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A MATHEMATICAL MODEL OF ENTERPRISE COMPETITIVE ABILITY AND PERFORMANCE THROUGH EMDEN-FOWLER EQUATION FOR SOME ENTERPRISES*

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Abstract In this paper, we work with the ordinary differential equation $n^2u\,(n)'' = u(n)^p$ and obtain some interesting phenomena concerning, boundedness, blow-up, blow-up rate, life-span of solutions to those equations.

Key words estimate; life-span; blow-up; blow-up rate; performance; competitive ability

2010 MR Subject Classification 34C

1 Introduction

How to improve the performance and competitiveness of the company is the critical issue of Industrial and Organizational Psychology in Taiwan. We try to design an appropriate mathematical model of the competitiveness and the performance of the 293 benchmark enterprises out of 655 companies. Unexpectedly, we discover the correlation of performance and competitiveness is extremely high. Some benchmark enterprises present the following phenomena:

Competitive ability (force, $F(P\,(n))$) is a power function of the performance ($P\,(n)$); that is, there exist positive performances $p > 0$ and a constant k so that

$$F\,(P\,(n)) = P\,(n)^p\, n^k,$$

where n is the surveying rod enterprise's composition department number or the main unit commanders counts, the performance $P\,(n)$ of the rod enterprise's is lager than P_0 and F is proportional to the second derivative of P with respect to n. For $F(P\,(n)) = M\,(n)\,\frac{\mathrm{d}^2P(n)}{\mathrm{d}n^2}$, normally, $M\,(n) = n$, we obtain a stationary one dimensional semilinear wave equation with initial condition

$$\begin{cases} nP\,(n)'' - P\,(n)^p\, n^k = 0, \quad n \ge n_0, \\ P\,(n_0) = P_0 \ge 0, \quad P'\,(n_0) = P_1. \end{cases} \tag{*}$$

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It is clear that the function $P(n)^p n^k$ is locally Lipschitz function in P, hence by the standard theory, the local existence of classical solutions is applicable to equation $(*)$.

We would use our methods used in [1–19] to discuss problem $(*)$ for $k=-1$; that is,

$$\begin{cases} n^2 P(n)'' - P(n)^p = 0, \quad n \ge n_0 > 1, \\ P(n_0) = P_0 \ge 0, \quad P'(n_0) = P_1. \end{cases} \tag{1.1}$$

In papers [Li 1–4, 6–8], the semi-linear wave equation $\Box u + f(u) = 0$ under some conditions, some interesting results on blow-up, blow-up rate and estimates for the life-span of solutions were obtained. We want to study the case of Emden-Fowler type wave equation in 0-dimension form, i.e., to consider the equation $n^2 P(n)_{nn} - \triangle P(n) = P(n)^p$ with zero-space dimension. For $p>1$, these functions $n^{-2}P(n)^p$ are locally Lipschitz, the local existence and uniqueness of solutions of equation (1.1) for $p>1$, can be obtained through the standard arguments. Consider the transformation $n=\mathrm{e}^s$, $P(n)=u(s)$, then $n^2P''(n) = -u_s(s) + u_{ss}(s)$, $u(s)^p = -u_s(s) + u_{ss}(s)$ and $u(0)=P(1)=P_0; u_s(0)=P'(1)=P_1$. Therefore, equation (1.1) can be transformed into the form

$$\begin{cases} u_{ss}(s) - u_s(s) = u(s)^p, \qquad p \in \mathbb{N}, \\ u(0) = P_0, \quad u_s(0) = P_1. \end{cases} \tag{1.2}$$

Thus, the local existence of solution u for (1.2) in $(0,S)$ is equivalent to the local existence of solution P for (1.1) in $(1,\ln S)$. In this paper, we have estimated the life-span S^* of positive solution u of (1.2) under three different cases. The main results are as follows:

(a) $P_1 = 0, P_0 > 0:$

$$N^* \le \mathrm{e}^{k_1},\ k_1 := s_0 + \frac{2(p+3)}{8-\epsilon}\frac{2}{p-1}u(s_0)^{\frac{1-p}{2}},\ \varepsilon \in (0,1).$$

