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柏瑞圖係數與所得分配

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摘要

國家的所得分配呈現明顯雙峰分佈，包含兩個不對稱的柏瑞圖分配。傳統不均度分析不考慮實際的所得分配形式，以整體的單一係數衡量不均度。本研究的目的是考慮國家間所得分佈的實際型態，分析所得不均。考量兩個不對稱的柏瑞圖分配，依所得分佈分組，分別估計柏瑞圖係數以衡量各組的國家群組間的不均度，進而分析不均度與經濟成長的關係。研究顯示兩組國家的不均特質以及與國民所得之關係有顯著的差異，忽略實際組間差異而整體考量可能會造成偏誤結果。

關鍵詞：柏瑞圖係數、雙峰分佈、所得分配

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Pareto Coefficient in Income Distribution

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Abstract

The distribution of income among countries is not symmetrical; rather it is close to Pareto type distribution. It shows consistent bi-modal shape between 1980 and 2007; and shifts to the original Pareto distribution after global financial crisis since 2008. A bi-modal type world income distribution implies a more severe inequality situation between groups of countries rather than among countries. The traditional inequality indexes measured as a whole is an aggregate result. Ignoring the shape of income distributions and difference between groups may overlook some essential features. The purpose of this paper is to investigate the world country income distribution considering its true distribution: bi-modal shape. We suggest to classify countries considering its bi-modal distribution and to estimate the Pareto coefficient of each groups as a complement measure of world income inequality.

The result shows that the Pareto coefficient of higher income group is much larger than that of lower income group. Changes of inequality within two groups have significantly different or even opposite directions. The relation between income growth and inequality in the higher income group demonstrates quite different result from the lower income group. And further, the direction of the influences of the mean income on various inequality indexes are opposite between two groups. This study shows that income distributions between two groups possess distinctive features; to measure and discuss the inequality matters as a whole without considering the possible differences between groups may lead to a vague or biased result.

Keywords: Bi-modal income distribution, Inequality, Pareto coefficient

I. Introduction

The pattern of world income distribution across countries matters greatly not only in the sense of economic performance and well being, but also for understanding the world's growth. Many growth models generate a single peak in the long run. Quah (1993,1997) and Jones (1997) found that the world distribution of incomes per capita across countries seemed to evolve toward a "twin peaked" (bi-modal) distribution. Quah (1997) suggests economic and statistical dependence would manifest both dynamically and cross-sectionally. Jones (1997) suggested there were substantial changes in the distribution of GDP per worker around the world after World War II. This change of income resulted in a movement of world distribution from a likely normal distribution in 1960 to a bi-modal "twin-peaks" distribution. It means more rich countries and fewer poor countries.

Kremer, Onatski, and Stock (KOS) (2001) emphasized the fragility of bi-modal distribution. They suggest convergence to a single peak at high incomes, with a prolonged transition. Both views suggest that the global poor are substantial and will continue so. Anand and Segal (2008) reviewed the recent literature on global income inequality and found there is insufficient evidence to determine the direction of change in global inequality in recent decades. The direction of global income inequality is not robust across different estimation methods and datasets. Most studies are ambiguous regarding the distribution of income. However the representative of mean is closely related to the distribution of income. Atkinson and Brandolini (2010) explored a new measure which allows poverty and inequality to be considered in the same framework. Mean of the distribution of income is applied in this new measure.

The relation between income inequality and economic performance has been examined; however recent empirical work has inconclusive results. Positive relationships between income inequality and economic growth have been found by Partridge (1997, 2005), Forbes (2000) and Frank (2009). Frank (2009) finds a long-run positive relationship between inequality and growth which is driven principally by the concentration of income in the upper end of the income distribution.

Little or no significant relationship is found in the work of Panizza(2002), Quah (2001) and Deininger, K. and Squire, L. (1996). Barro(2000) has found a positive relationship between inequality and growth among wealthier countries but a negative relationship among low-income countries. The results spear to be extremely sensitive to the econometric specification. Works of Frank (2009) and Barro(2000) show that the relationship and the source of driven force between inequality and growth closely depends on the level of income.

Meschi and Vivarelli (2009) have found that trade with richer countries worsen income distribution in developing countries due to technological upgrading. Segal and Anand (2008) have found global income inequality are not robust across different estimation methods and data sets, and conclude that the direction of change in global inequality in recent decades is uncertain. Xavier Sala-i-Martin (2006) finds that Gini coefficient decreases from 1980 to 2000. Steve Dowrick and Muhammad Akmal (2005) find that the change of Gini is varied by conversion factors. In the contrary, Milanovic (2005) finds an opposite result. Several studies use various measure of inequality. The Theil L is measured in Chotikapanich, Valenzuela, and Rao (1997), Milanovic (2002, 2005), and Dikhanov and Ward (2002). The Theil T entropy is measured in Bourguignon and Morrison (2002), Dowrick and Akmal (2005), Korzeniewicz and Moran (1997), and Sala-i-Martin (2006). Bourguignon and Morrison (2002) find differences in change of inequality in different measures. Uchida and Cook (2008) use Gini coefficients to measure inequality and find that the effects of structural change on income distribution vary across industries.

The traditional Kuznets hypothesis (Kuznets, 1955) postulated a nonlinear relationship between a measure of income distribution and the level of economic development (Bulir, 2001). Galor, O. and Zeira, J.(1993) show that initial distribution of wealth affects aggregate output and investment. Perotti (1993) finds an inverted-U relation between levels of inequality and levels of income in cross-sections.

Firebaugh and Glenn (1999) employed a general formula for inequality indexes to study inter-country income inequality. They found that the trend toward rising inequality leveled off from 1960 to 1989. The distribution of income in a country is traditionally assumed to shift from relative equality to inequality and back to greater equality as the country develops. The polarization theorists place the greatest emphasis on the mechanisms by which nations at the top become richer at the expense of nations at the bottom (Korzeniewicz and Moran, 1997). Income inequalities result from economic growth differentials that occurred between different cities over time. Yorukoglu and Mehmet (2002) studied the relationship between population density and income inequality across countries and found population density and income inequality are closely linked.

