國立政治大學經濟學系博士論文

R&D 經濟成長模型：資本稅之探討

Capital Taxation in R&D Growth Models

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摘要

本論文有系統性地利用不同的 R&D 模型，分析了資本所得稅對經濟成長及社會福利的效果。第二章利用第一代 R&D 模型，發現資本所得稅對經濟成長於長短期下具有顯著不同的效果。第三章建立半內生成長(semi-endogenous) R&D 模型，重新檢驗 Chamely-Judd 命題是否成立。我們發現最適的資本所得稅應大於零，並且檢驗最適資本所得稅在對應不同的 R&D 外部性程度下，正資本所得稅率結論是否仍然成立。第四章利用第二代 R&D 成長模型，在具有廠商家數內生化的特性下，重新檢視第二章的結論，並發現其資本所得稅對經濟成長的效果，在長短期下仍然具有顯著不同的效果。

Abstract

This dissertation has provided a systematic analysis on the growth and welfare effects of capital taxation within distinct R&D-based growth models. In Chapter 2, we employ a first-generation R&D-based growth model to examine the effcts of capital taxation on innovation and economic growth, and find that capital taxation has drastically different effects in the short run and in the long run. In Chapter 3, we set up a semi-endogenous growth model, and examine whether the Chamley-Judd result of a zero optimal capital income tax is valid. We find that the optimal capital income tax is positive, and this result is robust with respect to varying the degrees of various types of R&D externalities. In Chapter 4, we build up a second-generation R&D-based growth model which features endogenous market structure. In line with Chapter 2, we also find that capital taxation has drastically different effects in the short run and in the long run.

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CHAPTER 1

INTRODUCTION

The linkage among capital taxation, economic growth, and social welfare has been one of the central issues in the literature on growth economics. As far as we know, in the real world the estimated effective average tax rates on capital income are around 40% in the United States and 30% in EU countries. In some countries, such as the United Kingdom and Japan, the capital income tax rates are even up to near 60%. However, how much should the capital income be taxed is another important issue that will never cease being debated by economists and policymakers.

One of the major topics in the literature on growth economics is whether capital taxation boosts or impedes economic growth. The answer is hardly conclusive from both empirical and theoretical perspectives. On the empirical side, a number of studies have found that capital taxation, such as corporate profit tax and capital gains tax, is harmful to economic growth (see e.g., Lee and Gordon, 2005; Hungerford, 2010; Arnold et al., 2011; Dahlby and Ferede, 2012), whereas other studies have found a neutral or positive growth effect of capital taxation (see e.g., Mendoza et al., 1997; Angelopoulos et al., 2007; ten Kate and Milionis, 2015). On the theoretical side, earlier studies employing an AK-type endogenous growth model show that the impact of raising the capital tax rate on long-run economic growth is negative (Judd, 1985; Chamley, 1986; King and Rebelo, 1990; Rebelo, 1991; Jones et al.,1993; Pecorino, 1993, 1994; Devereux and Love, 1994; Milesi-Ferretti and Roubini, 1998), although the quantitative magnitude could be negligibly small (Lucas, 1990; Stokey and Rebelo, 1995). However, the point to observe is that these theoretical studies are unanimously centering on capital taxation in capital-driven growth models.

Another one of the major topics is optimal capital taxation. The pioneering work by Judd (1985) and Chamley (1986) proposes that the government should only tax labor income and leave capital income untaxed in the long run. The idea of a zero optimal capital tax has then been dubbed the Chamley-Judd result, which turns out to be one of the most well-established and important benchmarks in the optimal taxation literature. A number of subsequent studies, including Chari et al. (1994), Jones et al. (1997), Atkinson et al. (1999), and Chari and Kehoe (1999), relax key assumptions in Judd (1985) and Chamley (1986), and find that their result to be quite robust. A common fact in these studies is that they focus on AK-type (capital-based) growth models.

In general, the existing studies on the linkage among capital taxation, economic growth, and social welfare can be classified into two strands of literature. The first strand emphasizes the growth engine of capital accumulation. The relevant literature in this strand includes Judd (1985), Chamley (1986), King and Rebelo (1990), Jones et al. (1993), Devereux and Love (1994), and Milesi-Ferretti and Roubini (1998), just to mention a few. The second strand instead highlights the growth engine of R&D investment. *Up till now, to the best of our knowledge, only a few studies including Lin and Russo (1999), Zeng and Zhang (2002), Haruyama and Itaya (2006), Aghion et al. (2013), Yilmaz (2013), and Chen et al. (2016) falls into this strand. As a consequence, it is obvious that, within the context of R&D-based growth models, the issue on the growth and welfare effects of capital taxation is still an area that is less discussed.* Moreover, as reported by Aghion and Howitt (2009), technological progress driven by innovation and R&D acts as a more important engine of economic growth compared to capital accumulation.[1] In view of

[1]Aghion and Howitt (2009, p.108) report that "TFP growth accounts for about two-thirds of

the fact that innovation is a crucial factor to drive up economic growth, overlooking this facet may lead to inadequate design of tax policies. This dissertation thus aims to fill this gap.

The dissertation provides a systematic analysis regarding the growth and welfare effects of capital taxation with distinct R&D-based growth models. The models this dissertation deal with include the first-generation R&D-based growth model developed by Romer (1990), the semi-endogenous growth model developed by Jones and Williams (2000), and the second-generation R&D-based growth model developed by Dinopoulos and Thompson (1998) and Peretto (1998). This dissertation is composed of five chapters, including this Introduction. The main content of each chapter can be briefly described as follows.

Chapter 2 makes an extension of the seminal workhorse R&D-based growth model developed by Romer (1990), and discusses how capital taxation affects economic growth in the short run and in the long run. We find that an increase in the capital income tax rate has both a consumption effect and a tax-shifting effect on the equilibrium growth rates of technology and output. In the long run, the tax-shifting effect dominates the consumption effect, thereby yielding an overall positive effect of capital taxation on steady-state economic growth. However, in the short run, the consumption effect becomes the dominant force causing an initial negative effect of capital taxation on the equilibrium growth rates. These contrasting effects of capital taxation at different time horizons may provide a plausible explanation for the mixed evidence in the empirical literature on capital taxation and economic growth.

Chapter 3 sets up an innovation-based growth model (semi-endogenous growth

economic growth in OECD countries, while capital deepening accounts for one third."

model) developed by Jones and Williams (2000), and uses it to examine whether the Chamley-Judd result of a zero optimal capital income tax is valid. It is found that the optimal capital income tax is more likely to be positive if labor supply is endogenous and the government size is relatively large. Moreover, by calibrating our model to the US economy, it is also found that the result of a positive optimal capital income tax is robust with respect to varying the degrees of various types of R&D externalities.

Chapter 4 constructs a second-generation R&D-based growth model developed by Dinopoulos and Thompson (1998) and Peretto (1998). The main salient feature of the second-generation R&D-based growth model is that both vertical and horizontal innovations are present simultaneously. In the vertical dimension, each of incumbent firms engages in in-house R&D to improve the quality of their specific product. In the horizontal dimension, firms enter the market through the creation of new products. It is found that, in response to a change in the capital tax rate, the long-run and short-run responses of the economic growth rate exhibit distinct patterns. In the short run where the number of firms is fixed, a higher capital income tax rate is harmful to economic growth. During the transitional process, with the number of firms adjust endogenously, economic growth keeps on rising as each of the in-house R&D firms continues to expand its market size. In the long run, with the equal counteracting strength between the short run and the transition period, capital taxation is neutral with economic growth. As a result, the same as Chapter 2, this provides a plausible explanation for the mixed empirical observations between capital taxation and economic growth.

Finally, some concluding remarks are provided in Chapter 5.

CHAPTER 2

SHORT-RUN AND LONG-RUN EFFECTS OF CAPITAL TAXATION ON INNOVATION AND ECONOMIC GROWTH

2.1 Introduction

In this chapter, we examine the effects of capital taxation on innovation and economic growth. In the literature of endogenous growth, one of the major issues is whether capital taxation stimulates or impedes growth. Earlier studies employing an AK-type endogenous growth model show that the impact of raising the capital tax rate on long-run economic growth is negative (Judd, 1985; Chamley, 1986; King and Rebelo, 1990; Rebelo, 1991; Jones *et al.*, 1993; Pecorino, 1993, 1994; Devereux and Love, 1994; Milesi-Ferretti and Roubini, 1998), although the quantitative magnitude could be negligibly small (Lucas, 1990; Stokey and Rebelo, 1995).[1] The intuition of this negative growth effect of capital taxation is that a higher capital tax rate discourages the accumulation of physical capital and is therefore detrimental to economic growth.

On the empirical side, the results are rather inconclusive. A number of empirical studies have found that capital taxation, such as corporate profit tax and capital gains tax, can be harmful to economic growth (see e.g., Lee and Gordon, 2005; Hungerford, 2010; Arnold *et al.*, 2011; Dahlby and Ferede, 2012), whereas other empirical studies have found a neutral or even positive effect of capital taxation on growth (see e.g., Mendoza *et al.*, 1997; Angelopoulos *et al.*, 2007; ten Kate and Milionis, 2015). Therefore, although the abovementioned theoretical prediction is

[1] Other than focusing on the long-run growth effect, Frankel (1998) studies the dynamics of capital taxation during the transition process.

consistent with some of the empirical studies, it seems to contrast other empirical findings in the literature.

While capital accumulation is undoubtedly an important engine of economic growth, technological progress driven by innovation and R&D also acts as an important driver for growth; see Aghion and Howitt (2009, p.109) for a discussion on data from OECD countries.[2] Recently, R&D-based growth models pioneered by Romer (1990) and Aghion and Howitt (1992) have been used to explore the interrelation between capital taxation, innovation and economic growth; see e.g., Lin and Russo (1999), Zeng and Zhang (2002), Haruyama and Itaya (2006) and Aghion *et al.* (2013). This chapter contributes to the literature by providing an analysis of both the short-run and long-run effects of capital taxation on innovation and economic growth within the seminal innovation-driven growth model in Romer (1990), which is a workhorse model in R&D-based growth theory. In our analysis, we consider different tax-shifting schemes. Specifically, we examine the growth effects of capital taxation with tax shifting from lump-sum tax and also labor income tax to capital income tax.

In the case of tax shifting from lump-sum tax to capital income tax, an increase in the capital tax rate leads to a *decrease* in the steady-state equilibrium growth rate via a *consumption* effect of capital taxation. Intuitively, a higher capital tax rate causes households to decrease their saving rate and increase their consumption rate, which in turn leads to an increase in leisure and a decrease in labor supply. Given that labor is a factor input for R&D, a smaller labor supply gives rise to a lower growth rate of technology, which in turn determines the long-run growth rates of output and capital.

[2] Aghion and Howitt (2009, p.108) report that "TFP growth accounts for about two-thirds of economic growth in OECD countries, while capital deepening accounts for one third."

In the case of tax shifting from labor income tax to capital income tax, an increase in the capital tax rate leads to an *increase* in the steady-state equilibrium growth rate via a *tax-shifting* effect of capital taxation. Intuitively, an increase in the capital income tax rate allows the labor income tax rate to decrease, which in turn leads to a decrease in leisure and an increase in labor supply. The larger labor supply gives rise to higher growth rates of technology, output and even capital despite the lower capital-investment rate caused by the higher capital tax rate. Although the previously mentioned consumption effect of capital taxation is also present, it is dominated by the tax-shifting effect in the long run. However, we find that the relative magnitude of these two effects becomes very different in the short run.