(b) $P_1 > 0, P_0 > 0:$

$$\text{i)} \quad E(0) \ge 0, \quad N^* \le \mathrm{e}^{k_2}, \quad k_2 := \frac{2}{p-1}\sqrt{\frac{p+1}{2}}P_0^{\frac{1-p}{2}}.$$

$$\text{ii)} \quad E(0) < 0, \quad N^* \le \mathrm{e}^{k_3}, \quad k_3 := \frac{2}{p-1}\frac{P_0}{P_1}.$$

(c) $P_1 < 0, P_0 \in \left(0, (-P_1)^{\frac{1}{p}}\right):$

$$P(n) \le (P_0 - P_1 - P_0^p) + (P_1 + P_0^p)\,n - P_0^p \ln n.$$

2 Notation and Fundamental Lemmas

For a given function u in this work we use the following abbreviations

$$a(s) = u(s)^2,\ E(0) = P_1^2 - \frac{2}{p+1}P_0^{p+1},\ J(s) = a(s)^{-\frac{p-1}{4}}.$$

By some calculation we can obtain the following Lemma 1 and Lemma 2, we omit these argumentations on the proof of Lemma 1.

Lemma 1 Suppose that $u \in C^2([0,S])$ is the solution of (1.2), then

$$E(s) = u_s(s)^2 - 2\int_0^s u_s(r)^2\,\mathrm{d}r - \frac{2}{p+1}u(s)^{p+1} = E(0), \tag{2.1}$$

$$(p+3)\,u_s(s)^2=(p+1)\,E(0)+a''(s)-a'(s)+2\,(p+1)\int_0^s u_s(r)^2\,\mathrm{d}r, \tag{2.2}$$

$$J''(s)=\frac{p^2-1}{4}J(s)^{\frac{p+3}{p-1}}\left(E(0)-\frac{a'(s)}{p+1}+2\int_0^s u_s(r)^2\,\mathrm{d}r\right) \tag{2.3}$$

and

$$\begin{aligned}J'(s)^2=&\,J'(0)^2+\frac{(p-1)^2}{4}E(0)\left(J(s)^{\frac{2(p+1)}{p-1}}-J(0)^{\frac{2(p+1)}{p-1}}\right)\\&+\frac{(p-1)^2}{2}J(s)^{\frac{2(p+1)}{p-1}}\int_0^s u_s(r)^2\,\mathrm{d}r.\end{aligned} \tag{2.4}$$

Lemma 2 For $P_0>0$, the positive solution u of equation (1.2), we have

$$\text{i)}\quad P_1\geq 0,\ \text{then}\ u_s(s)>0\ \ \text{for all}\ s>0. \tag{2.5}$$

$$\text{ii)}\quad P_1<0, P_0\in\left(0,(-P_1)^{\frac{1}{p}}\right),\ \text{then}\ u_s(s)<0\ \ \text{for all}\ s>0. \tag{2.6}$$

Proof i) $u_{ss}(0)=P_1+P_0^p>0$, we know that $u_{ss}(s)>0$ in $[0,s_1)$ and $u_s(s)$ is increasing in $[0,s_1)$ for some $s_1>0$. Moreover, since u and u_s are increasing in $[0,s_1)$,

$$u_{ss}(s_1)=u_s(s_1)+u(s_1)^p>u_s(0)+u(0)^p>0$$

for all $s\in[0,s_1)$ and

$$u_s(s_1)>u_s(s)>0$$

for all $s\in[0,s_1)$, we know that there exists a positive number $s_2>0$, such that $u_s(s)>0$ for all $s\in[0,s_1+s_2)$.

Continuing such process, we obtain $u_s(s)>0$ for all $s>0$.

ii) According to $u_{ss}(0)=u_s(0)+u(0)^p=P_1+P_0^p<0$, there exists a positive number $s_1>0$ such that $u_{ss}(s)<0$ in $[0,s_1)$, $u_s(s)$ is decreasing in $[0,s_1)$; therefore,

$$u_s(s)<u_s(0)=u_1<0$$

for all $s\in[0,s_1)$ and $u(s)$ is decreasing in $[0,s_1)$.