Numerous works have applied various indexes in studying the features of aggregate inequality. There are different indexes for measuring income inequality including the Gini concentration ratio (Reardon and Bischoff, 2011; Jappelli and Pistaferr, 2010; McCall and Percheski, 2010 ; Frank, 2009; Meschi and Vivarelli 2009; Heidenreich and Wunder 2008; Cook and Uchida 2008; Cowell 1995; Galor, O. and Zeira, J. 1993), Theil's inequality index(Reardon and Bischoff, 2011), the Generalized Entropy measure (Shorroks, 1980) and the Oshima index (Meschi and Vivarelli 2009; Kuznets 1963; Sen 1973; Morduch and Sicular 1996). Both the Gini concentration ratio and Theil inequality index measure the dispersion of income in terms of the divergence to population share. The Oshima index takes the ratio of the top 20 percent average income to the lowest 20 percent average income. Additionally, the coefficient of variance offers a simple means of measuring difference. Kawachi, I (1997) found income inequality measured by the Robin Hood Index leads to increased mortality. Milanovic (2002) used the Gini index to calculate the income distribution among individuals around the world, and found that inequality increased from 1988 to 1993. Solt, F (2009) provides Gini indices of gross and net income inequality in the Standardized World Income Inequality Database (SWIID).

McCall and Percheski (2010) discussed the recent trend of income inequality, which is measured by income share, Gini and median household income. Meschi and Vivarelli (2009) estimated the impact of trade on within-country income inequality. They used Theil index and Gini index as base in measuring income inequality. Heidenreich and Wunder (2008) investigated the income inequalities in Europe both within-nation and between-nation. The inequalities of income are measured by Gini-coefficient and the mean logarithmic deviation. Cook and Uchida (2008) examined the relationship between structural change and income distribution applying Gini coefficient in measuring inequality in developing countries.

To sum up, most of the studies in inequality applying Gini index and other traditional index; and further they measure the inequality as a whole without considering the true distribution before measuring.

Previous studies support that distribution of income may be related to the level of income, output, investment and economic development. The measure of inequality is essential to the related researches including the causes and consequences of income inequality. The distribution of global income is neither equal nor symmetric with the mean income: there are many more countries with incomes below the mean income than above it. The distribution is skewed toward right and approximately consists of two Pareto distributions rather than a single one.

Due to the influence of the measure of inequality, and most popular indexes of inequality do not consider the distribution of income, this paper attempts to study income inequality according to the nature of income distribution. A different index (Pareto coefficient) is estimated to measure recent trends of the global income distribution. Pareto coefficient has been applied in investigating the distribution of sizes of cities.[2] Yet, the unsymmetrical feature of income distribution and the corresponding Pareto coefficient have not been empirically applied in interpreting the aggregate inequality.

The purpose of this paper is to investigate the world country income distribution considering the bi-modal distribution of world income. We suggest to classify countries according to the bi-modal type income distribution and to estimate the Pareto coefficient of each groups as a measure of the degree of inequality.

Various formulas for inequality indexes including the proposed Pareto coefficient are measured inter temporally, not only in the world as a whole, but also within two groups of countries. This is to investigate the features of income distribution overall and among groups of countries.

[2] See Soo (2005) and Nitsch (2005).

II. The indicators of inequality

The world income inequality is inequality among countries. The general formulas for inequality indexes are as follows:

1. The Gini coefficient (G)

The Gini coefficient is derived from the Lorenz curve, a cumulative frequency curve that compares the distribution of a specific variable with the uniform distribution that represents equality. It ranges from 0 to 1, reflecting the level of inequalities of the specific variable corresponding to the distribution of population. A value of 0 represents perfect equality and a value of 1 total inequality. The coefficient can be written as

$$G = (1/\mu)\sum_{i=1}^{n} \sum_{j=1}^{n} p_i p_j \left|Y_i - Y_j\right| \cdots\cdots(1)$$

where Y_i represents income in country i, μ is the world average income, p_i: refers to the population share of country *I*, and *n* is the number of countries.

2. The Theil inequality index (J)

The Theil inequality index (1967) is derived from the concept of relative entropy (known as the Kullback-Leibler divergence). It measures the distance between the distribution of population and the distribution of a specific variable. The divergence of the specific variable and the population is measured by the corresponding proportion and weighted by the population share. This inequality index is given by

$$J = \sum_{i=1}^{n} p_i \ln(p_i / y_i) = \sum_{i=1}^{n} p_i \ln j_i \cdots\cdots(2)$$

$$J' = \sum_{i=1}^{n} p_i [\ln(p_i / y_i)]^2 = \sum_{i=1}^{n} p_i (\ln j_i)^2 \cdots\cdots(3)$$

where y_i is the income share of country *i*. This index measures the level of inequality relative to the distribution of population, which is similar to the Gini coefficient. It can be decomposed into inequality between regions (J_{br}) and inequality within region (J_r).

$$J_{br} = \sum_{r=1}^{m} p_r \ln j_r \cdots\cdots(4)$$

$$J_r = \sum p_{ir} \ln j_{ir} \cdots\cdots(5)$$

$$J = J_{br} + \sum p_r J_r \cdots\cdots(6)$$

where parameter m refers to the number of subregions, p_{ir} represents the population share of country i in the subregion r.

3. The Generalized Entropy indicator (GE)

This indicator often called the mean logarithmic deviation is the transformed Theil inequality index. It is defined as the sum of the log of the world average income relative to the country income, weighted by the country's population share as in the Theil index. The divergence of the specific variable is measured according to its own mean value rather than the population share as in the Theil index.

$$GE = \sum_{i=1}^{n} p_i \ln(\mu / Y_i) \qquad (7)$$

$$GE' = \sum_{i=1}^{n} p_i [\ln(\mu / Y_i)]^2 \qquad (8)$$

where the variables are defined as in the previous indicators.