We calibrate the model to aggregate data in the US to provide a quantitative analysis on the dynamic effects of capital taxation on economic growth. We consider the case of tax shifting from labor income tax to capital income tax and find that an increase in the capital tax rate leads to a short-run *decrease* in the equilibrium growth rates of technology and output and a gradual convergence to the higher long-run growth rates of technology and output. The reason for these contrasting short-run and long-run effects is that the consumption effect of capital taxation is relatively strong in the short run. Intuitively, an increase in the capital income tax rate leads to a decrease in the steady-state equilibrium capital-technology ratio. Before the economy reaches this new steady-state capital-technology ratio, households drastically cut down their saving rate below its new steady-state level, which in turn increases their consumption rate substantially. This substantial increase in consumption leads to a substantial increase in leisure and a substantial decrease in labor supply, which in turn reduces temporarily the equilibrium growth rates of technology and output. In the long run, the effect of a lower wage-income tax rate becomes the dominant force and instead raises the supply of labor, which in turn increases the steady-state equilibrium growth rates of technology and output.

Our paper is most closely related to recent studies on taxation and economic growth in the R&D-based growth model. Zeng and Zhang (2002) show that the long-run growth rate is independent of labor income tax and consumption tax but decreasing in capital income tax. In contrast, Lin and Russo (1999) analyze how the taxation of different sources of capital income affects long-run growth and find that a higher capital income tax rate for innovative firms could be growth-enhancing if the tax system permits tax credits for R&D spending. Moreover, by focusing on the stability analysis of equilibria, Haruyama and Itaya (2006) also show that the growth effect of taxing capital income is positive when the economy exhibits indeterminacy. Although these two papers find that capital taxation and economic growth may exhibit a positive relationship, our paper departs from them in highlighting the contrasting dynamic effects of capital taxation on economic growth. More recently, Aghion *et al.* (2013) and Hong (2014) adopt a quality-ladder R&D-based growth model to investigate optimal capital taxation. Their primary focus, however, is on the normative analysis with respect to the Chamley-Judd (Chamley 1986; Judd 1985) result (i.e., the optimal capital tax is zero), while the present paper focuses on the positive analysis regarding the growth effect of capital taxation. Furthermore, their analysis does not deal with the case in which innovation is driven by R&D labor (e.g., scientists and engineers). When R&D uses labor as the factor input, we find that the effects of capital taxation are drastically different at different time horizons. This finding may provide a plausible explanation for the mixed evidence in the empirical literature on capital taxation and economic growth.

The remainder of this chapter is organized as follows. In Section 2.2, we describe the basic model structure. In Section 2.3, we investigate the growth effects of capital taxation. In Section 2.4, we calibrate the model to provide a quantitative analysis of capital taxation. Finally, some concluding remarks are discussed in Section 2.5.

2.2 The model

The model that we consider is an extension of the seminal workhorse R&D-based growth model from Romer (1990).[3] In the Romer model, R&D investment creates new varieties of intermediate goods. We extend the model by introducing endogenous labor supply and distortionary income taxes. In what follows, we describe the model structure in turn.

2.2.1 *Household*

The economy is inhabited by a representative household. Population is stationary and normalized to unity. The household has one unit of time that can be allocated between leisure and production. The representative household's lifetime utility is given as:[4]

$$U = \int_0^\infty e^{-\rho t}[\ln C + \theta \frac{(1-L)^{1-\eta}}{1-\eta}]\, dt, \tag{1}$$

[3] In the case of extending the model into a scale-invariant semi-endogenous growth model as in Jones (1995), the long-run growth effect of capital taxation simply becomes a level effect. In other words, instead of increasing (decreasing) the growth rate of technology, capital taxation increases (decreases) the level of technology in the long run.

[4] For notational simplicity, we drop the time subscript.

where the parameter $\rho > 0$ is the household's subjective discount rate and the parameter $\theta > 0$ determines the disutility of labor supply. $\eta \geq 0$ determines the Frisch elasticity of labor supply. The utility is increasing in consumption C and decreasing in labor supply $L \in (0, 1)$.

Two points regarding the utility function in eq. (1) should be noted. First, to make our analysis tractable, the household is specified to have a quasi-linear utility function. In the quantitative analysis in Section 2.4, we will consider a more general utility function in order to examine the robustness of our results. Second, as pointed out by Hansen (1985) and Rogerson (1988), the linearity in work hours in the utility function can be justified as capturing indivisible labor.

The representative household maximizes its lifetime utility subject to the budget constraint:

$$\dot{K} + \dot{a} = (r_A + \dot{V}/V)a + (1 - \tau_K)r_K K + (1 - \tau_L)wL - C - Z. \tag{2}$$

The variable K denotes the stock of physical capital. The variable $a\,(= VA)$ denotes the value of equity shares of monopolistic firms, in which A is the number of monopolistic firms and V is the market value of an invented variety, w is the wage rate. r_A is the rate of dividends, $\dot{V}/V$ is the rate of gain or loss of an invented variety, and r_K is the capital rental rate.[5] The policy instrument Z is a lump-sum tax.[6] The other policy instruments $\{\tau_L, \tau_K\} < 1$ are respectively the labor and capital income tax rates.[7]

[5]For simplicity, we assume zero capital depreciation rate.

[6]We allow for the presence of a lump tax simply to explore the implications of different tax-shifting schemes. Our main results focus on the more realistic case of $Z = 0$.

[7]In our analysis, we focus on the case in which $\tau_K > 0$; see for example Zeng and Zhang (2007) and Chu *et al.* (2016), who examine the effects of subsidy policies in the R&D-based growth model.

The rates of return on the two assets, physical capital and equity shares, must follow a no-arbitrage condition at any time:

$$r_A + \dot{V}/V = (1 - \tau_K) r_K \tag{3a}$$

We denote the common net return on both assets as r, i.e., $r \equiv r_A + \dot{V}/V = (1 - \tau_K) r_K$.

By solving the household's optimization problem, we can easily derive the typical Keynes-Ramsey rules:

$$\frac{\dot{C}}{C} = (1 - \tau_K) r_K - \rho, \tag{3b}$$

and also the optimality condition for labor supply, which is in the form of a horizontal labor supply curve given the quasi-linear utility function in eq. (1):

$$\theta (1 - L)^{-\eta} = \lambda (1 - \tau_L) w. \tag{4}$$

2.2.2 *Final goods*

There is a single final good Y, which is produced by combining labor and a continuum of intermediate goods, according to the following aggregator:

$$Y = L_Y^{1-\alpha} \int_0^A x_i^{\alpha} di, \tag{5}$$

where L_Y is the labor input in final goods production, x_i for $i \in [0, A]$ is the intermediate good of type i, and A is the number of varieties of intermediate goods. The final good is treated as the *numeraire*, and hence in what follows its price is normalized to unity. We assume that the final goods sector is perfectly competitive. Profit maximization of the final goods firms yields the following conditional demand functions for labor input and intermediate goods:

$$L_Y = (1-\alpha)Y/w, \tag{6}$$

$$x_i = L_Y \left(\alpha/p_i\right)^{\frac{1}{1-\alpha}}, \tag{7}$$

where p_i is the price of x_i relative to final goods.

2.2.3 *Intermediate goods*

Each intermediate good is produced by a monopolist who owns a perpetually protected patent for that good. Following Romer (1990), capital is the factor input for producing intermediate goods, and the technology is simply a linear one-to-one function. That is, the production function is expressed as $x_i = k_i$, where k_i is the capital input used by intermediate firm i. Accordingly, the profit of intermediate goods firm i is:

$$\pi_i = p_i x_i - r_K k_i. \tag{8}$$

Profit maximization subject to the conditional demand function for intermediate goods firm i yields the following markup-pricing rule:

$$p_i = \frac{r_K}{\alpha} > r_K. \tag{9}$$

Equation (9) implies that the level of price is the same across intermediate goods firms. Based on eq. (7) and the production function $x_i = k_i$, we have a symmetric equilibrium among intermediate firms; i.e., $x_i = x$ and $k_i = k$. Then, we can obtain the following profit function of intermediate goods firms:

$$\pi_i = \pi = \frac{(1-\alpha)\alpha Y}{A}. \tag{10}$$

2.2.4 *R&D*

In the R&D sector, the familiar no-arbitrage condition for the value of a variety V is:

$$rV = \pi + \dot{V}. \tag{11}$$

Equation (11) states that, for each variety, the rate of return on an invention must be equal to the sum of the monopolistic profit and capital gain (or loss) . As in Romer (1990), labor is the factor input of R&D. The innovation function of new varieties is given by:

$$\dot{A} = \phi A L_A, \tag{12}$$

where $\phi > 0$ is the R&D productivity parameter and L_A denotes R&D labor. Given free entry into the R&D sector, the zero-profit condition of R&D is

$$\dot{A}V = wL_A \Leftrightarrow \phi AV = w. \tag{13}$$

2.2.5 *Government*

The government collects taxes, including capital income tax, labor income tax, and lump-sum tax, to finance its public spending. At any instant of time, the government budget constraint can be expressed as:

$$\tau_K r_K K + \tau_L wL + Z = G. \tag{14}$$

The variable G denotes government spending, which is assumed to be a fixed proportion $\beta \in (0, 1)$ of final output such that

$$G = \beta Y. \tag{15}$$

2.2.6 *Aggregation*

Since the intermediate firms are symmetric, the total amount of capital is $K = Ak_i = Ak$. Given $x_i = k_i$, $x_i = x$, $k_i = k$, and $K = Ak$, the final output production function in eq. (5) can then be expressed as:

$$Y = A^{1-\alpha} K^{\alpha} L_Y^{1-\alpha}. \tag{16}$$

After some calculations using eqs (2), (6), (7), (11)-(14), and (16), we can derive the resource constraint in this economy:

$$\dot{K} = Y - C - G. \tag{17}$$

2.2.7 *Decentralized equilibrium and the balanced-growth path*

The decentralized equilibrium is a sequence of allocations $\{C, K, A, Y, L, L_Y, L_A, x, G\}_{t=0}^{\infty}$, prices $\{w, r, r_K, p_i, V\}_{t=0}^{\infty}$, and policies $\{\tau_K, \tau_L, Z\}$, such that at any instant of time:

a. households maximize lifetime utility (1) taking prices and policies as given;

b. competitive final goods firms choose $\{x, L_Y\}$ to maximize profit taking prices as given;

c. monopolistic intermediate firms $i \in [0, A]$ choose $\{k_i, p_i\}$ to maximize profit taking r_K as given;

d. R&D firms choose L_A to maximize profit taking $\{V, w\}$ as given;

e. the market for final goods clears, i.e., $\dot{K} = Y - C - G$;

f. the labor market clears, i.e., $L = L_A + L_Y$;

g. the government budget constraint is balanced, i.e., $\tau_K r_K K + \tau_L w L + Z = G$.

