more carefully, we implement the FM.

Let $U$ be the universe set, $L = \{L_1, L_2, \ldots, L_k\}$ be a set of $k$-linguistic variables on $U$, and $\{FS_i, \quad i = 1, 2, \ldots, n\}$ be a sequence of random fuzzy sample on $U$ (data with discrete fuzzy number). The following properties aim to illustrate the useful applications for the presented definition of FM and discuss some valuable properties. We will also compare these two types of modes with the traditional ones.

*Property 3.1.* If the maximum membership $m_{ij}$ in each $FS_i$ is larger than the significant level $\alpha$ and located at $L_i$, then the FM is consistent with the traditional mode.

*Property 3.2.* If there exists a membership $m_{ij}$ in each $FS_i$ with values $m_{ij} > 0.5$, then FM consists with the traditional mode for any significant level $\alpha \geq 0.5$.

*Property 3.3.* If there are samples whose maximum membership falls on two or more of $L = \{L_1, L_2, \ldots, L_k\}$ values. Then, we cannot compute the traditional mode without discarding these samples. However, by choosing an appropriate significant level $\alpha$, we can compute it by:

$$S_j = \sum_{i=1}^{n} I_{ij} \text{ and } \mathrm{FM} = \{L_j | S_j = \max_{1 \leq i \leq k} S_i\} \text{ to get the fuzzy mode.}$$

*Property 3.4.* We can adjust the values of $S_j = \sum_{i=1}^{n} I_{ij}$ and then change the location of FM by choosing the different significant level $\alpha$.

*Note.* If the significant level $\alpha$ is chosen to be too large, the value of FM will be low. If we lower the significant level $\alpha$, the value of FM will increase. So, the degree of significant level $\alpha$ is an important point that will influence the state of FM. The prior experience can help us to choose an appropriate significant level $\alpha$ according to the human thought or social utility.

*3.3 A $\chi^2$-test for fuzzy categorical data*

Consider a $K$-cell multimomial vector $n = \{n_1, n_2, \ldots n_k\}$ with $\sum_i n_i = n$. The Pearson $\chi^2$ test ($\chi^2 = \sum_i \sum_j (n_{ij} - e_{ij})/e_{ij}$) is a well-known statistical test for investigating the significance of the differences between observed data arranged in $K$ classes and the theoretically expected frequencies in the $K$ classes. It is clear that the large discrepancies between the observed data and expected cell counts will result in larger values of $\chi^2$.

However, a somewhat ambiguous question is whether (quantitative) discrete data can be considered categorical. For example, if a child is asked how you love your sister? If the sample of responses is a fuzzy number, the traditional $\chi^2$-test that are useful for pure categorical data are certainly inappropriate. Despite this, there are still advantages to recognizing the multi-value nature of the sample, since such analyses can be more effective and addressing the specific characteristics of such fuzzy sample. Thus, we will present a $\chi^2$-test for fuzzy categorical data as follows.

Procedure for $\chi^2$-test with fuzzy categorical data:

- *Sample*. Let $U$ be the universal set (a discussion domain), $L = \{L_1, L_2, \ldots, L_k\}$ be a set of $k$-linguistic variables on $U$, $\{A_1, A_2, \ldots, A_m\}$ and $\{B_1, B_2, \ldots, B_n\}$ be two set come from categorical populations with numbers on $U$. For each sample in $\{A_i, B_i\}$, assign a linguistic variable $L_j$ a normalized membership $m_{ij}\left(\sum_{j=1}^{k} m_{ij} = 1\right)$, $Fn_{ij} = \sum_{i \in A,B} Ln_{ij} \quad i \in A, B;\ j = 1, 2, \ldots, k$, be the total memberships in the cell $ij$.
- *Hypothesis*. Two populations have the same distribution ratio.
- *Statistics*. $\chi^2 = \sum_{i \in A,B} \sum_{j=1}^{c} ([Fn_{ij}] - e_{ij})^2 / e_{ij}$. (In order to perform the $\chi^2$ test for fuzzy data, we transfer the decimal fractions of $Fn_{ij}$ in each cell of fuzzy category into the integer $[Fn_{ij}]$ by counting 5 and higher fractions as 1 and discard the rest.)
- *Decision rule*. Under significance level $\alpha$, if $\chi^2 > \chi^2_\alpha(k-1)$, then we reject $H_0$.