4. The Oshima index (S_y)

This index is defined as the ratio of the top 20 percent average income (Y^H) to the lowest 20 percent average income (Y^L). It is a rough measure of the divergence of a specific variable regardless of the distribution of population.

$$S_y = Y^H / Y^L \qquad (9)$$

5. Pareto coefficient β

Pareto (1896) found that the upper-tail distribution of the number of people with an income greater than x is proportional to $1/x^{\beta}$. The general Pareto distribution has the power function form:

$$F(x) = Ax^{-\beta} \qquad (10)$$

where $F(x)$ is the cumulative distribution function, and the number of observations is at least as large as x. It is positively skewed with a long tail of high values and consists of an ever larger number of ever smaller values. Consequently, the rank for x is inversely proportional to size x with a constant exponent β $(\beta > 0)$, known as the Pareto coefficient. This coefficient measures the level of diversification. If we order the countries by income, the country with the largest income is ranked 1; the corresponding income is the size of this country. The plot of log of rank versus log of size approximates a straight line.[3]

$$\log(Rank) = \log(A) - \beta \log(Size) \qquad (11)$$

According to France (1995), John (1985), Singer (1936) and Zipf (1949), larger values of the Pareto coefficient imply more evenly distributed sizes of x. In contrast, smaller coefficient values represent more diverse sizes of x. For example, when Pareto coefficient is applied in explaining the distribution of population sizes of cities, larger values of the coefficient means more equally sized cities;

[3] Berry (1961), Carroll (1982), Chesire (1999), Dobkins & Ioannides (2000), Krugman (1996), Fujita, M., Krugman, P., Venables, A.J. (1999) , Gabaix, X. (1999a,b), Rosen & Resnick, 1980.

smaller value of coefficient implies more diversified city sizes.[4] When Pareto coefficient β equals one, the distribution follows Zipf's law.

Zipf's law which is named after the American linguist George Kingsley Zipf (1902-1950) is one of a family of discrete power law probability distributions. Zipf's law states that the frequency is inversely proportional to its rank. It is easily observed by plotting the data on a log-log graph as in equation (11) with slope equals one. Gabaix (1999b) applied Gibrat's law to explain Zipf's law. Gibrat's law suggests that homogeneous growth processes lead the distribution converging into a Zipf pattern. In other words, if the distribution in terms of the mean and the variance of the growth rate of a unit is independent of its size, the limiting distribution follows Zipf's law.

Ⅲ. Data and measurement

The data of world income is based on the real Gross Domestic Product per capita from the Penn World Table 7.1 for up to 189 countries in a fifty year period (1960~2010). Figure 1 presents the world country income distribution. The income is measured by PPP GDP per capita from 1960 to 2010. Similar to the distribution of wealth among individuals, the distribution of income among countries is not symmetrical; rather it is close to a Pareto distribution in 1960. A larger portion of the income is in a smaller percentage of the countries in the world. The fraction of low income countries is high; the probability decrease steadily as income increases.

The world income distribution shows bi-modal “twin-peaks” distribution in 1980. This bi-modal type of distribution is consistent between 1980 and 2007. It remains bi-modal during the Asian financial crisis in 1997. The world income distribution shifts from bi-modal distribution to the original Pareto distribution after global financial crisis in 2008.

The bi-modal type world income distribution implies a more severe inequality situation between groups of countries rather than among countries.

It is not proper to measure the degree of inequality by variance if the distribution is Pareto distribution; and the Pareto coefficient is an appropriate index to measure the inequality of the distribution. The world income distribution becomes bi-model from 1980 to 2007. This bi-model type distribution can be divided into two Pareto distributions.

Pareto coefficient is estimated by regressing log of rank of country income by log of country income. A Pareto distribution shows linear relation between log of rank and log of income. And the estimated Pareto coefficient (the estimated slope) is a measure of the degree of inequality of country income. Diagrams of log of rank versus log of income from 1950 to 2010 are shown in Figure 2. The kinked point is getting more observable after 1980. Since the world income distribution is bi-modal type which consists of two Pareto distributions, The graphs look smooth before 1970; it become kinked after

[4] See Soo (2005) and Nitsch (2005).

1970, and is getting more obvious. It looks approximating two connected straight lines especially during 1990~1999 period. Income distribution between two groups of countries becomes more diverse in the last decade. The more a distribution closes to a Pareto distribution, the more the log plot closes to linear relation. The graph looks like two straight lines connected at some kinked point; this implies that world income distribution consists of two Pareto distributions. The steeper line refers to a low income group and a flatter line refers to a high income group. Countries are classified into two groups according to the income level at the kinked point. Group 1 contains nearly the top 16 percent higher income nations and Group 2 contains the remaining 84 percent lower income nations.[5] There are strong negative linear correlations of log of rank versus log of income in both groups verified by Meta analysis; consequently, this linearity feature characterizes fractal distributions.[6] The absolute value of the slope, the Pareto coefficient in Section 2, is a measure of the level of inequality. The slope of line of the lower income group is significantly flatter than that of the higher income group. This is statistically examined by the Paired *t* test.[7] Table 1 is the estimated Pareto Coefficients (β) for each year derived by regressing log of rank on log of GDP per capita for each year by group. The Pareto coefficient of higher income group is much larger than it of lower income group. Income among high income countries diversified more in late 1990s; it becomes more evenly distributed especially after Asia Financial Crisis in 2008. The Pareto coefficient of low income group is gradually diminishing through time. This implies that income distribution becomes more concentrate in the long run among low income countries.