The balanced growth path is characterized by a set of constant growth rates of all economic variables. Let γ denote the growth rate of technology and a "$\sim$" over the variable denote its steady-state value. Along the balanced growth path, we have

$$\frac{\dot{K}}{K} = \frac{\dot{Y}}{Y} = \frac{\dot{C}}{C} = \frac{\dot{w}}{w} = \frac{\dot{A}}{A} = \tilde{\gamma},\ \dot{L} = \dot{L}_Y = \dot{L}_A = \dot{V} = 0. \tag{17a}$$

2.3 Long-run growth effects of capital taxation

We now turn to examine the long-run growth effects of the capital tax rate. In this section to obtain analytical solutions, we assume that $\eta = 0$. As a result, to maintain a constant proportion of government spending, raising the capital tax must be accompanied by a reduction in another tax. As revealed in eq. (14), this can be either a reduction in the lump-sum tax (if it is available) or a reduction in the labor income tax (if the lump-sum tax is not available). In the analysis that follows, we deal with each of the two scenarios in turn.

2.3.1 *Tax shifting from lump-sum tax to capital income tax*

Equipped with the definition of the decentralized equilibrium in Section 2.2.7,and defining $\omega = w/A$, $c = C/A$, and $z = Z/A$, we can express the steady-state equilibrium conditions as follows:

$$
\begin{aligned}
\tilde{\gamma} &= (1-\tau_K)\tilde{r}_K - \rho, & \text{(18a)}\\
\tilde{\omega} &= \theta\tilde{c}/(1-\tau_L), & \text{(18b)}\\
\tilde{L}_Y &= (1-\alpha)\tilde{x}^{\alpha}\tilde{L}_Y^{1-\alpha}/\tilde{\omega}, & \text{(18c)}\\
\tilde{x} &= \tilde{L}_Y(\alpha^2/\tilde{r}_K)^{1/(1-\alpha)}, & \text{(18d)}\\
\tilde{r} &= \phi\alpha\tilde{L}_Y, & \text{(18e)}\\
\tilde{r} &= (1-\tau_K)\tilde{r}_K, & \text{(18f)}\\
\tilde{\gamma} &= \phi\tilde{L}_A, & \text{(18g)}\\
\tilde{L} &= \tilde{L}_Y + \tilde{L}_A, & \text{(18h)}\\
\tilde{\gamma} &= (1-\beta)\tilde{x}^{\alpha-1}\tilde{L}_Y^{1-\alpha} - \tilde{c}/\tilde{x}, & \text{(18i)}\\
\tau_K\tilde{r}_K\tilde{x} + \tau_L\tilde{\omega}\tilde{L} + \tilde{z} &= \beta\tilde{x}^{\alpha}\tilde{L}_Y^{1-\alpha}, & \text{(18j)}
\end{aligned}
$$

in which ten equations are used to solve ten unknowns $\tilde{\gamma}$, $\tilde{r}_K$, $\tilde{L}_Y$, $\tilde{L}_A$, $\tilde{L}$, $\tilde{\omega}$, $\tilde{c}$, $\tilde{x}$, $\tilde{r}$ and $\tilde{z}$. We briefly discuss how we obtain eqs (18). Equation (18a) is derived from the usual Keynes-Ramsey rule (3b). Equation (18b) is derived from the optimality condition for labor supply (4). Equations (18c) and (18d) are respectively the demand functions for final-goods labor and intermediate goods, (6) and (7). Equation (18e) is derived from inserting $\dot{V} = 0$ into the no-arbitrage condition in the R&D sector (11), and by using eqs (6), (10) and (13). Equation (18f) is the no-arbitrage condition of asset. Equation (18g) is derived from the innovation function of varieties (12). Equation (18h) is the labor-market clearing condition. Equation (18i) is derived from dividing both sides of the resource constraint (17) by A and using the condition $Ax = K$. Equation (18j) is derived from dividing both sides of the government constraint (14) by A and using the condition $G = \beta Y$.

We first use eqs (18a), (18e) and (18f)-(18h) to eliminate $\{\tilde{r}, \tilde{\gamma}, \tilde{r}_K\}$ and express

$\{\tilde{L}_Y, \tilde{L}_A\}$ as functions of $\tilde{L}$ given by

$$\tilde{L}_Y = \frac{\tilde{L} + \rho/\phi}{1+\alpha},$$

$$\tilde{L}_A = \frac{\alpha\tilde{L} - \rho/\phi}{1+\alpha}.$$

These two equations indicate a positive relationship between $\{\tilde{L}_A, \tilde{L}_Y\}$ and $\tilde{L}$. Moreover, from the previous condition for $\tilde{L}_A$, we can derive the condition $\tilde{\gamma} = (\alpha\phi\tilde{L} - \rho)/(1+\alpha)$, which shows that the steady-state equilibrium growth rate of technology is increasing in $\tilde{L}$. Thus, we have

$$sgn\left(\frac{\partial\tilde{\gamma}}{\partial\tau_K}\right) = sgn\left(\frac{\partial\tilde{L}_A}{\partial\tau_K}\right) = sgn\left(\frac{\partial\tilde{L}}{\partial\tau_K}\right). \tag{19}$$

Accordingly, to investigate the growth effect of the capital tax rate, it is convenient to draw an inference from examining the effect of the capital tax rate on labor $\tilde{L}$.

We now derive an equilibrium expression of labor $\tilde{L}$. By using eqs (8) and (9), we have $\pi = (\frac{1}{\alpha} - 1)\tilde{r}_K K/A$. This expression together with eq. (10) implies that $r_K K = \alpha^2 Y$. Then, dividing both sides of eq. (17) by Y yields

$$\tilde{\gamma}\frac{K}{Y} = 1 - \beta - \frac{C}{Y}.$$

By inserting $C/Y = (1-\tau_L)(1-\alpha)/(\theta\tilde{L}_Y)$, which is derived from eqs (4) and (6), and $r_K K = \alpha^2 Y$ into the above equation and using eqs (18e), (18f), and (18g) along with the conditions for $\tilde{L}_Y$ and $\tilde{L}_A$, we can obtain the following equation with one unknown $\tilde{L}$:

$$\left[1 - \frac{\rho(1+\alpha)}{\alpha\phi(\tilde{L} + \rho/\phi)}\right]\alpha^2(1-\tau_K) = 1 - \beta - \frac{(1-\tau_L)(1-\alpha)(1+\alpha)}{\theta(\tilde{L} + \rho/\phi)}.$$

Simplifying this equation yields

$$\tilde{L} = \frac{1}{1 - \Phi(\tau_K)}\left[\frac{1-\tau_L}{\theta} - \frac{\alpha(1-\tau_K)}{(1-\alpha)}\frac{\rho}{\phi}\right] - \frac{\rho}{\phi}, \tag{20}$$

where $\Phi(\tau_K) \equiv (\beta - \alpha^2\tau_K)/(1-\alpha^2)$ is a composite parameter and τ_L is an exogenous policy parameter. Then, from eq. (20), we can obtain the following relationship:

$$\frac{\partial \tilde{L}}{\partial \tau_K} = -\frac{\alpha^2}{(1-\alpha^2)[1-\Phi(\tau_K)]^2}\left[\frac{1-\tau_L}{\theta} - \frac{\alpha(1-\tau_K)}{(1-\alpha)}\frac{\rho}{\phi} - [1-\Phi(\tau_K)]\frac{1+\alpha}{\alpha}\frac{\rho}{\phi}\right],$$

which can be further simplified to[8]

$$\frac{\partial \tilde{L}}{\partial \tau_K} = -\frac{\alpha[(1+\alpha)\tilde{L}_A + 2\alpha\rho/\phi]}{(1-\alpha^2)[1-\Phi(\tau_K)]} < 0. \tag{21}$$

From eqs (19) and (21), we have established the following proposition:

Proposition 1 *In the case of tax shifting from lump-sum tax to capital income tax, raising the capital income tax rate reduces the steady-state equilibrium growth rate.*

Equation (19) is the key to understanding Proposition 1. It essentially says that the effect of the capital tax rate on long-run growth hinges on its effect on labor $\tilde{L}$. When the capital tax rate is higher, households tend to reduce their investment rate and increase their consumption rate. The increase in consumption raises leisure and reduces labor supply (by shifting up the horizontal labor supply curve). Therefore, a higher capital tax rate reduces the equilibrium levels of labor input, R&D labor and economic growth.

[8]The following reasoning ensures that $1-\Phi(\tau_K) = [1-\beta-\alpha^2(1-\tau_K)]/(1-\alpha^2) > 0$. The steady-state consumption-output ratio is $C/Y = 1-\beta-\alpha^2(1-\tau_K)+\alpha^2(1-\tau_K)\rho/(\tilde{\gamma}+\rho)$. Therefore, $\lim_{\rho\to 0} C/Y = 1-\beta-\alpha^2(1-\tau_K)$. In other words, one can restrict $1-\Phi(\tau_K) > 0$ by appealing to the fact that $C/Y > 0$ for all values of ρ.

2.3.2 *Tax shifting from labor income tax to capital income tax*

A lump-sum tax is not a realistic description in most economies. In this subsection, we therefore set aside the possibility of a lump-sum tax and deal with the more realistic case in which a rise in the capital tax rate is coupled with a reduction in another distortionary tax. This kind of tax shifting has been extensively investigated in the literature on factor taxation; see e.g., Judd (1985), Chamley (1986), Niepelt (2004), Aghion *et al.* (2013) and Chen and Lu (2013). Under such a situation we drop $\tilde{z}$ from the model in this subsection. Thus, eq. (18j) is rewritten as:

$$\tau_K \tilde{r}_K \tilde{x} + \tilde{\tau}_L \tilde{\omega} \tilde{L} = \beta \tilde{x}^{\alpha} \tilde{L}_Y^{1-\alpha}. \tag{22}$$

It is useful to note that in eq. (22) the labor income tax rate $\tilde{\tau}_L$ becomes an endogenous variable because it needs to adjust in response to a change in the capital tax rate.

The macroeconomy is now described by eqs (18a)-(18i) and (22) from which we solve for ten unknowns $\tilde{\gamma}$, $\tilde{r}_K$, $\tilde{L}_Y$, $\tilde{L}_A$, $\tilde{L}$, $\tilde{\omega}$, $\tilde{c}$, $\tilde{x}$, $\tilde{r}$ and $\tilde{\tau}_L$. By arranging eq. (22) with eqs (6), (16), (18c), and the condition $r_K K = \alpha^2 Y$, we can obtain

$$\tilde{\tau}_L = \frac{(\beta - \alpha^2 \tau_K)}{1-\alpha} \frac{\tilde{L}_Y}{\tilde{L}} = \left(1 + \frac{\rho}{\phi \tilde{L}}\right) \Phi(\tau_K),$$

where the second equality uses $\tilde{L}_Y = (\tilde{L} + \rho/\phi)/(1+\alpha)$. Using the above condition and eq. (20), we can solve the two unknowns $\{\tilde{L}, \tilde{\tau}_L\}$ and obtain the following quadratic equation:

$$\frac{\phi}{\rho}\tilde{L}^2 - \left[\frac{\phi}{\rho\theta} - 1 - \frac{\alpha(1-\tau_K)}{(1-\alpha)[1-\Phi(\tau_K)]}\right]\tilde{L} + \frac{\Phi(\tau_K)}{[1-\Phi(\tau_K)]\theta} = 0.$$