## 4. Empirical studies on children's acquisition of conditionals and theory of mind

In this section, partial data from two empirical studies by Wu (2000, 2004) are reanalyzed using fuzzy statistical analysis. The results are compared side-by-side to the ones by the original (traditional) analysis to demonstrate the differences.

### *4.1 A study on children's acquisition of conditional reasoning and expressions across languages*

In Wu's (2004) study on children's acquisition of conditional reasoning and expressions across languages, she studied 56 Mandarin-speaking Taiwanese children with a mean age of 48 months or four-years old and 22 English-speaking US children with a mean age of 48.9 months old. The study aims to answer the following two research questions:

*RQ1*. How do Mandarin- and English-speaking children's conditional reasoning and expressions develop over time?

*RQ2*. Is language difference, such as English versus Chinese, related to children's understanding of conditionals?

Among the testing stimuli, six conditional questions with different degrees of hypotheticality were asked based on a picture book to the two groups of children in their native language, respectively. The questions are listed in Table III.

*4.1.1 Measurement and coding*. Children's responses to the conditional questions were coded as "conditional," "indeterminate" or "non-conditional." The conditional nature of a response was judged by whether or not a child accepted the premise in the if-clause (antecedent) and answered the question in the consequent clause accordingly. For instance, if a child responded to the question: "Which animal would you like to be if you were the piglet?" with "I want to be a giraffe," it was an indication that the child was able to engage in a conditional mode of thinking to derive the conclusion. An answer like "No, I'm not the piglet" was considered as a refusal of the stated premise (If you were the piglet), and were thus scored as "non-conditional." If a child reacted to the question with silence or some indeterminate answers such as "I don't know," "The piglet was an animal," his or her answer would be coded as "indeterminate."

| Conditional questions | Examples |
|---|---|
| 1. Future open conditionals | If you ask your Mom whether she loves you, what will she say? |
| 2. Present open conditionals | If somebody bites you, does it hurt? |
| 3. Past open conditionals | There are lions in the zoo. If I have been to the zoo, would I see the lions? |
| 4. Imaginative conditionals | Which animal would you like to be if you were the piglet? Why would you want to be a ___? |
| 5. Present counterfactuals | The mother pig is afraid after the piglet becomes a lion because the lion might bite her with its sharp teeth. What if the lion didn't have sharp teeth? (The experimenter covered up the teeth in the book with her fingers.) |
| 6. Past counterfactuals | The piglet was a lion before. But he changed back to be a piglet again at the end. What if the piglet had not changed back to himself, what would the mommy pig have done then? |

**Table III.**
Types of conditional questions and examples

Indeterminate answers did not clearly reflect a conditional understanding of the question.

*4.1.2 Analyses and results*. The dependent variables of this study are the children's responses to the conditional questions. The major independent variables include:

- language group (Chinese versus English);
- age (three-, four-, and five-year olds); and
- categories of conditionals including degree of hypotheticality (open, imaginative and counterfactual) and temporal references (future, present and past).

Both traditional and fussy statistical analyses using $\chi^2$ test were conducted, the significance level was set at 0.1. The results are contained in the following Table IV.

From Table IV, we find that the null hypotheses testing for questions 1, 4 and 5 are invalid in the traditional analysis. While in the fuzzy category analysis, only question 3 is invalid for calculating a $\chi^2$ test. Statistically, since fuzzy analysis makes a finer differentiation of the data value, it tends to be more robust and reliable in hypothesis testing than the traditional ones. Both analyses generated the same results on questions 2 and 3, but not on question 6. In the traditional analysis, the Chinese-speaking children gave significantly fewer conditional responses to this question than the English-speaking children did. However, in the fuzzy analysis, there was no difference between these two groups. This does not mean that the fuzzy analysis is less effective in rejecting the null hypothesis. On the contrary, it indicates precisely how traditional analysis using a binary logic tends to over-simplify the behavior under investigation with discrete levels. Statistically, more crude categories are more likely to reject a null hypothesis, but the result may be less reliable. The fuzzy analysis, on the other hand, uses more differentiated categories and tends to reflect a more truthful picture of the data.