Trends of GDP per capita for all countries by groups are displayed in Figure 3. The disparity between the higher income group (group 1) and the lower income group (group 2) is getting severe with the development of economy overall.

Inequality indexes are measured overall and within groups. Ignoring the differences of income distributions between two groups of countries and measure the inequality index as a whole would lead to overestimate of the degree of inequality of global income. The traditional overall index is an aggregate result. It may disregard the significant differences of the degree of inequality between two groups of countries. In other words, the overall degree of global inequality could be divided into two parts: inequality between groups and inequality within groups.

We observe the distribution of income and measure the degree of inequality accordingly. This can also be applied to national income distribution. Income inequality within Group 2 deviates more than that in Group1 except for the Pareto coefficient. Measures of inequalities overall are much closer to the measures of Group 2. This may be due to that the number of nations in Group 2 represents larger part of the total countries.

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[5] Nations in Group 1 belong to High-income OECD members except for Singapore, Hong Kong, and Macao.

[6] Meta analysis tests the hypothesis that the correlation coefficient of log of rank versus log of income in each group is significantly smaller than -0.9 at the 0.01 significance level. The Fisher's Z statistics of Group 1 is -34.36, and the Fisher's Z statistics of Group 2 is -30.01 (Hedges and Olkin,1985; Lipsey and Wilson ,2001; Wolf, 1986).

[7] Paired t test concludes that the estimated Pareto coefficient of the higher income group is significantly larger than that of the lower income group at the 0.01 significance level. The test statistics is 23.65.

Various indexes rate the scale of inequalities from different aspects. The level of income inequalities measured by all indexes is significantly higher in Group 2 than that in Group 1. This is examined by the Paired *t* test. The Pareto coefficient (β) is significantly larger in Group 1 than in Group 2. All the other inequality indexes are significantly smaller in Group 1 than those in Group 2. Two groups of countries reveal distinct features in terms of scale and the trend of income inequality.

The correlation matrix of all indexes is shown in Table 2. The Pareto coefficient (β) is negatively correlated to Gini and the coefficient of variation. GE and J indexes overall and in Group 2 are highly linearly correlated; in Group 1, the relation is trivial and insignificant.

The relation between inequality and growth is examined by the regressions of various inequality indexes on the average income and mean income growth rate. The result is shown in Table 3. In general, the mean income growth rate does not have significant effects on most inequality indexes in Group 2. On the contrary, in group 1, mean income growth rate has a significant influence on most indexes except J and J'.

Furthermore, in Group 1 (higher income), the relationship between the Pareto coefficient and mean income growth is significant positive. The more the mean income growth, the larger the Pareto coefficient. This implies that the higher the mean income growth, the more evenly distributed of the income in higher income group. The distribution of income is not normal; rather, it is close to the Pareto distribution. Pareto coefficient is more suitable to describe the degree of inequality of income than any other inequality index. The average income has a significant effect on almost all indexes in Group 1 and Group 2 except for GE in group 1. The direction of the effect of the average income on various inequality indexes in group 1 is opposite to that in group 2.

In group 2 (lower income), the relationship between the Pareto coefficient and mean income is significant negative. The more the mean income, the less the Pareto coefficient. This implies that the larger the mean income, the more diversified the income distribution in Group 2. This may be shown by the diminishing Pareto coefficient of Group 2 in Table 1 as the development of economics through time. However, in Group 1 (higher income), the relationship between the Pareto coefficient and mean income is significant positive. The more the mean income, the larger the Pareto coefficient. This implies that the higher the mean income, the more evenly distributed of the income in higher income group.

In short, Table 3 shows that the relation between income growth and inequality in the higher income group demonstrates quite different result from the lower income group. And further, the direction of the influence of the mean income on various inequality indexes is opposite between higher income and lower income groups.

This again verifies that income distributions between two groups possess distinctive features. To measure and discuss the inequality matters as a whole without considering the possible differences between groups may lead to a vague or biased result. The policy implication is to consider the true distribution before measuring and analyzing inequality matters may be more appropriate.

Ⅳ. Concluding remarks

The traditional inequality index is measured globally as a whole; it is an aggregate result. This may disregard the possible differences of the degree of inequality among groups of countries. The consequences of ignoring the possible differences of income distributions between groups may lead to an overestimate of the degree of inequality and losing the detail information.

This paper examines the features of income distribution from the aspect of Pareto distribution, and groups countries according to the character of income distribution. The income inequality is divided into two parts: inequality between groups and inequality within groups. The purpose of this paper is to suggest a different view and method to investigate the measure of income inequality based on descriptive statistics perspective.

The Pareto coefficient of higher income group is much larger than it of lower income group. Income among high income countries diversified more in late 1990s; it becomes more evenly distributed especially after Asia Financial Crisis in 2008. The Pareto coefficient of low income group is gradually diminishing through time. This implies that income distribution becomes more diversified in the long run among low income countries. Changes of inequality within two groups have significantly different or even opposite directions. Using Gini to measure overall income disparity may blurs the tendencies of income inequalities between two substantially different groups. Referring to the Gini and Pareto coefficient measures, income disparity in the higher income group, on the contrary, tends towards greater equality in the beginning and changes towards greater inequality. In general, the mean income growth rate does not have significant effects on most inequality indexes in Group 2. On the contrary, in group 1, mean income growth rate has a significant influence on most indexes except J and J'.

This study shows that the relation between income growth and inequality in the higher income group demonstrates quite different result from the lower income group. And further, the direction of the influences of the mean income on various inequality indexes are opposite between higher income and lower income groups. It proves that income distributions between two groups possess distinctive features. To measure and discuss the inequality matters as a whole without considering the possible differences between groups may lead to a vague or biased result. The policy implication is to consider the true distribution before measuring and analyzing inequality matters may be a more suitable way.