This quadratic equation has two solutions, denoted as $\tilde{L}_1$ and $\tilde{L}_2$, which are given by:

$$\tilde{L}_1 = \frac{B(\tau_K) + \sqrt{B(\tau_K)^2 - 4\Phi(\tau_K)\phi/\{[1-\Phi(\tau_K)]\rho\theta\}}}{2\phi/\rho}, \tag{23a}$$

$$\tilde{L}_2 = \frac{B(\tau_K) - \sqrt{B(\tau_K)^2 - 4\Phi(\tau_K)\phi/\{[1-\Phi(\tau_K)]\rho\theta\}}}{2\phi/\rho}, \tag{23b}$$

where $B(\tau_K) \equiv \phi/(\rho\theta) - 1 - \alpha(1-\tau_K)/\{(1-\alpha)[1-\Phi(\tau_K)]\}$ is a composite parameter.[9]

To ensure that $\tilde{L}$ is positive, we assume that the set of parameters jointly satisfies the condition $B > \sqrt{4\Phi\phi/[(1-\Phi)\rho\theta]}$. Moreover, we restrict our analysis to the case where an increase in the capital tax rate is coupled with a decrease in the labor tax rate. By doing so, we can show that $\tilde{L}_1$ is the only possible solution to this system.[10] From eq. (23a), we can derive the relationship:

$$\frac{\partial \tilde{L}_1}{\partial \tau_K} = \frac{\rho}{2\phi}\left\{\frac{\partial B}{\partial \tau_K} + \frac{B\partial B/\partial\tau_K + 2\phi\alpha^2/[(1-\alpha^2)(1-\Phi)^2\rho\theta]}{\sqrt{B^2 - 4\Phi\phi/[(1-\Phi)\rho\theta]}}\right\} > 0 \tag{24}$$

where $\partial B/\partial\tau_K = \alpha\left[1-\Phi+\alpha^2(1-\tau_K)/(1-\alpha^2)\right]/\{(1-\alpha)(1-\Phi)^2\} > 0$. The result in eq. (24) leads us to establish the following proposition:

Proposition 2 *In the case of tax shifting from labor income tax to capital income tax, raising the capital income tax rate increases the steady-state equilibrium growth rate.*

It would not be difficult to understand the intuition underlying the positive growth effect given that we have already shown the importance of equilibrium

[9] For notational simplicity, we suppress the argument of $\Phi(\tau_K)$ and $B(\tau_K)$ in the following equations.

[10] Based on the definition of tax shifting, an increase in one tax rate should be coupled with a fall in another tax rate. In Appendix 2.A, we will show that when $L = \tilde{L}_2$, to hold a constant proportion of the government spending, the labor tax rate actually increases in response to an increase in the capital tax rate. In this paper, we rule out this unrealistic case and only focus on the solution $L = \tilde{L}_1$.

labor input on economic growth from previous discussion. In the present case, there are two conflicting effects on labor supply. The first is the consumption effect that we discussed in Proposition 1; i.e., raising the capital tax rate induces the households to lower the investment rate and increase the consumption rate, which in turn reduces labor supply. The second effect emerges from the channel of shifting taxes from labor income to capital income. A rise in the capital income tax rate leads to a reduction in the labor income tax rate, which tends to boost labor supply. In particular, this latter tax-shifting effect has a more powerful direct impact on the labor market so that it dominates the former one. As a result, the net effect is positive such that a higher capital income tax rate stimulates economic growth in the long run.

2.4 Quantitative analysis

To examine the robustness of our results, we generalize the utility function as follows:

$$U = \int_0^\infty \left[\ln C + \theta \frac{(1-L)^{1-\eta}}{1-\eta} \right] e^{-\rho t} dt, \tag{25}$$

where $\eta \geq 0$ determines the Frisch elasticity of labor supply. Equation (25) nests eq. (1) as a special case when $\eta = 0$. The model features 7 parameters: $\{\rho, \alpha, \eta, \beta, \theta, \phi, \tau_K\}$. We consider the following standard parameter values or empirical moments in the literature. First, we set the discount rate to $\rho = 0.04$ and the capital share to $\alpha = 0.30$. Second, we set $\eta = 1.67$, which implies a Frisch elasticity of 1.2; see Chetty *et al.* (2011). Third, in line with Belo *et al.* (2013), the government spending ratio is set to $\beta = 0.20$. Fourth, to obtain a leisure time of two-thirds (i.e., $L = 1/3$), we set $\theta = 1.17$. Fifth, to generate a steady-state output growth rate of 1.92%, which is the per capita long-run growth rate of the

US economy, we set $\phi = 0.65$. Finally, the benchmark value of the capital tax rate is set to $\tau_K = 0.36$; see for example Lucas (1990). The parameter values are summarized below.

Table 2.1: Calibrated parameter values

ρ	θ	η	α	ϕ	β	τ_K
0.04	1.17	1.67	0.30	0.65	0.20	0.36

Figure 2.1 presents the growth effects of varying the capital income tax rate from 0 to 0.6. We can clearly see that, as the capital tax rate increases, the steady-state equilibrium growth rate increases. From this illustrative numerical exercise, we find that if the government raises the capital tax rate from the benchmark value of 36% to a hypothetical value of 50%, the steady-state equilibrium growth rate increases from 1.92% to 2.02%. The intuition can be explained as follows. Although an increase in the capital tax rate exerts a negative effect on economic growth by depressing capital accumulation, it also causes a fall in the labor income tax rate, which boosts labor supply and thus is beneficial to R&D and economic growth. In the long run, the latter effect dominates. Consequently, the steady-state equilibrium growth rate increases in response to a rise in the capital income tax rate.[11]

In the rest of this section, we simulate the transition dynamics of an increase in the capital income tax rate. The dynamic system is presented in Appendix 2.B. We consider the case of an increase in the capital income tax rate by one percentage point (i.e., from 36% to 37%).[12] First of all, the higher rate of capital taxation

[11] Our simulation result is robust if we introduce dividend income taxes into our model, see Appendix 2.C.

[12] In the case of a larger increase in the capital income tax rate, the qualitative pattern of the transitional paths of variables remains the same. Results are available upon request.

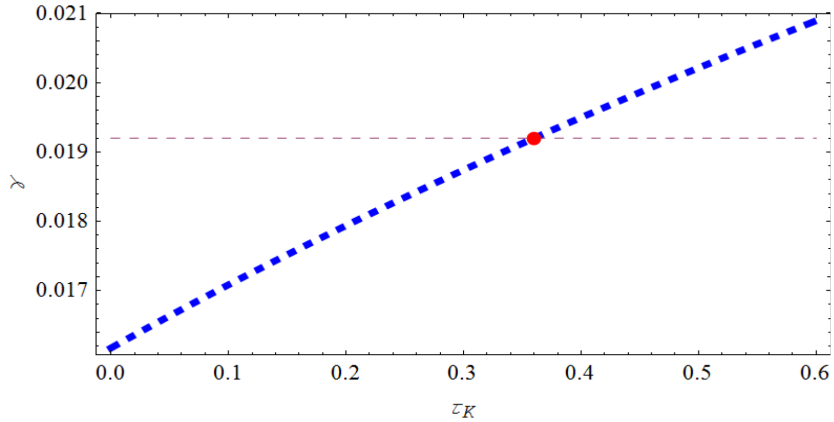


Figure 2.1: Long-run growth effect of capital taxation

leads to a decrease in the investment rate and an increase in the consumption rate as shown in Figures 2.2 and 2.3, where investment $I = \dot{K}$.

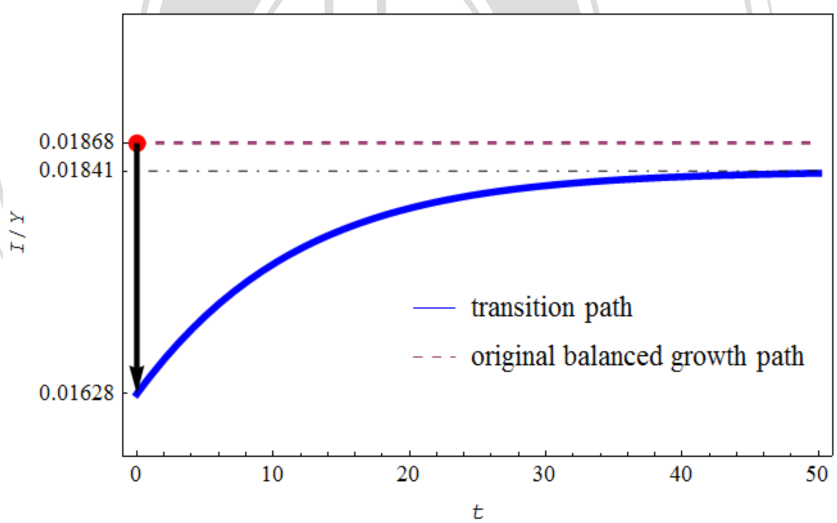


Figure 2.2: Transition path of the investment rate

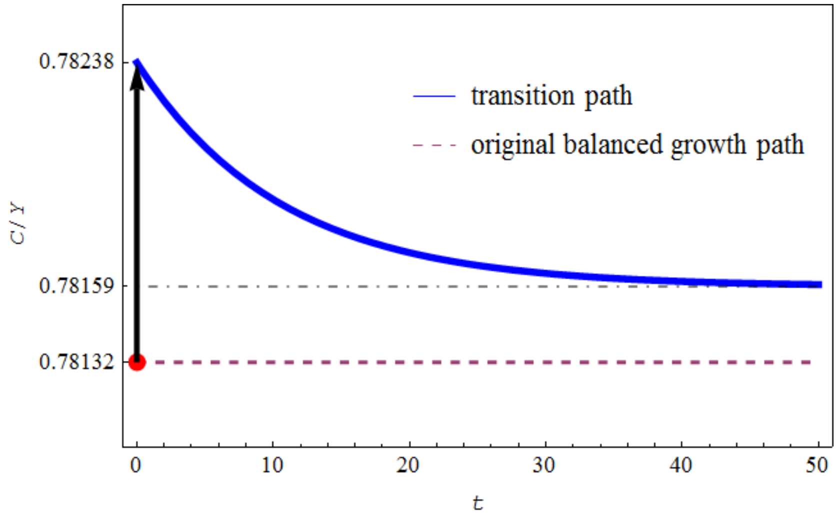


Figure 2.3: Transition path of the consumption rate

The lower capital-investment rate gives rise to an initial fall in the capital growth rate as shown in Figure 2.4, which contributes to an initial fall in the output growth rate as we will show later. The rise in the consumption rate increases leisure and decreases labor supply as shown in Figure 2.5. This decrease in labor supply reduces the amount of factor input available for R&D. As a result, the growth rate of technology also decreases initially as shown in Figure 2.6.