Moreover, since u and u_s are decreasing in $[0,s_1)$,

$$u_{ss}(s)=u_s(s)+u(s)^p<u_s(0)+u(0)^p<0$$

for all $s\in[0,s_1)$ and $u_s(s_1)<u_s(s)<0$ for all $s\in[0,s_1)$, we know that there exists a positive number $s_2>0$, such that $u_s(s)<0$ for all $s\in[0,s_1+s_2)$.

Continuing such process, we obtain $u_s(s)<0$ for all $s>0$. □

3 Estimates for the Life-Span of Positive Solution u of (1.2) under $P_1=0, P_0>0$

In this section we want to estimate the life-span of positive solution u of (1.2) under $P_1=0$, $P_0>0$. Here the life-span S^* of u means that u is the solution of equation $(*)$ and u exists only in $[0,S^*)$ so that problem (1.2) possesses the positive solution $u\in C^2[0,S^*)$ for $S<S^*$.

Theorem 3 For $P_1=0$, $P_0>0$, the positive solution u of (1.2) blows up in finite time; that is, there exists a bound number S^* so that

$$u(s)^{-1}\to 0\quad\text{for}\quad s\to S^*.$$

Remark The phenomena of blow-up of $u(s)$ at $s = S^*$ (or $P(n)$ at $n = \mathrm{e}^{S^*}$) means that such benchmark enterprises attain their maximum of performance and competitiveness.

Proof By (2.5), we know that $u_s(s) > 0, a'(s) > 0$ for all $s > 0$ under $P_1 = 0$, $P_0 > 0$.

By Lemma 1,

$$a''(s) - a'(s) = 2\left(u_s(s)^2 + u(s)^{p+1}\right),$$

$$\left(a'(s)\mathrm{e}^{-s}\right)' = \mathrm{e}^{-s}\left(a''(s) - a'(s)\right) = 2\mathrm{e}^{-s}\left(u_s(s)^2 + u(s)^{p+1}\right),$$

$$a'(s)\mathrm{e}^{-s} = 2\int_0^s \mathrm{e}^{-r}\left(u_s(r)^2 + u(r)^{p+1}\right)\mathrm{d}r \geq 4\int_0^s \mathrm{e}^{-r}u_s(r)\,u(r)^{\frac{p+1}{2}}\,\mathrm{d}r$$

and $a'(0) = 0$, we have

$$\begin{aligned} a'(s)\mathrm{e}^{-s} &\geq \frac{8}{p+3}\left(u(r)^{\frac{p+3}{2}}\mathrm{e}^{-r}\,\Big|_{r=0}^{s} + \int_0^s u(r)^{\frac{p+3}{2}}\mathrm{e}^{-r}\mathrm{d}r\right) \\ &= \frac{8}{p+3}\left(u(s)^{\frac{p+3}{2}}\mathrm{e}^{-s} - u(0)^{\frac{p+3}{2}}\right) + \frac{8}{p+3}\int_0^s u(r)^{\frac{p+3}{2}}\mathrm{e}^{-r}\mathrm{d}r. \end{aligned}$$

Since $a'(s) > 0$ for all $s > 0$, u is increasing in $(0,\infty)$ and

$$\begin{aligned} a'(s)\mathrm{e}^{-s} &\geq \frac{8}{p+3}\left(u(s)^{\frac{p+3}{2}}\mathrm{e}^{-s} - u(0)^{\frac{p+3}{2}}\right) + \frac{8}{p+3}\int_0^s u(0)^{\frac{p+3}{2}}\mathrm{e}^{-r}\mathrm{d}r \\ &= \frac{8}{p+3}\left(u(s)^{\frac{p+3}{2}}\mathrm{e}^{-s} - u(0)^{\frac{p+3}{2}}\right) + \frac{8}{p+3}u(0)^{\frac{p+3}{2}}\left(1 - \mathrm{e}^{-s}\right), \end{aligned} \tag{3.1}$$

$$a'(s) \geq \frac{8}{p+3}\left(u(s)^{\frac{p+3}{2}} - u(0)^{\frac{p+3}{2}}\right) = \frac{8}{p+3}\left(u(s)^{\frac{p+3}{2}} - P_0^{\frac{p+3}{2}}\right).$$