When we combined the six questions together and compared the traditional versus the fuzzy analysis as a whole across the two language groups, the above pattern of difference between the two analyses was found again. The traditional analysis rejects the null hypothesis, i.e. there is a significant difference in conditional responses from the Chinese- and English-speaking children; the fuzzy analysis fails to reject the hypothesis, i.e. there is no difference between these two groups.

**Table IV.** A comparison of the traditional and fuzzy statistical analyses on Mandarin- and English-speaking children's responses to six conditional questions

| *H0*. Language difference does not affect children's understanding of conditionals | | Traditional category analysis: Conditional response | Indeterminate response | Non-conditional response | $\chi^2$ test | Fuzzy category analysis: Conditional response | Indeterminate response | Non-conditional response | $\chi^2$ test[a] |
|---|---|---|---|---|---|---|---|---|---|
| 1. Future open conditionals | Chinese | 31 | 25 | 0 | $\chi^2 = 16.18$[b] | 29.8 | 21.2 | 5 | $\chi^2 = 3.71$, $p = 0.16$, accept |
| | English | 21 | 0 | 1 | | 16.8 | 4.4 | 0.8 | |
| 2. Present open conditionals | Chinese | 19 | 28 | 9 | $\chi^2 = 12.2$, $p = 0.002$, reject | 20.6 | 22.4 | 12.6 | $\chi^2 = 4.68$, $p = 0.096$, reject |
| | English | 17 | 3 | 2 | | 14.2 | 5.6 | 2.2 | |
| 3. Past open conditionals | Chinese | 21 | 30 | 5 | $\chi^2 = 1.99$, $p = 0.37$, accept | 22.8 | 23.2 | 10 | $\chi^2 = 0.55$, $p = 0.76$, accept |
| | English | 12 | 9 | 1 | | 11.4 | 8 | 2.6 | |
| 4. Imaginative conditionals | Chinese | 52 | 3 | 1 | $\chi^2 = 0.43$[b] | 42.2 | 12.4 | 1.4 | $\chi^2 = 0.41$[b] |
| | English | 21 | 1 | 0 | | 17 | 4.8 | 0.2 | |
| 5. Present counterfactuals | Chinese | 30 | 5 | 1 | $\chi^2 = 0.35$[b] | 25 | 9.2 | 1.8 | $\chi^2 = 0.34$, $p = 0.84$, accept |
| | English | 17 | 4 | 1 | | 14.4 | 6 | 1.6 | |
| 6. Past counterfactuals | Chinese | 25 | 28 | 3 | $\chi^2 = 16.18$, $p = 0$, reject | 25.6 | 22.4 | 8 | $\chi^2 = 2.76$, $p = 0.25$, accept |
| | English | 13 | 2 | 7 | | 10.8 | 5.2 | 6 | |
| Total | Chinese | 178 | 119 | 19 | $\chi^2 = 23$, $p = 0$, reject | 166.2 | 110.8 | 39 | $\chi^2 = 5.82$, $p = 0.054$, reject |
| | English | 101 | 19 | 12 | | 84.6 | 33 | 13.4 | |

**Notes:** [a]In order to perform the $\chi^2$ test for fuzzy data, we transfer the decimal fractions in each cell of fuzzy category into the integer by counting 5 and higher fractions as 1 and discard the rest; [b]More than two cells numbers are $\leq 1$, $\chi^2$ test is invalid

*4.2 A study on children's acquisition of false belief understanding*

Wu's (2000) study on children's acquisition of theory of mind (TOM) investigates how children start to understand their own and other people's internal states such as feelings, beliefs, knowledge, and so forth. The term "children's theory of mind" is worth some clarification. It does not mean that the child is really developing a theory in its scientific sense, but rather a way of thinking and talking about self and others relating to mental states.