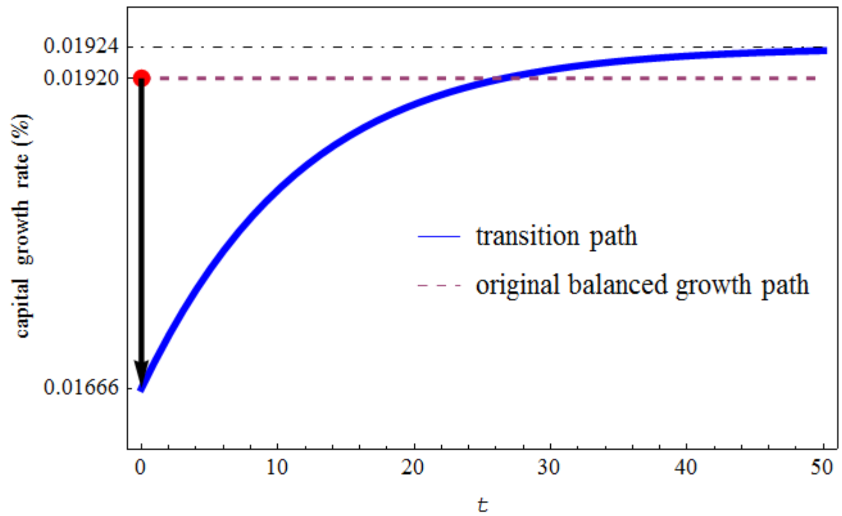


Figure 2.4: Transition path of the capital growth rate

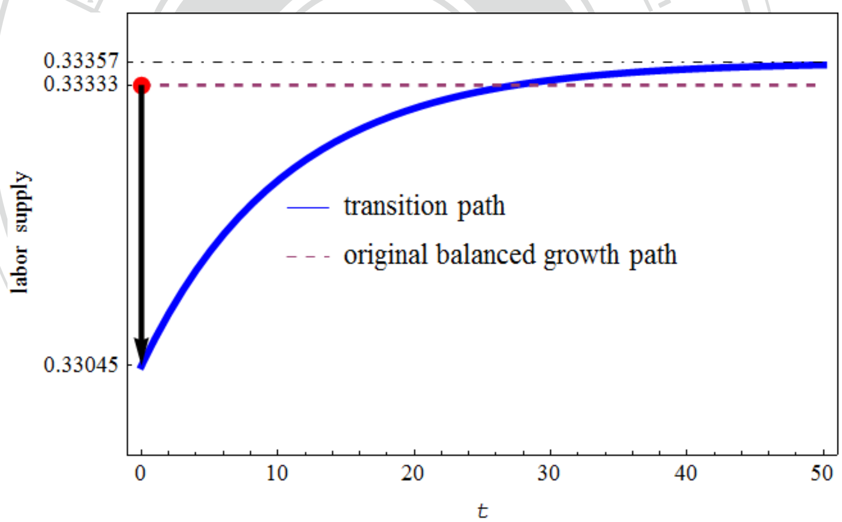


Figure 2.5: Transition path of labor supply

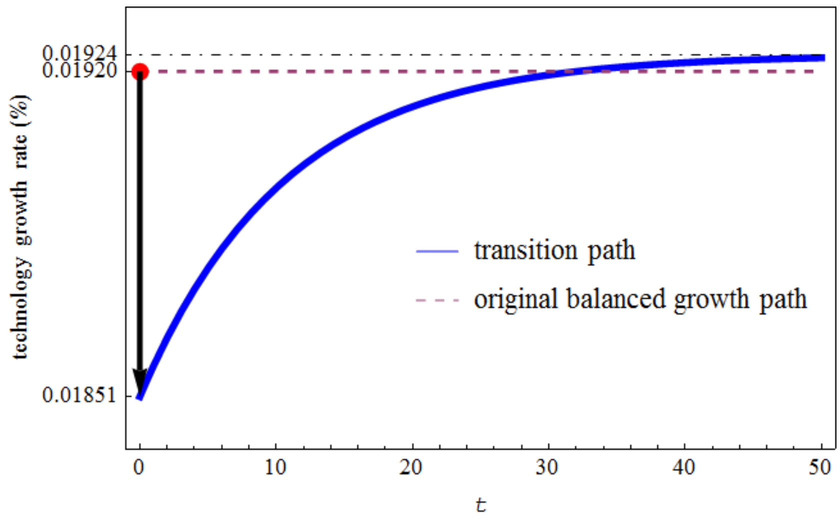


Figure 2.6: Transition path of the technology growth rate

Although tax shifting resulting from a higher capital income tax rate gives rise to a lower labor income tax rate, this effect is weak in the short run. However, it becomes a stronger force in the long run as shown in Figure 2.7. As a result, labor supply eventually rises above the original level, which in turn leads to a higher steady-state equilibrium growth rate of technology. Therefore, the initial drop in the growth rates of output and capital is followed by a subsequent increase. In the long run, the steady-state equilibrium growth rate of output is higher than the initial steady-state equilibrium growth rate as shown in Figure 2.8. To sum up, the reason for the contrasting short-run and long-run effects of capital taxation on economic growth is that the consumption effect is stronger (weaker) than the tax-shifting effect in the short (long) run.

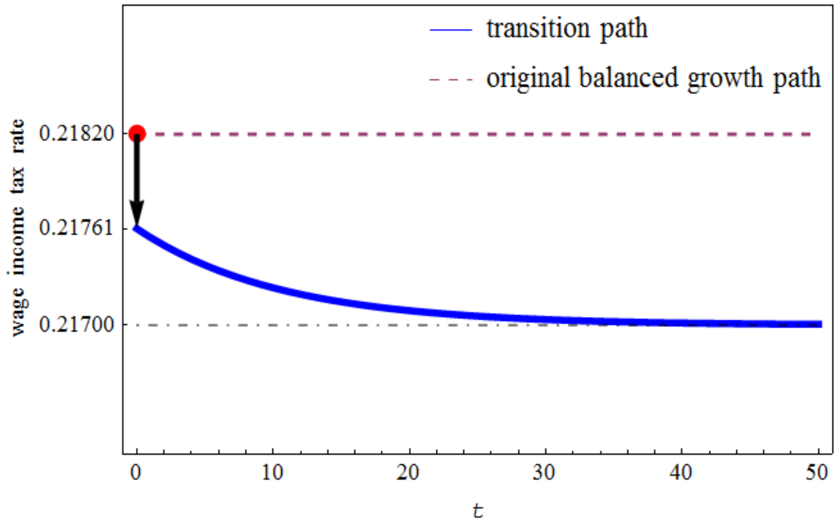


Figure 2.7: Transition path of the labor tax rate

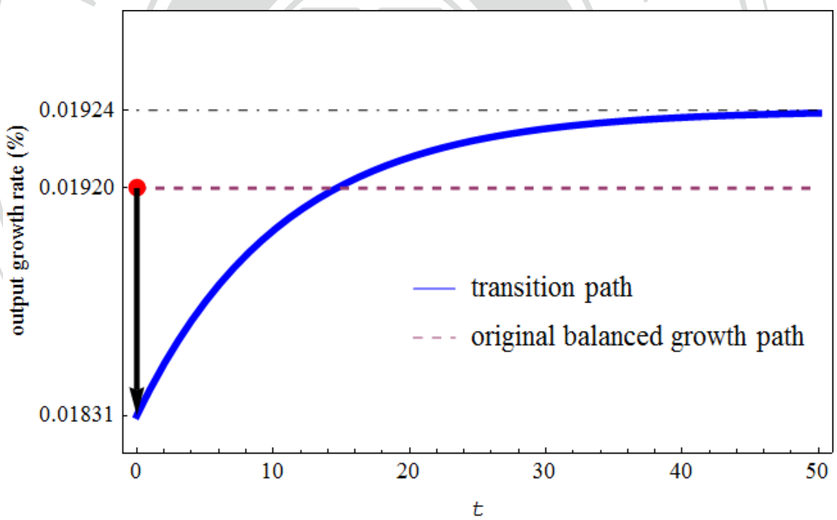


Figure 2.8: Transition path of the output growth rate

2.5 Conclusion

In this chapter, we have explored the short-run and long-run effects of capital taxation on innovation and economic growth. Our results can be summarized as follows. An increase in the capital income tax rate has both a positive tax-shifting effect and a negative consumption effect on innovation and economic growth. In the long run, increasing the capital tax rate has an unambiguously positive effect on the steady-state equilibrium growth rate because the positive tax-shifting effect strictly dominates the negative consumption effect. However, along the transitional path, increasing the capital tax rate first decreases the equilibrium growth rates of technology and output before these growth rates converge to a higher steady-state equilibrium level. These contrasting implications of capital taxation on economic growth suggest that a complete empirical analysis of capital taxation and economic growth needs to take into consideration the possibility that the effects of capital taxation change sign at different time horizons.

Appendix 2.A

The system has ten equations, (18a)-(18i), and (22). After some calculations, we can derive the following expressions for $\tilde{L}$ and $\tilde{\tau}_L$:

$$\frac{\phi}{\rho}\tilde{L}^2 - \left[\frac{\phi}{\rho\theta} - 1 - \frac{\alpha(1-\tau_K)}{(1-\alpha)(1-\Phi)}\right]\tilde{L} + \frac{\Phi}{(1-\Phi)\theta} = 0, \tag{A1}$$

$$\tilde{\tau}_L = [1 + \rho/(\phi\tilde{L})]\Phi, \tag{A2}$$

where $\Phi \equiv (\beta - \alpha^2\tau_K)/(1-\alpha^2)$. Equation (A1) gives the two solutions for $\tilde{L}$:

$$\tilde{L}_1 = \frac{\rho\left(B + \sqrt{B^2 - 4\Phi\phi/[(1-\Phi)\rho\theta]}\right)}{2\phi}, \tag{A3}$$

$$\tilde{L}_2 = \frac{\rho\left(B - \sqrt{B^2 - 4\Phi\phi/[(1-\Phi)\rho\theta]}\right)}{2\phi}, \tag{A4}$$

where $B \equiv \phi/(\rho\theta) - 1 - \alpha(1-\tau_K)/[(1-\alpha)(1-\Phi)]$.

As mentioned in the main text, our analysis focuses on the case where the notion of tax shifting is sustained. That is, we impose the condition $\partial\tilde{\tau}_L/\partial\tau_K < 0$. We can then show that the condition $\partial\tilde{\tau}_L/\partial\tau_K < 0$ does not hold if $L = \tilde{L}_2$.

By plugging $\tilde{L}_2$ into eq. (A2) and differentiating it with respect to τ_K yields:

$$\left.\frac{\partial\tilde{\tau}_L}{\partial\tau_K}\right|_{L=\tilde{L}_2} = \left[\frac{\alpha(1-\alpha+\alpha(1-\rho\theta/\phi))}{1-\alpha^2} + \frac{\partial\Lambda}{\partial\tau_K}\right]/2. \tag{A5}$$

where $\Lambda \equiv \sqrt{B^2 - 4\Phi\phi/[(1-\Phi)\rho\theta]}$ and

$$\frac{\partial \Lambda}{\partial \tau_K} = \frac{1}{\Lambda}\left(B\frac{\partial B}{\partial \tau_K} + \frac{2\alpha^2\phi}{(1-\alpha^2)(1-\Phi)^2\rho\theta} \right) > 0, \tag{A6}$$

where $\partial B/\partial \tau_K > 0$. It is clear from eq. (A5) that $\partial \tilde{\tau}_L/\partial \tau_K|_{L=\tilde{L}_2}$ is positive, which contradicts the assumption of tax shifting. Therefore, we should rule out the possibility $\tilde{L} = \tilde{L}_2$.