Using $P_1 = 0$ and integrating (1.2), we obtain

$$u_s(s) = u(s) - P_0 + \int_0^s u(r)^p\,\mathrm{d}r,$$

$$u_s(s) \geq u(s) - P_0 + \int_0^s u(0)^p\,\mathrm{d}r = u(s) - P_0 + P_0^p s,$$

$$\left(\mathrm{e}^{-s}u(s)\right)_s = \mathrm{e}^{-s}\left(u_s(s) - u(s)\right) \geq \mathrm{e}^{-s}\left(P_0^p s - P_0\right), \tag{3.2}$$

$$a'(s) \geq \frac{8}{p+3}\left(u(s)^{\frac{p+3}{2}} - u(0)^{\frac{p+3}{2}}\right) = \frac{8}{p+3}\left(u(s)^{\frac{p+3}{2}} - P_0^{\frac{p+3}{2}}\right).$$

According to (3.2) and $u'(s) > 0$,

$$u(s)^{\frac{p+3}{2}} \geq \left(P_0 + P_0^p\left(\mathrm{e}^s - 1 - s\right)\right)^{\frac{p+3}{2}}$$

and for all $\epsilon \in (0,1)$, we get that

$$\epsilon u(s)^{\frac{p+3}{2}} \geq \epsilon\left(P_0 + P_0^p\left(\mathrm{e}^s - 1 - s\right)\right)^{\frac{p+3}{2}},$$

$$\begin{aligned} \epsilon u(s)^{\frac{p+3}{2}} - 8P_0^{\frac{p+3}{2}} &\geq \epsilon\left(P_0 + P_0^p\left(\mathrm{e}^s - 1 - s\right)\right)^{\frac{p+3}{2}} - 8P_0^{\frac{p+3}{2}} \\ &\geq \epsilon\left(P_0^{\frac{p+3}{2}} + P_0^{\frac{p(p+3)}{2}}\left(\mathrm{e}^s - 1 - s\right)^{\frac{p+3}{2}}\right) - 8P_0^{\frac{p+3}{2}} \\ &= (\epsilon - 8)\,P_0^{\frac{p+3}{2}} + \epsilon P_0^{\frac{p(p+3)}{2}}\left(\mathrm{e}^s - 1 - s\right)^{\frac{p+3}{2}}. \end{aligned}$$

Now, we want to find a number $s_0 > 0$ such that

$$e^{s_0} - s_0 = 1 + \left(\frac{8-\epsilon}{\epsilon} P_0^{\frac{p+3}{2}(1-p)} \right)^{\frac{2}{p+3}}. \tag{3.3}$$

This means that there exists a number $s_0 > 0$ satisfying (3.3) with $\epsilon \in (0,1)$ such that

$$\epsilon u(s)^{\frac{p+3}{2}} - 8P_0^{\frac{p+3}{2}} \geq 0 \quad \text{for all } s \geq s_0.$$

From (3.1), it follows that

$$\begin{aligned} a'(s) &\geq \frac{8}{p+3} u(s)^{\frac{p+3}{2}} - \frac{8}{p+3} P_0^{\frac{p+3}{2}} \\ &= \frac{8-\epsilon}{p+3} u(s)^{\frac{p+3}{2}} + \frac{\epsilon u(s)^{\frac{p+3}{2}} - 8P_0^{\frac{p+3}{2}}}{p+3} \\ &\geq \frac{8-\epsilon}{p+3} u(s)^{\frac{p+3}{2}} \quad \text{for all } s \geq s_0. \end{aligned}$$