A key measure of children's TOM development is their ability to understand that another person may hold a false belief, i.e. others' beliefs or ideas may not match reality or facts as perceived by the children. Most studies (Goetz, 1999; Gopnik and Astington, 1988) on children's false belief understanding typically follow the false belief protocol of Wimmer and Perner (1983). Usually, presented in a story, false belief tasks ask children to make a prediction as to how one of the characters will behave at the end of the story and the children's judgment will reveal their ability to engage in mental perspective taking, i.e. to think about what goes on in another person's mind. There are two prototypical false-belief tasks that are generally considered as the "definitive" tests of false belief understanding:

(1) unexpected transfer; and

(2) unexpected content.

This paper will focus on the first task only.

*4.2.1 Unexpected transfer task*. In unexpected transfer or change of location tasks, a character (a toy or a doll) puts an object in one of two locations. While the character is away the item is removed from the original location. The child tested is asked where the character will look when he/she comes back. Most of these types of tasks follow the prototypical change of location task in Wimmer and Perner's (1983) study in which a character named Maxi puts his chocolate in the drawer and his mother moves it to the cupboard when Maxi is out. The child tested is asked: "Where will Maxi look for his chocolate, in the drawer or in the cupboard?"

An appropriate or correct response to the unexpected transfer task depends on the child's understanding of what Maxi will be thinking. By answering that Maxi will look in the original location (drawer), the child demonstrates that he or she realizes Maxi's belief about the location of the chocolate is different from his or her own. Correct answers to these two false belief tasks will suggest whether the child has acquired TOM or not. The proper prediction reveals that this child has an understanding that other people's actions are related to their internal states and that their internal states can be different from one's own. This understanding is related to two abilities in successful second-order thinking:

(1) the ability to take a different perspective; and

(2) the ability to make an accurate assessment of others' internal states.

*4.2.2 Participants and materials.*

4.2.2.1 Participants. In total, 68 Chinese-speaking children from three kindergartens and preschools in Taipei and Taoyuan, Taiwan were interviewed and tested. The children's ages ranged from three-and-half years (three years and four months) to five-and-half years old, with a mean age of four-and-half years old (four years and five months). They were divided into two age groups for analysis and comparison: 37

younger (three-and-half to four-and-half years old) and 31 older (four-and-half to five-and-half years old) children.

4.2.2.2 Procedures. Each child was interviewed individually for a 15- to 20-minute session. A story was acted out in front of the child using toy animals. Typically, the story was told in Chinese as below:

> Exper: (Dino and the reindeer were playing with the ball, and they were having great fun. The mother dinosaur came and told Dino: "Dino, it's time for school. Let's go." Dino put away his ball in his toy box and went with his mother. While they were out, they did not know what was going on at home, right? At home, the reindeer took the ball from Dino's toy box and played by himself. Oh no! He kicked the ball into the ditch and the ditch was too deep for him to reach the ball. Afraid that Dino would be mad, the reindeer went to hide. Later, when Dino came back from school, he wanted to play with his ball.)
>
> *Test question 1:*
>
> *Exper:* Now, listen to me carefully. Where will Dino look for his ball?
>
> *Child:* The ditch/toy box.
>
> *Test question 2:*
>
> *Exper:* Why would Dino go to the ditch/toy box to look for it?

4.2.2.3 Coring of false belief task. Different from the dichotomous scoring (0 – incorrect versus 1 – correct) of false belief understanding tasks in the previous studies (Astington, 1993; Astington *et al.*, 1988; Goetz, 1999; Gopnik and Astington, 1988; Wimmer and Perner, 1983), a more elaborate four-point scoring scheme (from 0 to 4) was developed with a view to fully reflecting the variability of children's performance patterns in the data. A child would get two points for correctly predicting the location where the dinosaur would look for his ball, namely, in the toy box, by either pointing to it or telling it verbally, or both pointing and telling.

The child would get another two points for giving a reasonable explanation for the dinosaur's action, for example: "He looked there because he put it there in the morning when he left/because he didn't know the ball was kicked into the ditch by the reindeer." Thus, four points were given for correct prediction and appropriate explanation or justification. If the child made the correct prediction but did not give an appropriate explanation (e.g. by replying: "don't know" or being silent), he/she would get two points only. Two points were also given if a child gave a good explanation but made the wrong prediction, but it was considered unlikely, and would be treated as an exception.