Appendix 2.B

This appendix solves the dynamic system of the model under *tax shifting from labor income taxes to capital income taxes* ($Z = 0$). The set of equations under the model is expressed by:

$$
\begin{aligned}
1/C &= \lambda && \text{(B1)}\\
\theta(1-L)^{-\eta} &= \lambda(1-\tau_L)w && \text{(B2)}\\
r &= (1-\tau_K)r_K && \text{(B3)}\\
\dot{C}/C &= (1-\tau_K)r_K - \rho && \text{(B4)}\\
wL_Y &= (1-\alpha)Y && \text{(B5)}\\
x &= L_Y(\alpha^2/r_K)^{1/(1-\alpha)} && \text{(B6)}\\
r_K K &= \alpha^2 Y && \text{(B7)}\\
A\pi &= \alpha(1-\alpha)Y && \text{(B8)}\\
r &= \frac{\pi}{V} + \frac{\dot{V}}{V} && \text{(B9)}\\
G &= \beta Y && \text{(B10)}\\
\beta Y &= \tau_K r_K K + \tau_L w L && \text{(B11)}\\
Y &= K^{\alpha}(AL_Y)^{1-\alpha} && \text{(B12)}\\
\dot{K} &= Y - C - G && \text{(B13)}\\
\dot{A} &= \phi A L_A && \text{(B14)}\\
V &= \frac{w}{\phi A} && \text{(B15)}\\
L &= L_Y + L_A && \text{(B16)}
\end{aligned}
$$

in which 16 equations are used to solve 16 unknowns endogenous variables $\{C, L, A, K, L_Y, x, r_K, \pi, r, G, \tau_L, Y, \lambda, L_A, V, w\}$, where λ denotes the Hamiltonian multiplier. Based on $K = Ax$, eqs (B1), (B2), (B5), and (B12) and let $f = C/K$ be the ratio between consumption and capital, we can obtain:

$$L = 1 - [(1/(\theta f))(1-\tau_L)(1-\alpha)(1/L_Y)(L_Y/x)^{1-\alpha}]^{-1/\eta}. \tag{B17}$$

Based on eqs (B5), (B7), and (B11), we have:

$$\tau_L = \frac{\beta - \alpha^2 \tau_K}{1-\alpha}(\frac{L_Y}{L}). \tag{B18}$$

We now turn to deal with the transitional dynamics of the model. By using $x = K/A$, eqs (B16), (B17), and (B18), we can infer the following expression:

$$L = L(x, f, L_Y; \tau_K), \tag{B19}$$

where

$$\frac{\partial L}{\partial x} = \frac{(1-\alpha)}{x(-\frac{\eta}{1-L} + \frac{\tau_L}{(1-\tau_L)L})}, \tag{B20a}$$

$$\frac{\partial L}{\partial f} = \frac{1}{f(-\frac{\eta}{1-L} + \frac{\tau_L}{(1-\tau_L)L})}, \tag{B20b}$$

$$\frac{\partial L}{\partial L_Y} = \frac{(\frac{\beta-\alpha^2\tau_K}{1-\alpha}\frac{L_Y}{L})/(1-\tau_L) + \alpha}{L_Y(-\frac{\eta}{1-L} + \frac{\tau_L}{(1-\tau_L)L})}, \tag{B20c}$$

$$\frac{\partial L}{\partial \tau_K} = \frac{(\alpha^2\frac{L_Y}{L})/((1-\alpha)(1-\tau_L))}{(-\frac{\eta}{1-L} + \frac{\tau_L}{(1-\tau_L)L})}. \tag{B20d}$$

Based on eqs (B14) and (B15), we have:

$$\frac{\dot{V}}{V} = \frac{\dot{w}}{w} - \frac{\dot{A}}{A}. \tag{B20e}$$

From $K = Ax$, eqs (B5), and (B12), we can further obtain:

$$\dot{w}/w = \dot{A}/A - \alpha \dot{L}_Y/L_Y + \alpha \dot{x}/x. \tag{B20f}$$

Additionally, substituting eq. (B20f) into eq. (B20e) yields:

$$\frac{\dot{V}}{V} = \alpha(\dot{x}/x - \dot{L}_Y/L_Y). \tag{B20g}$$

Combining eqs (B3), (B5), (B7), (B8), (B9), (B12), and (B15), we can obtain:

$$(1 - \tau_K)\alpha^2(\frac{L_Y}{x})^{1-\alpha} = \alpha\phi L_Y + \frac{\dot{V}}{V}. \tag{B21a}$$

Substituting eq. (20g) into eqs (B21a), (B21a) can be rearranged as:

$$\dot{L}_Y/L_Y = \phi L_Y + \dot{x}/x - (1 - \tau_K)\alpha(\frac{L_Y}{x})^{1-\alpha}. \tag{B21b}$$

Based on $x = K/A$, we have the result:

$$\dot{x}/x = \dot{K}/K - \dot{A}/A. \tag{B21c}$$

Substituting $f = C/K$, eqs (B10), (B13), (B14), and (B16) into eq. (B21c), we have:

$$\dot{x}/x = (1 - \beta)(\frac{L_Y}{x})^{1-\alpha} - f - \phi(L - L_Y). \tag{B21d}$$

From eqs (B21b) and (B21d), we can obtain:

$$\dot{L}_Y/L_Y = (1 - \beta - \alpha(1 - \tau_K))(\frac{L_Y}{x})^{1-\alpha} - f - \phi(L - 2L_Y). \tag{B21e}$$

Moreover, from eqs (B3), (B4), (B5), (B7), and (B12) we can obtain:

$$\dot{C}/C = (1-\tau_K)\alpha^2(\frac{L_Y}{x})^{1-\alpha} - \rho. \tag{22a}$$

Based on $f = C/K$, we have the following expression:

$$\dot{f}/f = \dot{C}/C - \dot{K}/K. \tag{B22b}$$

Substituting eqs (B10), (B12), (B13), and (B22a) into eq. (B22b), we can derive:

$$\dot{f}/f = ((1-\tau_K)\alpha^2 - (1-\beta))(\frac{L_Y}{x})^{1-\alpha} - \rho + f. \tag{B22c}$$

Based on eqs (B19), (B21d), (B21e), and (B22c), the dynamic system can be expressed as:

$$\begin{aligned}
\dot{x}/x &= (1-\beta)(\frac{L_Y}{x})^{1-\alpha} - f - \phi(L(x,f,L_Y;\tau_K) - L_Y), && \text{(B23a)}\\
\dot{f}/f &= ((1-\tau_K)\alpha^2 - (1-\beta))(\frac{L_Y}{x})^{1-\alpha} - \rho + f, && \text{(B23b)}\\
\dot{L}_Y/L_Y &= (1-\beta-\alpha(1-\tau_K))(\frac{L_Y}{x})^{1-\alpha} - f - \phi(L(x,f,L_Y;\tau_K) - 2L_Y). && \text{(B23c)}
\end{aligned}$$

Linearizing eqs (B23a), (B23b), and (B23c) around the steady-state equilibrium yields:

$$\begin{pmatrix} \dot{x} \\ \dot{f} \\ \dot{L}_Y \end{pmatrix} = \begin{pmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{pmatrix} \begin{pmatrix} x-\tilde{x} \\ f-\tilde{f} \\ L_Y-\tilde{L}_Y \end{pmatrix} + \begin{pmatrix} b_{14} \\ b_{24} \\ b_{34} \end{pmatrix} d\tau_K, \tag{B24}$$

where

$$b_{11} = -(1-\alpha)(1-\beta)(\tfrac{L_Y}{x})^{1-\alpha} - \phi x \tfrac{\partial L(x,f,L_Y;\tau_K)}{\partial x},$$

$$b_{12} = x(-1 - \phi(\tfrac{\partial L(x,f,L_Y;\tau_K)}{\partial f})),$$

$$b_{13} = (1-\alpha)(1-\beta)(\tfrac{L_Y}{x})^{-\alpha} - \phi x(\tfrac{\partial L(x,f,L_Y;\tau_K)}{\partial L_Y} - 1),$$

$$b_{14} = -\phi(\tfrac{\partial L(x,f,L_Y;\tau_K)}{\partial \tau_K}),$$

$$b_{21} = -(1-\alpha)((1-\tau_K)\alpha^2 - (1-\beta))(\tfrac{L_Y}{x})^{1-\alpha}(\tfrac{f}{x}),$$

$$b_{22} = f,$$

$$b_{23} = (1-\alpha)((1-\tau_K)\alpha^2 - (1-\beta))(\tfrac{L_Y}{x})^{-\alpha}(\tfrac{f}{x}),$$

$$b_{24} = -\alpha^2(\tfrac{L_Y}{x})^{1-\alpha} f,$$

$$b_{31} = -(1-\alpha)(1-\beta-\alpha(1-\tau_K))(\tfrac{L_Y}{x})^{2-\alpha} - \phi \tfrac{\partial L(x,f,L_Y;\tau_K)}{\partial x} L_Y,$$

$$b_{32} = -L_Y - \phi \tfrac{\partial L(x,f,L_Y;\tau_K)}{\partial f} L_Y,$$

$$b_{33} = (1-\alpha)(1-\beta-\alpha(1-\tau_K))(\tfrac{L_Y}{x})^{1-\alpha} - \phi(\tfrac{\partial L(x,f,L_Y;\tau_K)}{\partial L_Y} L_Y - 2L_Y),$$

$$b_{34} = \alpha L_Y(\tfrac{L_Y}{x})^{1-\alpha} - \phi(\tfrac{\partial L(x,f,L_Y;\tau_K)}{\partial \tau_K} L_Y).$$

Let ℓ_1, ℓ_2, and ℓ_3 be the three characteristic roots of the dynamic system. We do not analytically prove the saddle-path stability of the dynamic system; instead, we show that the dynamic system features two positive and one negative characteristic roots numerically. For expository convenience, in what follows let ℓ_1 be the negative root and ℓ_2, and ℓ_3 be the positive roots. From eq. (B24), the general solutions for x_t, f_t, and $L_{Y,t}$, can be described by:

$$x_t = \tilde{x} + D_1 e^{\ell_1 t} + D_2 e^{\ell_2 t} + D_3 e^{\ell_3 t}, \quad \text{(B25a)}$$

$$f_t = \tilde{f} + h_1 D_1 e^{\ell_1 t} + h_2 D_2 e^{\ell_2 t} + h_3 D_3 e^{\ell_3 t}, \quad \text{(B25b)}$$

$$L_{Y,t} = \tilde{L}_Y + \frac{\ell_1 - b_{11} - b_{12}h_1}{b_{13}} D_1 e^{\ell_1 t} + \frac{\ell_2 - b_{11} - b_{12}h_2}{b_{13}} D_2 e^{\ell_2 t} + \frac{\ell_3 - b_{11} - b_{12}h_3}{b_{13}} D_3 e^{\ell_3 t}. \quad \text{(B25c)}$$

where $h_1 = [(\ell_1 - b_{33})(\ell_1 - b_{11}) - b_{31}b_{13}]/[b_{32}b_{13} + b_{12}(\ell_1 - b_{33})]$, $h_2 = [(\ell_2 - b_{33})(\ell_2 - b_{11}) - b_{31}b_{13}]/[b_{32}b_{13} + b_{12}(\ell_2 - b_{33})]$, $h_3 = [(\ell_3 - b_{33})(\ell_3 - b_{11}) - b_{31}b_{13}]/[b_{32}b_{13} + b_{12}(\ell_3 - b_{33})]$, and D_1, D_2 and D_3 are undetermined coefficients.