For all $s \geq s_0$, $\epsilon \in (0,1)$, we obtain that

$$2u(s)\, u_s(s) \geq \frac{8-\epsilon}{p+3} u(s)^{\frac{p+3}{2}},$$

$$u(s)^{-\frac{p+1}{2}} u_s(s) \geq \frac{8-\epsilon}{2(p+3)},$$

$$\frac{2}{1-p} \left(u(s)^{\frac{1-p}{2}} \right)_s \geq \frac{8-\epsilon}{2(p+3)}$$

and

$$\left(u(s)^{\frac{1-p}{2}} \right)_s \leq \frac{8-\epsilon}{2(p+3)} \frac{1-p}{2}.$$

Integrating the above inequality, we conclude that

$$u(s)^{\frac{1-p}{2}} \leq u(s_0)^{\frac{1-p}{2}} - \frac{8-\epsilon}{2(p+3)} \frac{p-1}{2} (s - s_0).$$

Thus, there exists a finite number

$$S_1^* \leq s_0 + \frac{2(p+3)}{8-\epsilon} \frac{2}{p-1} u(s_0)^{\frac{1-p}{2}} := k_1,$$

such that $u(s)^{-1} \to 0$ for $s \to S_1^*$, that is,

$$P(n)^{-1} \to 0 \quad \text{for } n \to e^{k_1},$$

which implies that the life-span N^* of positive solution P is finite and $N^* \leq e^{k_1}$. □

4 Estimates for the Life-Span of Positive Solution u of (1.2) under $P_1 > 0, P_0 > 0$

In this section we start to estimate the life-span of positive solution u of (1.2) under $P_1 > 0$, $P_0 > 0$.

Theorem 4 For $P_1 > 0$, $P_0 > 0$, the positive solution u of (1.2) blows up in finite time; that is, there exists a bound number S^* so that

$$u(s)^{-1} \to 0 \quad \text{for} \quad s \to S^*.$$

Moreover, for $E(0) \geq 0$, we have

$$S^* \leq S_2^* = \frac{2}{p-1}\sqrt{\frac{p+1}{2}}P_0^{\frac{1-p}{2}}.$$

For $E(0) < 0$, we also have

$$S^* \leq S_3^* = \frac{2}{p-1}a(0)^{-\frac{p-1}{4}}\left(\frac{2}{p+1} + E(0)\,a(0)^{-\frac{p+1}{2}}\right)^{-\frac{1}{2}}.$$

Proof We separate the proof into two parts, $E(0) \geq 0$ and $E(0) < 0$.

i) $E(0) \geq 0$. By (2.1) and (2.5) we have

$$u_s(s)^2 - \frac{2}{p+1}u(s)^{p+1} \geq E(0)$$

and

$$u_s(s)^2 \geq \frac{2}{p+1}u(s)^{p+1} + E(0),$$

$$u_s(s) \geq \sqrt{\frac{2}{p+1}u(s)^{p+1} + E(0)}.$$

Under the condition $E(0) \geq 0$, we get

$$u_s(s) \geq \sqrt{\frac{2}{p+1}}u(s)^{\frac{p+1}{2}}, \quad u(s)^{-\frac{p+1}{2}}u_s(s) \geq \sqrt{\frac{2}{p+1}}$$

and

$$\left(u(s)^{\frac{1-p}{2}}\right)_s \leq \frac{1-p}{2}\sqrt{\frac{2}{p+1}}.$$

Integrating the above inequality, we obtain

$$u(s)^{\frac{1-p}{2}} \leq P_0^{\frac{1-p}{2}} + \frac{1-p}{2}\sqrt{\frac{2}{p+1}}s.$$