Income inequalities within the two groups of nations reveal distinct features in terms of scale, trend, and the relation to income growth and mean income. The properties of the lower income group are much closer to those of all countries as a whole. The disparity becomes critical between the two groups with the emergence of their divergent economic development and contrasts with the diminishing of inequality within the lower income countries. Increased integration promotes income disparities between groups; however, income disparity within the lower income group falls. Our results suggest that measuring income inequality disregarding the nature of income distribution and analyzing income distribution overall despite differences between groups may overlook some underlying features.

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Table 1　Parato Coefficient (β)

	Group 1	Group 2
2010	2.51*	.71*
2009	2.73*	.71*
2008	2.56*	.69*
2007	2.95*	.68*
2006	3.02*	.70*
2005	3.38*	.71*
2004	3.96*	.72*
2003	3.59*	.73*
2002	4.15*	.74*
2001	3.98*	.75*
2000	3.77*	.74*
1999	4.22*	.75*
1998	4.68*	.75*
1997	4.29*	.75*
1996	4.14*	.75*
1995	4.16*	.76*
1994	4.13*	.75*
1993	3.98*	.77*
1992	3.60*	.76*
1991	3.31*	.77*
1990	3.21*	.77*
1989	3.17*	.81*
1988	2.96*	.81*
1987	2.80*	.80*
1986	2.88*	.80*
1985	3.03*	.81*
1984	2.87*	.82*
1983	2.36*	.84*
1982	2.55*	.88*
1981	2.49*	.86*
1980	2.21*	.87*

Note: $\log(Rank) = \alpha - \beta \log(Income)$. Coefficient β is Pareto coefficient.

*Significant at the 1% level.

Source: Calculations by author (Data is from the Penn World Table 7.1).

Table 2 Correlation Matrix

Total countries

	Osh.	Gini	GE	GE'	J	J'	Cov.
Oshima	1.000	0.101	-0.354**	-0.588*	-0.245	-0.516*	0.696*
Gini	0.101	1.000	-0.214	-0.210	-0.304	-0.276	0.446*
GE	-0.354**	-0.214	1.000	0.936*	0.964*	0.960*	-0.573*
GE'	-0.588*	-0.210	0.936*	1.000	0.848*	0.976*	-0.664*
J	-0.245	-0.304	0.964*	0.848*	1.000	0.907*	-0.577*
J'	-0.516*	-0.276	0.960*	0.976*	0.907*	1.000	-0.711*
Cov.	0.696*	0.446*	-0.573*	-0.664*	-0.577*	-0.711*	1.000

Group 1

	$\|\beta\|$	Gini	GE	GE'	J	J'	Cov.
$\|\beta\|$	1.000	-0.965*	0.095	-0.803*	0.580*	0.347**	-0.966*
Gini	-0.965*	1.000	-0.137	0.746*	-0.520*	-0.274	0.963*
GE	0.095	-0.137	1.000	-0.266	0.009	-0.402**	-0.186
GE'	-0.803*	0.746*	-0.266	1.000	-0.799*	-0.505*	0.728*
J	0.580*	-0.520*	0.009	-0.799*	1.000	0.850*	-0.489*
J'	0.347**	-0.274	-0.402	-0.505*	0.850*	1.000	-0.248
Cov	-0.966*	0.963*	-0.186	0.728*	-0.489*	-0.248	1.000

Group 2

	$\|\beta\|$	Gini	GE	GE'	J	J'	Cov.
$\|\beta\|$	1.000	-0.537*	0.478*	0.434*	0.493*	0.527*	-0.935*
Gini	-0.537*	1.000	-0.194	-0.147	-0.231	-0.238	0.545*
GE	0.478*	-0.194	1.000	0.976*	0.975*	0.990*	-0.332**
GE'	0.434*	-0.147	0.976*	1.000	0.933*	0.963*	-0.264
J	0.493*	-0.231	0.975*	0.933*	1.000	0.985*	-0.373**
J'	0.527*	-0.238	0.990*	0.963*	0.985*	1.000	-0.399**
Cov	-0.935*	0.545*	-0.332**	-0.264	-0.373**	-0.399**	1.000

*Significant at the 1% level

**Significant at the 5% level.

Source: Calculations by author (Data is from the Penn World Table 7.1).

Table 3　Inequality and growth (Tobit)

Inequality Index R-square	Constant	b1	b2	
Group I: Higher Income				
β	3.27*	16.71*	0.0001*	0.40
	(6.7161)	(3.4302)	(5.3645)	
Gini	0.12*	-0.32*	-0.0000019*	0.49
	(16.8348)	(-4.3955)	(-6.2009)	
GE	-0.029*	0.11**	0.00000035	0.09
	(-5.7790)	(2.3239)	(1.7159)	
GE'	0.07*	-0.21*	-0.0000017*	0.64
	(14.2108)	(-4.5537)	(-8.7066)	
J	0.78*	0.07	0.0000048*	0.44
	(38.4698)	-0.3212	-5.5842	
J'	1.67*	-0.17	0.0000081*	0.14
	(23.8269)	(-0.2462)	-2.7753	
Cov	0.23*	-0.63*	-0.0000030*	0.36
	(14.6977)	(-4.0425)	(-4.6353)	
Group 2:Lower Income				
β	1.23*	0.03	-0.000068*	0.84
	(88.4452)	(0.0722)	(-14.1124)	
Gini	0.39*	0.43**	0.0000092*	0.26
	(60.6676)	(2.1119)	(4.1485)	
GE	0.36*	0.05	-0.000022*	0.5
	(36.2906)	(0.1714)	(-6.3597)	
GE'	0.07*	-0.99	-0.000077*	0.32
	(24.9483)	(-0.6609)	(-4.6861)	
J	2.57*	0.68	-0.0001*	0.56
	(100.8802)	(0.8400)	(-6.8003)	
J'	11.88*	0.63	-0.0005*	0.56
	(58.1665)	(0.0968)	(-7.1062)	
Cov	0.65*	-0.09	0.000032*	0.71
	(69.9070)	(-0.3181)	(9.7807)	
Total				
Oshima	11.24*	25.86*	0.0012*	0.96
	(68.9056)	(2.8624)	(31.1939)	
Gini	0.50*	0.23	0.00000011	-0.07
	(81.7313)	(0.6652)	(0.0810)	
GE	0.37*	0.11	-0.0000083*	0.40
	(51.7192)	(0.2740)	(-5.0832)	
GE'	1.96*	0.64	-0.000068*	0.58
	(46.8568)	(0.2782)	(-7.1382)	