The government changes the capital tax rate τ_K from τ_{K0} to τ_{K1} at t=0, based on eqs (B25a)-(B25c), we employ the following equations to capture the dynamic adjustment of x_t, f_t, and $L_{Y,t}$:

$$x_t = \begin{cases} \tilde{x}(\tau_{K0}); & t = 0^- \\ \tilde{x}(\tau_{K1}) + D_1 e^{\ell_1 t} + D_2 e^{\ell_2 t} + D_3 e^{\ell_3 t}; & t \geq 0^+ \end{cases} \quad \text{(B26a)}$$

$$f_t = \begin{cases} \tilde{f}(\tau_{K0}); & t = 0^- \\ \tilde{f}(\tau_{K1}) + h_1 D_1 e^{\ell_1 t} + h_2 D_2 e^{\ell_2 t} + h_3 D_3 e^{\ell_3 t}; & t \geq 0^+ \end{cases} \quad \text{(B26b)}$$

$$L_{Y,t} = \begin{cases} \tilde{L}_Y(\tau_{K0}); & t = 0^- \\ \tilde{L}_Y(\tau_{K1}) + \frac{\ell_1 - b_{11} - b_{12}h_1}{b_{13}} D_1 e^{\ell_1 t} + \frac{\ell_2 - b_{11} - b_{12}h_2}{b_{13}} D_2 e^{\ell_2 t} + \frac{\ell_3 - b_{11} - b_{12}h_3}{b_{13}} D_3 e^{\ell_3 t}; & t \geq 0^+ \end{cases} \quad \text{(B26c)}$$

where 0^- and 0^+ denote the instant before and after the policy implementation, respectively. The values for D_1, D_2, and D_3 are determined by:

$$
\begin{aligned}
x_{0^-} &= x_{0^+}, &\text{(B27a)}\\
D_2 &= D_3 = 0. &\text{(B27b)}
\end{aligned}
$$

Equation eq. (B27a) indicates that the level of intermediate goods remains unchanged at the instant of the policy implementation. Equation (B27b) is the stability condition which ensures that all x_t, f_t, and $L_{Y,t}$ converge to their new steady-state equilibrium. By using eqs (B27a) and (B27b), we can obtain:

$$
D_1 = \tilde{x}(\tau_{K0}) - \tilde{x}(\tau_{K1}). \tag{B28}
$$

Inserting eqs (B27b) and (B28) into eqs (B26a)-(B26c) yields:

$$
x_t = \begin{cases} \tilde{x}(\tau_{K0}); & t = 0^- \\ \tilde{x}(\tau_{K1}) + (\tilde{x}(\tau_{K0}) - \tilde{x}(\tau_{K1}))e^{\ell_1 t}; & t \geq 0^+ \end{cases} \tag{B29a}
$$

$$
f_t = \begin{cases} \tilde{f}(\tau_{K0}); & t = 0^- \\ \tilde{f}(\tau_{K1}) + h_1(\tilde{x}(\tau_{K0}) - \tilde{x}(\tau_{K1}))e^{\ell_1 t}; & t \geq 0^+ \end{cases} \tag{B29b}
$$

$$
L_{Y,t} = \begin{cases} \tilde{L}_Y(\tau_{K0}); & t = 0^- \\ \tilde{L}_Y(\tau_{K1}) + \frac{\ell_1 - b_{11} - b_{12}h_1}{b_{13}}(\tilde{x}(\tau_{K0}) - \tilde{x}(\tau_{K1}))e^{\ell_1 t}; & t \geq 0^+ \end{cases} \tag{B29c}
$$

Appendix 2.C

In the case of tax shifting from labor income taxes to capital income taxes, raising the capital income tax rate increases the steady-state equilibrium growth rate. This result relies on some assumptions. In this appendix, we will relax them and examine whether the Proposition 2 is still robust. Specifically, this appendix takes into account dividend income taxes. We will use a quantitative analysis to show that Proposition 2 still holds if we introduce dividend income taxes into our model.

In subsection 2.3.2 we deal with a tax-shifting from labor income taxes to capital income taxes. In this appendix we relax this assumption by considering tax-shifting from labor income taxes to both capital income taxes and divident income taxes. with this consideration, household's budget constraint reported in eq. (2) and government's budget constraint reported in eq.(22) can be respectively modified as follows:

$$\dot{K} + \dot{a} = ((1-\tau_A)r_A + \dot{V}/V)a + (1-\tau_K)r_K K + (1-\tau_L)wL - C, \tag{C1}$$

$$\tau_K r_K K + \tau_A r_A V A + \tau_L wL = \beta Y. \tag{C2}$$

where $r_A a$ is agent's total dividend income. The rates of return on the two assets, physical capital and equity shares, must follow a no-arbitrage condition at any time:

$$r \equiv (1-\tau_A)r_A + \dot{V}/V = (1-\tau_K)r_K \tag{C3}$$

Given that the government imposes the same tax rate on both capital income and dividend income ($\tau_K = \tau_A$) and the long-run market value of an invented variety

V is equal to constant since $V = w/(\phi A)$, and eq. (17a) hold in the long run, we then have:

$$r_K = r_A. \tag{C4}$$

From eqs (18a)-(18i), (C1), (C2), (C3), and (C4), after some tedious calculations as well as defining $\tilde{L}$ as the level of steady-state labor supply, we then have:

$$\theta(1-L)^{-\eta}\left[\frac{1-\beta-\alpha^2(1-\tau_K)}{1-\alpha^2}(\tilde{L}+\frac{\rho}{\phi})+(1-\tau_K)\frac{\alpha\rho}{(1-\alpha)\phi}\right] \tag{C3}$$
$$= \; 1-\frac{\beta-\alpha^2\tau_K\left(1+\frac{(1-\alpha)\tau_K}{\alpha(1-\tau_K)}\right)}{(1-\alpha^2)}\frac{\tilde{L}+\frac{\rho}{\phi}}{\tilde{L}}$$

From eq. (18g), $\tilde{L}_A = (\alpha\tilde{L} - \rho/\phi)/(1+\alpha)$, eq. (C3), and the following standard parameter values, we can obtain values of long-run growth rate with respect to varying the capital income tax rate. The parameter values are summarized below in Table 2.C.1

Table 2.C.1: Calibrated parameter values

ρ	θ	η	α	ϕ	β	τ_K
0.04	1.17	1.67	0.30	0.65	0.20	0.36

Figure 2.C.1 presents the growth effects of varying the capital income tax rate from 0 to 0.6. We can clearly see that, as the capital tax rate increases, the steady-state labor supply increases (see Figure 2.C.2), and thus the growth rate increases (see eq.(19)). The intuition can be explained as follows. Although an increase in the capital tax rate and dividend income tax rate exerts a negative effect on economic growth by depressing household's saving, it causes dramatic fall in the labor income taxes, which boosts labor supply and thus is beneficial to R&D and economic growth. In the long run, the latter effect dominates. Compared with tax-shifting from labor income taxes to capital income taxes, tax-shifting from

labor income taxes to both capital income taxes and dividend income taxes leads to lower wage income taxes and hence results in higher labor supply (see Figure 2.C.2). However, our result is robust if we consider tax-shfiting from labor taxes to both capital income taxes and dividend income taxes.

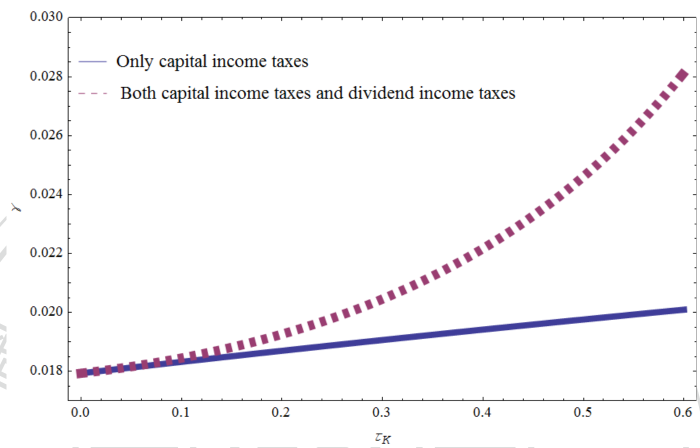


Figure 2.C.1

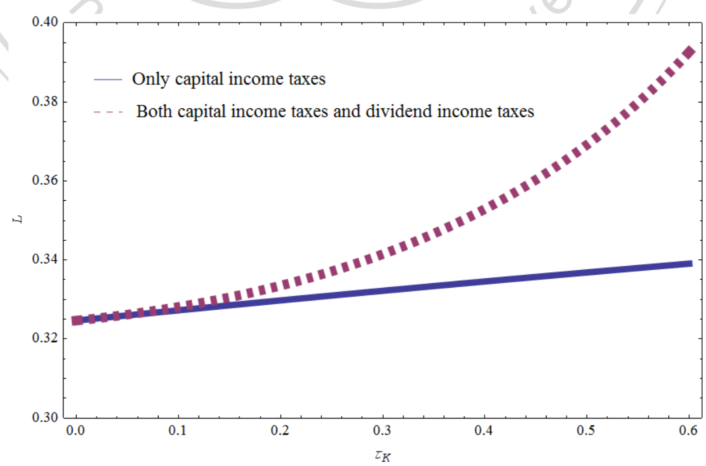


Figure 2.C.2

CHAPTER 3

OPTIMAL CAPITAL TAXATION AND R&D EXTERNALITIES

3.1 Introduction

Capital income is taxed worldwide. The estimated effective average tax rates on capital income are around 40% in the United States and 30% in EU countries. In some countries, such as the United Kingdom and Japan, the capital income tax rates are even up to near 60%. From the perspective of welfare maximization, whether these capital tax rates are too high or too low is a question that will never cease being debated by economists and policymakers.

Despite that capital taxes are commonly levied in the real world, a striking theory put forth by Judd (1985) and Chamley (1986) suggests that the government should only tax labor income and leave capital income untaxed in the long run. A number of subsequent studies, including Chari et al. (1994), Jones et al. (1997), Atkinson et al. (1999), and Chari and Kehoe (1999), relax key assumptions in Judd (1985) and Chamley (1986), and find their result to be quite robust. The idea of a zero optimal capital tax has then been dubbed the Chamley-Judd result, which turns out to be one of the most well-established and important benchmarks in the optimal taxation literature.

In this paper, we revisit the Chamley-Judd result in an innovation-based growth model. There are several reasons with regard to why we choose this environment to study optimal taxation. First, as stressed by Aghion et al. (2013), the consideration of growth seems not to play much of a role in the debate of the Chamley-Judd result. However, given that the recent empirical evidence suggests a significant impact of the tax structure on economic growth (e.g., Arnold et al., 2011), it is more plausible to bring the role of growth into the picture. Second, along the line of the optimal taxation literature, production technology is treated as exogenously given. The role of endogenous technological change driven by R&D has thus been neglected in previous models. In view of the fact that innovation is a crucial factor in economic development as well as in the improvement of human well-being, overlooking this facet could lead to inadequate design of tax policies. Our study thus aims to fill this gap. Third, as pointed out by Domeij (2005), a key premise in early contributions supporting the Chamley-Judd result is that there exist no inherent distortions (externalities) in the economy. If incomplete markets are present, the optimal capital income tax might be different from zero. Thus, we introduce an innovation market featured with various R&D externalities put forth by Jones and Williams (2000). Within this framework we can study how the optimal capital taxation and R&D externalities interact in ways not heretofore understood.