A score of 1 or 3 was given for partially correct answers. A child would get one point for providing a plausible, if not reasonable, explanation such as: "Because he went to the toy box first to take a look." Children would get three points if they made the correct prediction but provided only a partial or incomplete explanation such as: "Because he just knew."

4.2.2.4 Levels of general language ability. A number of studies have suggested that language ability is closely tied to performance of false-belief tasks (Goetz, 1999; Jenkins and Astington, 1996; Shatz *et al.*, 1995; Welch-Ross, 1997). Jenkins and Astington (1996) have found that general language ability and verbal memory are significant predictors of false belief understanding.

The level of linguistic maturity or proficiency of the children who participated in this study was also evaluated. It was not measured in any standardized language tests owing to a lack of appropriate language ability tests in Chinese. Information about the children's linguistic performance was requested from the teachers at the preschools. The teachers were asked to evaluate children's general language ability using a five-level (1-5) scale, level 5 representing superior language ability and level 1 standing for below average language development. The children's general language ability distribution is contained in the Table V.

*4.2.3 Analyses and results*. The dependent variable in this study is the children's performance of the false belief task, i.e. unexpected transfer task (0-4 points). The independent variable is children's age group (two levels, younger versus older) and their general language ability (five levels).

4.2.3.1 Conversion of the numbers. The five-level scores either for the false-belief task (0-4) or for the general language ability (1-5) are "new level." For fuzzy statistical analyses, these levels need to be converted to fuzzy numbers, which are based on the concept of the "spill-over" degrees of membership. For instance, the level 1 is converted into 0.6 for new level 2, and the remaining 0.4 is divided evenly with new level 1 (0.2) and with new level 3 (0.2). As for level 0 or 4, their fuzzy numbers are divided only with one bordering level. Thus, level 0 is converted to 0.8 to new level 1 and the remaining 0.2 goes to new level 2. Table VI illustrates such a division scheme and how fuzzy numbers are derived from.

Mathematically, the above fuzzification of the real numbers into fuzzy numbers is represented as below:

$$0 \to \frac{0.2}{1} + \frac{0.8}{2},\ 1 \to \frac{0.2}{1} + \frac{0.6}{2} + \frac{0.2}{3},\ 2 \to \frac{0.2}{2} + \frac{0.6}{3} + \frac{0.2}{4},\ 3 \to \frac{0.2}{3} + \frac{0.6}{4} + \frac{0.2}{5},$$
$$4 \to \frac{0.2}{4} + \frac{0.8}{5}.$$

**Table V.** The children's general language ability distribution

| Level | 1 | 2 | 3 | 4 | 5 | Total |
|---|---|---|---|---|---|---|
| Younger children (three-and-half to four-and-half years old) | 1 (3) | 4 (11) | 12 (32) | *15 (41)* | 5 (13) | 37 |
| Older children (four-and-half to five-and-half years old) | 1 (3) | 0 (0) | *11 (36)* | 10 (32) | 9 (29) | 31 |
| Total | 2 (3) | 4 (6) | 23 (34) | *25 (37)* | 14 (21) | 68 |

**Note:** The figures in parentheses are in percentages

**Table VI.** Fuzzification of five-level real numbers into fuzzy numbers

| | Real numbers | | | | |
|---|---|---|---|---|---|
| Level | 1 | 2 | 3 | 4 | 5 |
| 0 | 0.8 | 0.2 | | | |
| 1 | 0.2 | 0.6 | 0.2 | | |
| 2 | | 0.2 | 0.6 | 0.2 | |
| 3 | | | 0.2 | 0.6 | 0.2 |
| 4 | | | | 0.2 | 0.8 |

Second, the five-level test scores on the unexpected transfer task were later collapsed into two levels ("0" from the original 0 and 1, and "1" from the original 2, 3, and 4). The scoring was reverted back to a parsimonious scheme mainly for the purpose of making comparisons with the results of other studies in the literature.