Table 3　Inequality and growth (Tobit)

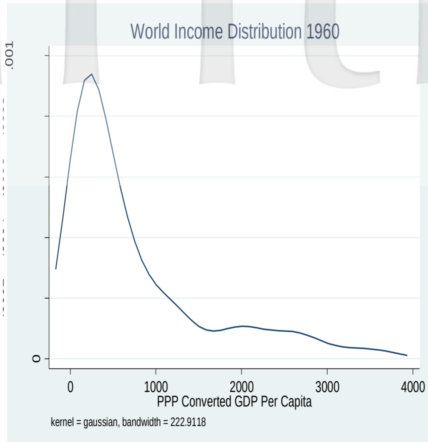
Inequality Index R-square	Constant	b1	b2	
J	2.43*	0.61	-0.000018*	0.29
	(123.817)	(0.5584)	(-3.9137)	
J'	11.89*	2.22	-0.0002*	0.51
	(72.0462)	(0.2429)	(-6.2475)	
Cov	0.91*	0.59**	0.0000074*	0.48
	(183.4586)	(2.1410)	(6.4796)	

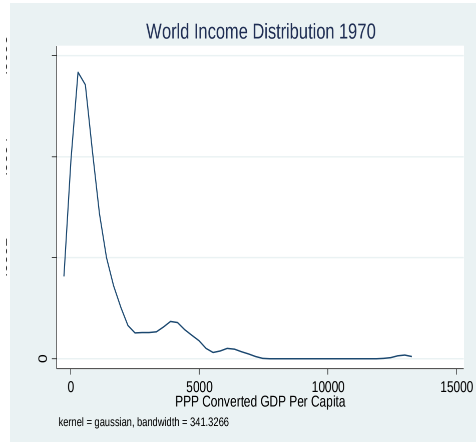
Note: $y = a + b_1 x_1 + b_2 x_2$, where y is inequality index; x_1 is the growth of mean income and x_2 is the mean income.　Numbers in parentheses are *t* values.

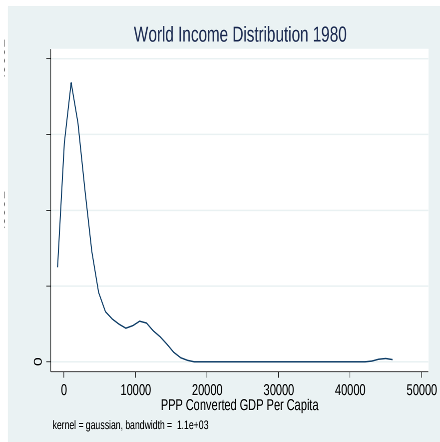
*Significant at the 1% level

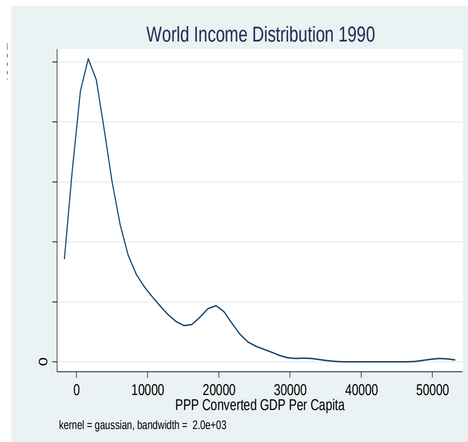
**Significant at the 5% level.

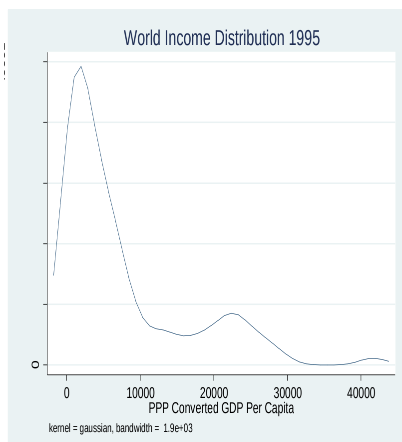
Source: Calculations by author (Data is from the Penn World Table 7.1).

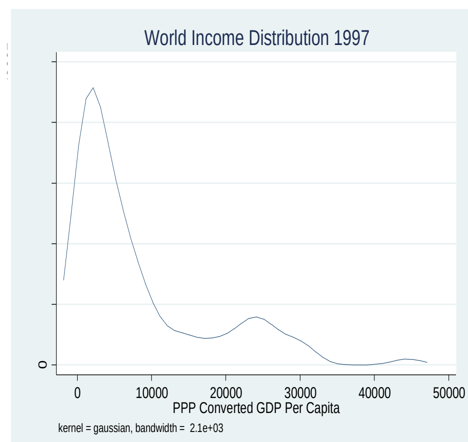

World Income Distribution 1960
.001
0
0
1000
2000
3000
4000
PPP Converted GDP Per Capita
kernel = gaussian, bandwidth = 222.9118


World Income Distribution 1970
0
0
5000
10000
15000
PPP Converted GDP Per Capita
kernel = gaussian, bandwidth = 341.3266


World Income Distribution 1980
0
0
10000
20000
30000
40000
50000
PPP Converted GDP Per Capita
kernel = gaussian, bandwidth = 1.1e+03