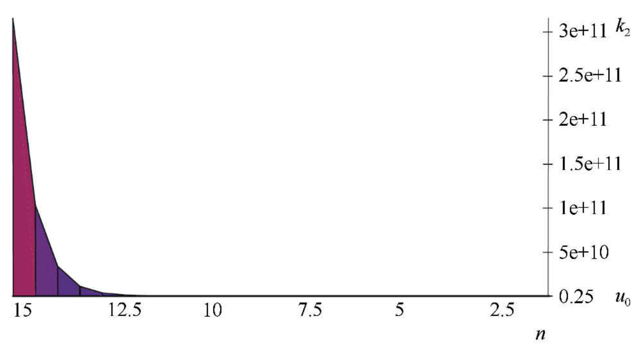
Thus, there exists a finite time

$$S_2^* \leq \frac{2}{p-1}\sqrt{\frac{p+1}{2}}P_0^{\frac{1-p}{2}} := k_2,$$

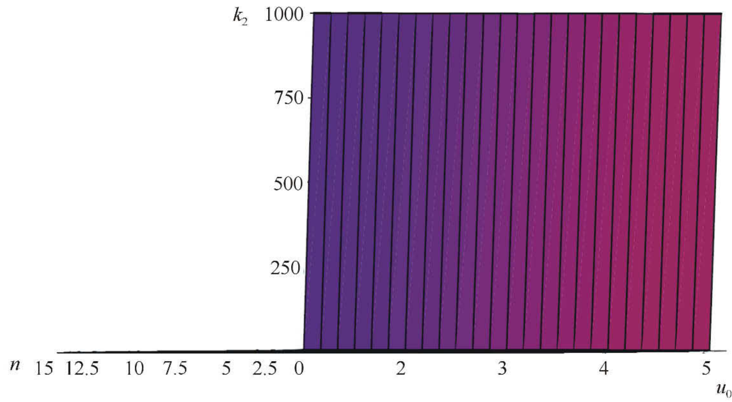
such that $u(s)^{-1} \to 0$ for $s \to S_2^*$, that is,

$$P(n)^{-1} \to 0 \quad \text{for} \quad n \to \mathrm{e}^{k_2},$$

which means that the life-span N^* of positive solution P is finite and $N^* \leq \mathrm{e}^{k_2}$.



Picture 1 graph of k_2, $u_0 \in [0.02, 1]$



Picture 2 graph of k_2, $u_0 \in [1,5]$

ii) $E(0) < 0$. From (2.1) and (2.5) we obtain that $J'(s) = -\frac{p-1}{4} a(s)^{-\frac{p+3}{4}} a'(s)$, $a'(s) > 0$, $u_s(s) > 0$ for all $s > 0$ and

$$\begin{aligned}
J'(s) &= -\frac{p-1}{2}\sqrt{\frac{2}{p+1} + E(0)\, a(s)^{-\frac{p+1}{2}} + 2a(s)^{-\frac{p+1}{2}} \int_0^s u_s(r)^2 \, \mathrm{d}r} \\
&\le -\frac{p-1}{2}\sqrt{\frac{2}{p+1} + E(0)\, a(s)^{-\frac{p+1}{2}}}, \\
J(s) &\le J(0) - \frac{p-1}{2}\int_0^s \sqrt{\frac{2}{p+1} + E(0)\, a(r)^{-\frac{p+1}{2}}} \mathrm{d}r.
\end{aligned}$$

Since $E(0) < 0$ and $a'(s) > 0$ for all $s > 0$, then

$$\begin{aligned}
J(s) &\le J(0) - \frac{p-1}{2}\int_0^s \sqrt{\frac{2}{p+1} + E(0)\, a(0)^{-\frac{p+1}{2}}} \mathrm{d}r \\
&= a(0)^{-\frac{p-1}{4}} - \frac{p-1}{2}\sqrt{\frac{2}{p+1} + E(0)\, a(0)^{-\frac{p+1}{2}}}\, s.
\end{aligned}$$

Thus, there exists a finite number

$$S_3^* \le \frac{2}{p-1} a(0)^{-\frac{p-1}{4}} \left(\frac{2}{p+1} + E(0)\, a(0)^{-\frac{p+1}{2}}\right)^{-\frac{1}{2}} := k_3,$$

such that $J(S_3^*) = 0$ and $a(s)^{-1} \to 0$ for $s \to S_3^*$, that is,

$$u(s)^{-1} \to 0 \quad \text{for} \quad s \to \mathrm{e}^{k_3}.$$

This means that the life-span S^* of u is finite and $S^* \le \mathrm{e}^{k_3}$. □

5 Estimates for the Life-Span of Positive Solution u of (1.2) under $P_1 < 0$

Finally, we estimate the life-span of positive solution u of (1.2) under $P_1 < 0$ in this section.