4.2.3.2 Comparison of the traditional and fuzzy statistical analysis. After the conversion of the scores, both traditional and fussy statistical analyses were conducted. We first made a comparison of the children's general language ability in real numbers and in fuzzy numbers, focusing on their modes, i.e. the language ability level most children received from the teachers. Table VII is the general language ability levels in fuzzy numbers; Table VIII shows how the mode in traditional analysis differs from that in fuzzy analysis.

As indicated by Table VIII, across groups most children received a level 4 for their general language ability both in the traditional and fuzzy analysis. However, in the traditional analysis, younger children were judged to have a higher level of proficiency (mode = 4) than that of the older children (mode = 3). The FM, however, shows a different pattern, i.e. most older children's language proficiency (mode = 4) is the same as that of the younger children's (mode = 4), a result more reasonable and plausible giving the usual positive correlation between children's age and their language proficiency.

Based on the two-level or binary analysis, there are significantly for older children passed the task than the younger ones (older 58 percent, $n = 31$, versus younger 32 percent, $n = 37$, $\chi^2 = 4.49$, df $= 1$, $p < 0.03$). This pattern indicates that there is a higher percentage of older children showing a fully developed understanding of another's false belief and the ability to explain it verbally than that of younger children who have not acquired both abilities. Table IX illustrates that the five-level coding has a better explanatory power than that of the two level dichotomies; and the fuzzy evaluation has the best explanatory efficacy.

| Level | 1 | 2 | 3 | 4 | 5 | Total |
|---|---|---|---|---|---|---|
| Younger children (three-and-half to four-and-half years old) | 1.6 (4) | 5 (14) | 11 (30) | *12.4 (34)* | 7 (19) | 37 |
| Older children (four-and-half to five-and-half years old) | 0.8 (3) | 2.4 (8) | 8.6 (25) | *10 (32)* | 9.2 (30) | 31 |
| Total | 2.4 (3) | 7.4 (9) | 19.6 (29) | *22.4 (35)* | 16.0 (24) | 68 |

**Note:** The figures in parentheses are in percentages

**Table VII.** The children's general language ability distribution in fuzzy numbers

| Level | Mode | FM |
|---|---|---|
| Younger children (three-and-half to four-and-half years old) | 4 | 4 |
| Older children (four-and-half to five-and-half years old) | 3 | 4 |
| Total | 4 | 4 |

**Table VIII.** A mode comparison of children's general language ability levels

**Table IX.**
Children's performance on the unexpected transfer task

| | Two-level | | Five-level | | | | | Five-level fuzzy numbers | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Rank | 0 | 1 | 0 | 1 | 2 | 3 | 4 | 0 | 1 | 2 | 3 | 4 |
| Younger children (three-and-half to four-and-half years old) | 25 | 12 | 18 | 7 | 1 | 4 | 7 | 15.8 | 8 | 2.8 | 4 | 6.4 |
| | *Mode = 0* | | | | *Mode = 0* | | | | | *FM = 0* | | |
| Older children (four-and-half to five-and-half years old) | 13 | 18 | 6 | 7 | 4 | 3 | 11 | 6.2 | 6.2 | 4.4 | 4.8 | 9.4 |
| | *Mode = 1* | | | | *Mode = 4* | | | | | *FM = 4* | | |
| Total | 38 | 30 | 24 | 14 | 5 | 7 | 18 | 22 | 14.2 | 7.2 | 8.8 | 15.8 |
| | *Mode = 0* | | | | *Mode = 0* | | | | | *FM = 0* | | |

## 5. Discussions and conclusion

This paper is an attempt to demonstrate the difference between categories based on binary logic (e.g. traditional multiple choices survey questions), and a more complicated yet more precise fuzzy membership function assessment. Relevant fuzzy statistical analysis such as FM, fuzzy $\chi^2$-test and fuzzy weight were proposed. We have shown that how these measures can be properly and easily applied in research to reveal the rich and complex nature of learning such as those in child language and cognitive developmental research. Through the analyses and comparisons, human cognitive functioning and its development may be perceived as natural and continuous flows instead of discrete data points.