World Income Distribution 1990
0
0
10000
20000
30000
40000
50000
PPP Converted GDP Per Capita
kernel = gaussian, bandwidth = 2.0e+03


World Income Distribution 1995
0
0
10000
20000
30000
40000
PPP Converted GDP Per Capita
kernel = gaussian, bandwidth = 1.9e+03


World Income Distribution 1997
0
0
10000
20000
30000
40000
50000
PPP Converted GDP Per Capita
kernel = gaussian, bandwidth = 2.1e+03

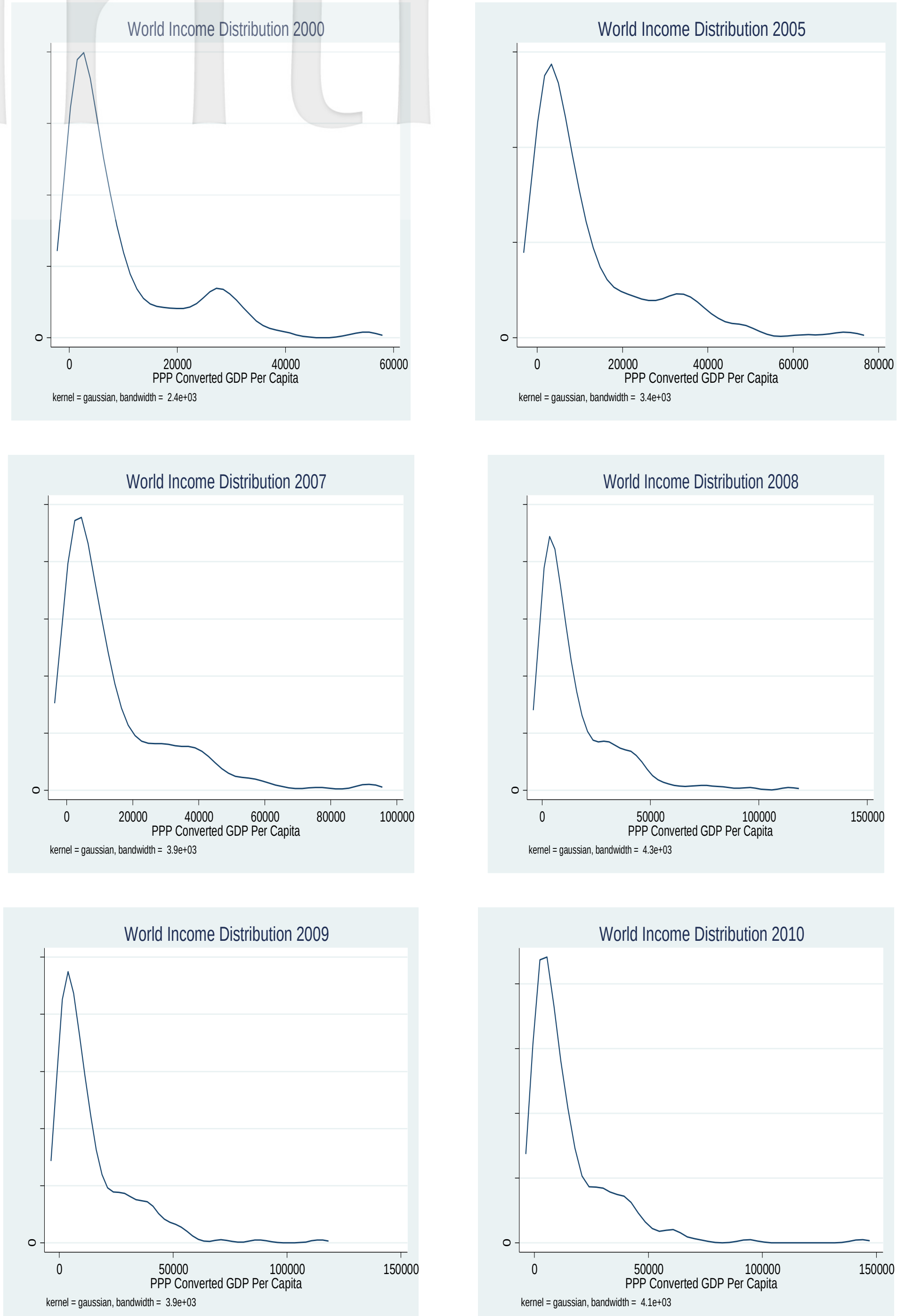


Figure 1　World Income Distribution

Source: Graph by author (Data is from the Penn World Table 7.1).