Theorem 5 For $P_1 < 0$, $P_0 \in \left(0, (-P_1)^{\frac{1}{p}}\right)$ we have

$$u(s) \le (P_0 - P_1 - P_0^p) + (P_1 + P_0^p)\, s - P_0^p \ln s.$$

And particularly, for $E(0)\ge 0$, then

$$u(s)\le\left(P_0^{\frac{1-p}{2}}+\frac{p-1}{2}\sqrt{\frac{2}{p+1}}\ln s\right)^{\frac{2}{1-p}}.$$

Remark This boundedness of u or P means that such enterprises will go to their minimum of performance and competitiveness if they enlarge their amount of the surveying rod enterprise's composition department number or the main unit commanders counts; if such number can not be well controlled, they will only go into bankruptcy one day.

Proof i) According to (1.2) and integrating this equation with respect to s, we get

$$u_s(s)=(P_1-P_0)+u(s)+\int_0^s u(r)^p\,\mathrm{d}r.$$

By (2.6), we have u is decreasing and

$$\begin{aligned}u_s(s)&\le(P_1-P_0)+u(s)+\int_0^s u(0)^p\,\mathrm{d}r\\&=(P_1-P_0)+u(s)+P_0^p s,\end{aligned}$$

$$\begin{aligned}\mathrm{e}^{-s}u(s)-P_0&\le(P_1-P_0)\int_0^s\mathrm{e}^{-r}\mathrm{d}r+P_0^p\int_0^s r\mathrm{e}^{-r}\mathrm{d}r\\&=(P_1-P_0)\left(1-\mathrm{e}^{-s}\right)+P_0^p\left(-s\mathrm{e}^{-s}-\mathrm{e}^{-s}+1\right);\end{aligned}$$

that is,

$$\begin{aligned}u(s)&\le(P_0-P_1)+P_1s+P_0^p(s-1-\ln s)\\&=(P_0-P_1-P_0^p)+(P_1+P_0^p)s-P_0^p\ln s.\end{aligned}$$

ii) $E(0)\ge 0$. By (2.1), we have

$$u_s(s)^2-\frac{2}{p+1}u(s)^{p+1}=E(0)+2\int_0^s u_s(r)^2\,\mathrm{d}r\ge E(0),$$

$$u_s(s)^2\ge E(0)+\frac{2}{p+1}u(s)^{p+1}\ge\frac{2}{p+1}u(s)^{p+1}.$$

By (2.6), we obtain that

$$-u_s(s)\ge\sqrt{\frac{2}{p+1}}u(s)^{\frac{p+1}{2}},$$

$$\frac{2}{p-1}\left(u(s)^{\frac{1-p}{2}}\right)_s\ge\sqrt{\frac{2}{p+1}}$$

and

$$\sqrt{\frac{2}{p+1}}s\le\frac{2}{p-1}\left(u(s)^{\frac{1-p}{2}}-u(0)^{\frac{1-p}{2}}\right),$$

$$u(s)^{\frac{1-p}{2}}\ge\left(P_0^{\frac{1-p}{2}}+\frac{p-1}{2}\sqrt{\frac{2}{p+1}}s\right).$$

Then, we know that $u(s)\le\left(P_0^{\frac{1-p}{2}}+\frac{p-1}{2}\sqrt{\frac{2}{p+1}}s\right)^{\frac{2}{1-p}}$ for all $s\ge 0$, that is,

$$u(s)\le\left(P_0^{\frac{1-p}{2}}+\frac{p-1}{2}\sqrt{\frac{2}{p+1}}\ln s\right)^{\frac{2}{1-p}}\quad\text{for all}\quad s\ge 1.$$

□

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