Fuzzy statistical analysis has grown to be a new analytical approach in response to the traditional analysis which may represent inadequately human behaviors under investigation. Our analyses and comparisons in this paper show that there are several advantages of fuzzy analysis:

- evaluation process is robust and consistent with a decreased degree of subjectivity in the part of the evaluator;
- self-potentiality is highlighted by indicating individual distinctions;
- respondents are given more flexible and stimulating choices for issues investigated;
- graded semantic meanings such as more or less, not so many, and so forth can be better reflected;
- FM tends to reveal the true consensus or agreement than the traditional mode; and
- the results from fuzzy analysis are more reliable and robust than those of the traditional analysis.

However, fuzzy statistical analyses are not without problems. Several issues need to be addressed for future analyses. First, we can further the research on data simulation so that we may understand features of the fuzzy linguistic, multi-facet assessment, and the balance of the moving consensus. Moreover, the choice of different significant $\alpha$-cut will influence the statistical results. An appropriate criterion for selecting a significant $\alpha$-cut should be investigated in order to reach the best common agreement.

Second, there are other types of membership functions we could explore in future research. For the FM of continuous type, we can extend the uniform and triangular types of membership functions to non-symmetric or multiple peaks types. A third issue is subtle and complex. It involves the relationship between the ambiguity in human thoughts and in human behaviors. How truthful can the fuzzy analysis reflect the cognitive functioning in human mind? In other words, how fuzzy is fuzzy enough to truly capture what is going on in human mind?

## References

Armstrong, S.L., Gleitman, L.R. and Gleitman, H. (1983), “What some concepts might not be”, *Cognition*, Vol. 13, pp. 263-308.

Astington, J.W. (1993), *The Child’s Discovery of the Mind*, Harvard University Press, Cambridge, MA.

Astington, J.W., Harris, P.L. and Olson, D.R. (1988), *Developing Theories of Mind*, Cambridge University Press, New York, NY.

Brown, R. (1986), "Language and thought", in Brown, R. (Ed.), *Social Psychology*, 2nd ed., The Free Press, New York, NY, pp. 467-94.

Dubois, D. and Prade, H. (1991), "Fuzzy sets in approximate reasoning, Part 1: Inference with possibility distributions", *Fuzzy Sets and Systems*, Vol. 40, pp. 143-202.

Goetz, P.J. (1999), "A comparison of 3- and 4-year-old English monolinguals, Mandarin monolinguals, and Mandarin-English bilinguals on a series of theory of mind-related tasks", unpublished doctoral dissertation, University of Michigan, Ann Arbor, MI.

Gopnik, A. and Astington, J.W. (1988), "Children's understanding of representational change and its relation to understanding of false belief and the appearance-reality distinction", *Child Development*, Vol. 59, pp. 26-37.

Guariso, G., Rizzoli, A. and Werthner, H. (1992), "Identification of model structure via qualitive simulation", *IEEE Trans. on Systems, Man, and Cybernetics*, Vol. 22 No. 5, pp. 1075-86.

Hwang, Z. and Wu, B. (1995), "Testing for the stability of Taiwan's short run money demand function: the application of fuzzy time series", *Annual Conference Proceedings of Taiwan Economic Association, National Taiwan University, Taipei, November*.

Jenkins, J. and Astington, J. (1996), "Cognitive factors and family structure associated with theory of mind development in young children", *Developmental Psychology*, Vol. 32, pp. 70-8.

Lowen, R. (1990), "A fuzzy language interpolation theorem", *Fuzzy Sets and Systems*, Vol. 34, pp. 33-8.

Nguyen, H.T. and Walker, E.A. (2000), *A First Course in Fuzzy Logic*, 2nd ed., Chapman & Hall/CRC, New York, NY.

Rosch, E. (1973), "On the internal structure of perceptual and semantic categories", in Moore, T.E. (Ed.), *Cognitive Development and the Acquisition of Language*, Academic Press, New York, NY, pp. 111-44.

Rosch, E., Mervis, C.B., Gray, W., Johnson, D. and Boyes-Braem, P. (1976), "Basic objects in natural categories", *Cognitive Psychology*, Vol. 8, pp. 382-439.

Ruspini, E. (1991), "Approximate reasoning: past, present, future", *Information Sciences*, Vol. 57, pp. 297-317.