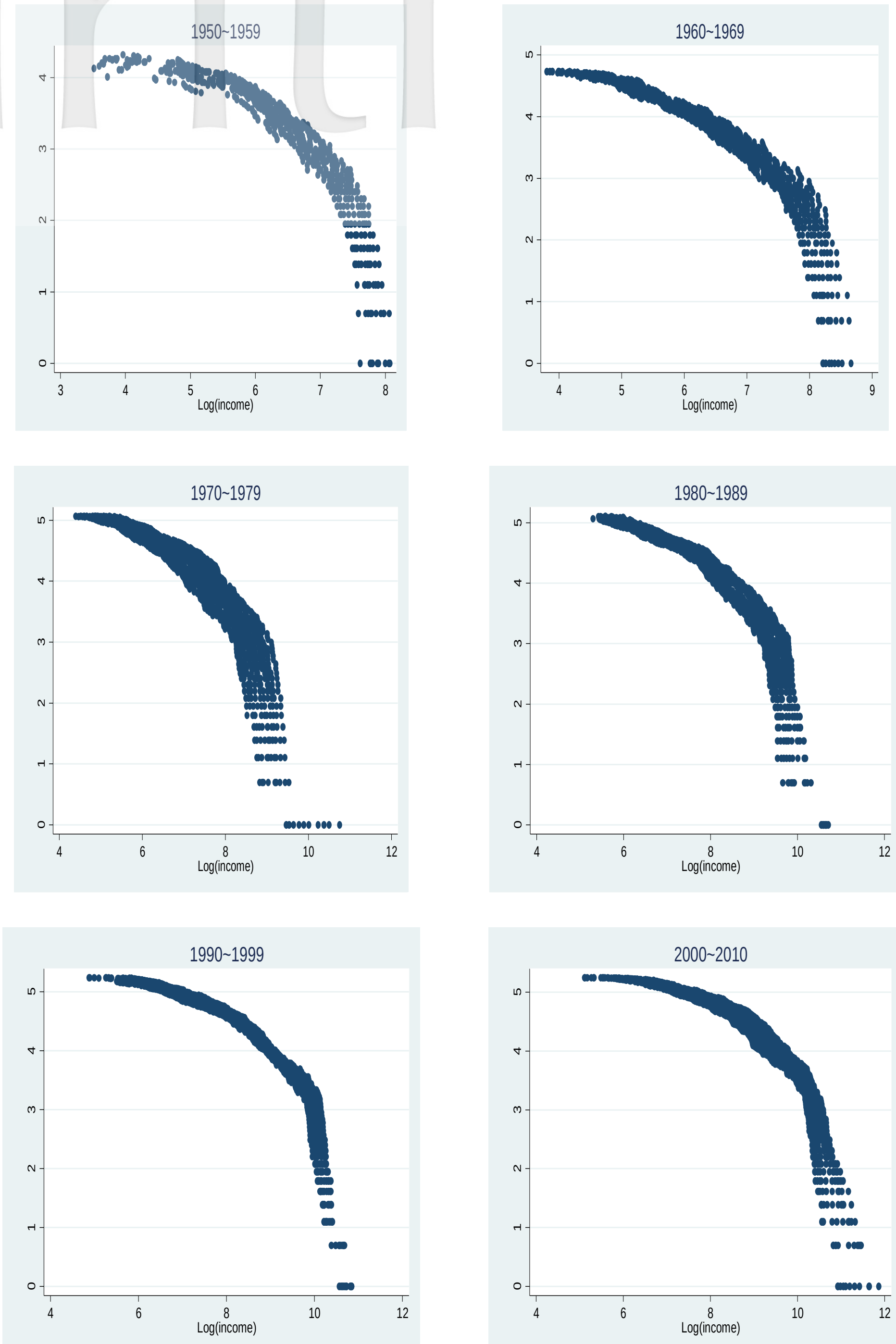


Figure 2.1　Log of rank versus log of income

Source: Graph by author (Data is from the Penn World Table 7.1).

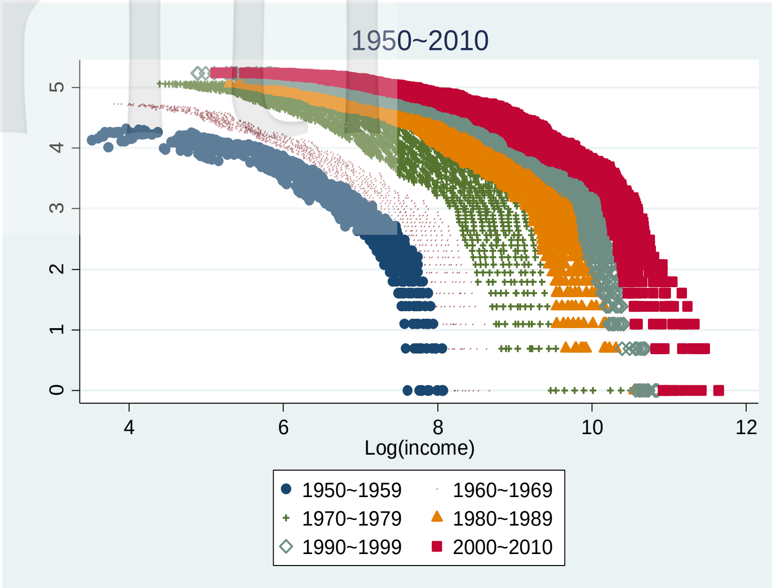


Figure 2.2 Log of rank versus log of income 1950~1960

Source: Graph by author (Data is from the Penn World Table 7.1).

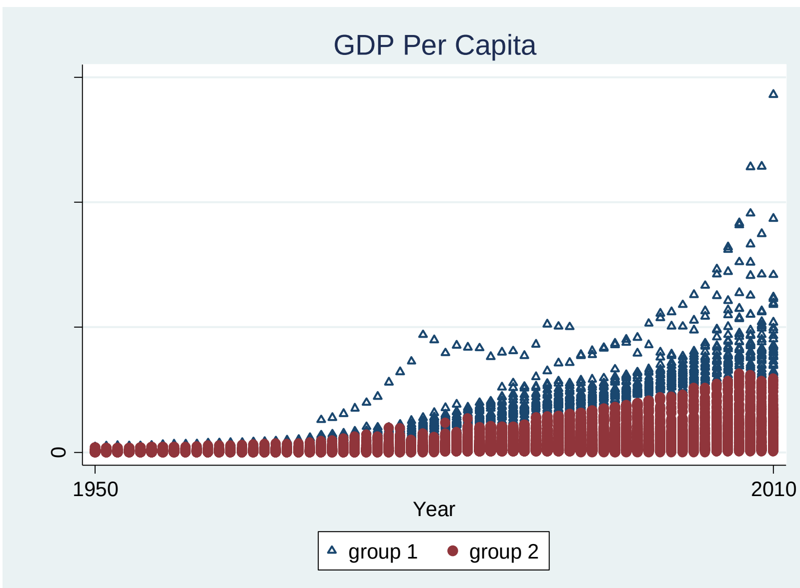


Figure 3 GDP per Capita by groups

Source: Graph by author (Data is from the Penn World Table 7.1).