Shatz, M., Martinez, I., Diesendruck, G. and Akar, D. (1995), "The influence of language on children's understanding of false belief", paper presented at the Society for Research in Child Development, Washington, DC, April.

Taylor, J.R. (1995), *Linguistic Categorization*, Oxford University Press, Oxford.

Tseng, T. and Klein, C. (1992), "A new algorithm for fuzzy multicriteria decision making", *International Journal of Approximate Reasoning*, Vol. 6, pp. 45-66.

Welch-Ross, M.K. (1997), "Mother-child participation in conversation about the past: relationships to preschoolers' theory of mind", *Developmental Psychology*, Vol. 33, pp. 618-29.

Wimmer, H. and Perner, W. (1983), "Beliefs about beliefs: representation and constraining function of wrong beliefs in young children's understanding of deception", *Cognition*, Vol. 13, pp. 103-28.

Wittgenstein, L. (1953), *Philosophical Investigations*, Macmillan, New York, NY.

Wu, B. and Hsu, Y.Y. (2004), "A new approach of bivariate fuzzy time series: with applications to the Stock Index forecasting", *International Journal of Uncertainty, Fuzziness and Knowledge-Based Systems*, Vol. 11 No. 6, pp. 671-90.

Wu, B. and Sun, C. (1996), "Fuzzy statistics and computation on the lexical semantics", *Language, Information and Computation*, Vol. 11, pp. 337-46.

Wu, B. and Sun, C. (2001), "Interval-valued statistics, fuzzy logic, and their use in computational semantics", *Journal of Intelligent and Fuzzy Systems*, Vol. 11, pp. 1-7.

Wu, B. and Tseng, N. (2002), "A new approach to fuzzy regression models with application to business cycle analysis", *Fuzzy Sets and Systems*, Vol. 130, pp. 33-42.

Wu, B. and Yang, W. (1998), "Fuzzy statistics and its applications in the social science research", in Yang, W. (Ed.), *Developing and Applications of Quantity Methods in Social Sciences*, Institute of Social Science, Academic Sinica, Taipei.

Wu, H.F. (2000), "Where will the dinosaur look for his ball? Children's false belief understanding and the mental state verb *yi3wei2* (以為)", *Wen Shan Review*, Vol. 1 No. 4, pp. 235-324.

Wu, H.F. (2004), "Children's acquisition of conditionals across languages", unpublished manuscript, Brandeis University, Waltham, MA.

Zadeh, L.A. (1965), "Fuzzy sets", *Information and Control*, Vol. 8, pp. 338-53.

## Further reading

Carlsson, C. and Fuller, R. (2000a), "Benchmarking in linguistic importance weighted aggregations", *Fuzzy Sets and Systems*, Vol. 114, pp. 35-41.

Carlsson, C. and Fuller, R. (2000b), "Multiobjective linguistic optimization", *Fuzzy Sets and Systems*, Vol. 115, pp. 5-10.

Chiang, D.A., Chow, L.R. and Wang, Y.F. (2000), "Mining time series data by a fuzzy linguistic summary system", *Fuzzy Sets and Systems*, Vol. 112, pp. 419-32.

Geert, P.V. and Dijk, M.V. (2003), "Ambiguity in child language: the problem of inter-observer reliability in ambiguous observation data", *First Language*, Vol. 23 No. 69, pp. 259-84.

Herrera, F. and Herrera-Viedma, E. (2000), "Linguistic decision analysis: steps for solving decision problems under linguistic information", *Fuzzy Sets and Systems*, Vol. 116, pp. 67-82.

Kosko, B. (1993), *Fuzzy Thinking: The New Science of Fuzzy Logic*, Hyperion, New York, NY.

Liu, W.Y. and Song, N. (2001), "The fuzzy association degree in semantic data models", *Fuzzy Sets and Systems*, Vol. 117, pp. 203-8.

Manton, K.G., Woodbury, M.A. and Tolley, H.D. (1994), *Statistical Applications using Fuzzy Sets*, Wiley, New York, NY.

Yager, R.R. and Zadeh, L.A. (1992), *An Introduction to Fuzzy Logic Applications in Intelligent Systems*, Kluwer Academic, Boston, MA.

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