By calibrating the model to the US economy, our numerical analysis shows that the optimal capital income tax is significantly positive at a rate of 12 percent. The reason for a positive optimal capital income tax in our R&D-based growth model can be briefly explained as follows. In essential, the Chamley-Judd result involves a tax shift between capital income tax and labor income tax. The basic rationale behind a zero optimal capital tax is that taxing capital generates more distortion than taxing labor, because taxing capital creates a dynamic inefficiency for capital accumulation. In our R&D-based growth model, by contrast, labor is considered as the main input of innovation, as typically specified in standard R&D-based growth models (e.g., Romer, 1990; Jones, 1995; Acemoglu, 1998). Under such a framework, taxing labor has a detrimental effect on the incentives to innovation and growth. This introduces a justification for taxing capital income instead of labor income. On this ground, it might be optimal to have a non-zero capital income tax rate.

The main contribution of this study is to link optimal capital taxation to the features of innovation process. We vary the parameters capturing important R&D externalities and see how the optimal capital income tax changes in response. Our main findings can be briefly summarized as follows. First, under most circumstances, the positive optimal capital income tax still holds. Second, when knowledge spillovers are strong and/or the duplication externalities are small (in which cases the underinvestment of R&D is more likely), it is more likely to have a positive optimal capital income tax rate. Third, when creative destruction is more important during the R&D process, the optimal capital income tax should be higher (smaller) if the monopolistic markup is constrained (unconstrained) by the parameter of creative destruction. Fourth, a higher government spending ratio pushes toward a positive optimal capital income tax.

Finally, it is well-known in the existing studies (e.g., Aiyagari, 1995; Judd, 1997, 2002; Coto-Martínez et al, 2007) that when the intermediate firms are imperfectly competitive, the production level is too low compared to social optimum. Accordingly, the government should subsidize capital to induce a higher level of production. This means that the optimal capital income tax tends to be negative, in particular when the monopolistic markup is higher. However, our results show that the optimal capital income tax and the markup display an inverse-U shaped relationship, meaning that there is another effect of the markup on the optimal capital tax, which we call the R&D effect. To be more precise, in an R&D-based growth model, the monopolistic rents go to the upstream R&D sector. A higher markup means that the R&D sector is more important. Under such a situation, subsidizing capital financed by taxing labor has a detrimental effect on the incentives to innovation, which reduces growth and welfare. Considering this R&D effect, an increase in the monopolistic markup does not necessarily result in a lower optimal capital income tax.

Our study related to a vast literature attempting to overturn the Chamley-Judd result and obtaining a positive optimal capital income tax (e.g., among others, Chamley, 2001; Erosa and Gervais, 2002; Domeij, 2005; Golosov et al., 2006; Conesa et al., 2009; Aghion et al., 2013; Chen and Lu, 2013; Piketty and Saez, 2013). This paper contributes to the literature by introducing the role of endogenous technological change. Two papers studying optimal factor tax within the framework of an endogenous growth model are closely related to the present paper. Chen and Lu (2013) consider a human capital-based endogenous growth model developed by Lucas (1988). They find that a switch from labor income taxes to capital income taxes always enhances growth and welfare. Thus, the government should tax capital income to a maximum level of 99%. Aghion et al. (2013) also introduce R&D-based growth into the debate of the Chamley-Judd result. However, our paper differs from Aghion et al. (2013) in the following ways. First, Aghion et al. (2013) consider a Schumpeterian quality-ladder growth model, while we adopt an expanding-variety R&D model (Romer, 1990) incorporating the feature of creative destruction by following Jones and Williams (2000). Second, Aghion et al. (2013) consider a lab-equipment innovation process (i.e., R&D uses final goods as inputs), while we assume a knowledge-driven innovation process (i.e., R&D uses labor as inputs). Under our setting, therefore, the welfare costs of taxing labor would be larger than that in their model. Third, in Aghion et al. (2013), the positive optimal capital income tax sustains when the government spending of output ratio exceeds about 38%, which is much larger than the empirical value.

In our analysis, by contrast, the optimal capital income tax is positive even if the government spending ratio is quite small (around 14%). Finally, Aghion et al. (2013) do not examine how the optimal capital income tax responds to various R&D externalities, which is the main focus of our analysis.

The remaining of the paper proceeds as follows. In Section 3.2 we describe the R&D-based growth model featuring creative destruction and various types of R&D externalities elucidated by Jones and Williams (2000). In Section 3.3 we analyze in the long run how capital tax changes affect the economy. In Section 3.4 we quantify the optimal capital income tax and examine how it interacts with R&D externalities. Section 3.5 provides concluding remarks.

3.2 The model

Our framework builds on the non-scale R&D-based growth model of the seminal work developed by Jones and Williams (2000). The main novelty of the Jones and Williams model is that it removes the scale effects and introduces several important dimensions of R&D into the original variety-expending R&D-based model of Romer (1990). In this paper, we extend their model by incorporating (i) elastic labor supply and (ii) factor taxes, namely the capital and labor income taxes. To conserve space, the familiar components of Romer's variety-based model will be briefly described, while the new features will be described in more detail.

3.2.1 *Households*

We consider a continuous-time economy that is inhabited by a unit continuum of identical infinitely-lived households. At time t, the population size of each household is N_t, which grows at an exogenous constant rate n. Each member of households is endowed with one unit of time that he/she can supply labor to a competitive market or enjoy leisure. The lifetime utility function of a representative household is given as:[1]

$$U = \int_0^\infty e^{-\beta t}\left[\ln c_t + \chi \ln(1 - l_t)\right] dt, \quad \beta > 0, \quad \chi \geq 0, \tag{1}$$

where c_t is per capita consumption and l_t is the supply of labor per capita. The parameters β and χ denote respectively the subjective rate of time preference and leisure preference. The representative household maximizes (1) subject to the following budget constraint:

$$\dot{k}_t + \dot{e}_t = [(1 - \tau_K) r_{K,t} - n - \delta] k_t + (r_t - n)\, e_t + (1 - \tau_{L,t}) w_t l_t - c_t, \tag{2}$$

where a dot hereafter denotes the rate of change with respect to time, k_t is physical capital per capita, δ is physical capital depreciation rate, e_t is the value of equity shares of R&D owned by each member, $r_{K,t}$ is the capital rental rate, r_t is real interest rate, w_t is the wage rate. The policy parameters $\tau_{K,t}$ and $\tau_{L,t}$ are respectively the capital and labor income tax rate.

Solving the dynamic optimization problem yields the following first-order conditions:

$$\frac{1}{c_t} = q_t, \tag{3}$$

$$(1 - \tau_{L,t}) w_t (1 - l_t) = \chi c_t, \tag{4}$$

[1] In line with Chu and Cozzi (2014) we assume that the utility function is based on per capital utility function.

$$r_t = (1 - \tau_K) r_{K,t} - \delta. \tag{5}$$

where q_t is the Hamiltonian co-state variable on eq. (2). Equations (3) and (4) are respectively the optimality conditions for consumption and labor supply, and eq. (5) is a no-arbitrage condition which states that the net returns on physical capital and equity shares must be equalized. We denote the common net return on both assets as r_t (i.e., $r_t = (1 - \tau_K) r_{K,t} - \delta$). The typical Keynes-Ramsey rules is:

$$\frac{\dot{c}_t}{c_t} = r_t - n - \beta. \tag{6}$$

3.2.2 *The final-goods sector*

A perfectly-competitive final-good sector produces a single final output Y_t (treated as the numéraire) by using labor and a continuum of intermediate capital goods, according to the CES technology:

$$Y_t = L_{Y,t}^{1-\alpha} \left(\sum_{i=1}^{A_t} x_t^{\alpha\rho}(i) \right)^{\frac{1}{\rho}}, 1 > \alpha > 0,\ 1/\alpha > \rho > 0, \tag{7}$$

where $L_{Y,t}$ is the labor input employed in final goods production, $x_t(i)$ ($i \in [0, A]$) is the ith intermediate capital good, and A_t is the number of varieties of the intermediate goods. As will be introduced later, intermediate goods and capital has a one-to-one relation. Therefore, in eq. (7) we have followed Jones and Williams (2000) and Comin (2004) to separate the capital share (α) and the elasticity of substitution across varieties ($\alpha\rho$).

Profit maximization yields the following conditional demand functions for the labor input and intermediate goods:

$$w_t = (1 - \alpha) \frac{Y_t}{L_{Y,t}}, \tag{8}$$

$$p_t(i) = \alpha L_{Y,t}^{1-\alpha} \left(\sum_{i=1}^{A_t} x_t^{\alpha\rho}(i) \right)^{\frac{1}{\rho}-1} x_t^{\alpha\rho-1}(i), \tag{9}$$

where $p_t(i)$ is the price of the ith intermediate good.

3.2.3 *The intermediate-goods sector*

Each intermediate good is produced by a monopolistic producer who owns a perpetually protected patent for that good. The producer needs to use one unit of physical capital to produce one unit of intermediate goods. Thus, the production function is $x_t(i) = v_t(i)$, where $v_t(i)$ denotes the capital input employed by monopolistic intermediate firm i. Accordingly, the profit of intermediate goods firm i is:

$$\pi_{x,t}(i) = p_t(i)x_t(i) - r_{K,t}v_t(i). \tag{10}$$

Profit maximization subject to the production function $x_t(i) = v_t(i)$ and eq. (9) yields the pricing rule:

$$p_t(i) = \frac{1}{\rho\alpha} r_{K,t}. \tag{11}$$

Let $\eta_t(i)$ denote the gross markup that the ith intermediate firm can charge over its marginal cost. Then, we have:

$$p_t(i) = \eta_t(i) r_{K,t}. \tag{12}$$

By some manipulations, the profit of the ith intermediate firm can be obtained as:

$$\pi_{x,t}(i) = \frac{\eta_t(i) - 1}{\eta_t(i)} \alpha \frac{Y_t}{A_t}. \tag{13}$$

It follows from eqs (11) and (12) that if the monopolistic intermediate firm freely sets the price, the markup would be equal to the elasticity of substitution between intermediate capital goods, i.e., $\eta_t(i) = 1/(\rho\alpha)$. This is the case of an "unconstrained" markup (Jones and Williams, 2000). Another scenario is that the markup is subject to an *adoption constraint*, which may happen if the new designs are linked together in the *innovation cluster*. This involves the property of the research process, which we will discuss in more detail in the next subsection.

3.2.4 *The R&D sector*

R&D creates new varieties of intermediate goods for final-good production. In line with Romer (1990) and Jones (1995), we assume that new varieties are developed by labor input (scientists). The production technology is given as:

$$(1+\psi)\dot{A}_t = \tilde{\varsigma}_t L_{A,t}, \quad \psi \geq 0, \tag{14}$$

where $L_{A,t}$ is the labor input used in the R&D sector, $\tilde{\varsigma}_t$ is the productivity of R&D which the innovators take as given. The meaning of the parameter ψ will be explained later.

We follow Jones (1995) to specify that the productivity takes the following function form:

$$\tilde{\varsigma}_t = \varsigma L_{A,t}^{\lambda-1} A_t^{\phi}, \quad \varsigma > 0, \quad 1 \geq \lambda > 0, \quad 1 > \phi > 0, \tag{15}$$

where ς is a constant productivity parameter. In addition to ς, eqs (14) and (15) contain three parameters λ, ϕ and ψ. These parameters capture salient features of R&D proposed by Jones and Williams (1998). We then discuss each of them.

First, the parameter $1 \geq \lambda > 0$ reflects a (negative) duplication externality or a congestion effect of R&D. It implies that the social marginal product of research labor can be less than the private marginal product. This may happen because of, for example, a patent race, or if two researchers accidentally work out a similar idea. Jones and Williams (1998) coin this negative duplication externality as the *stepping on toes effect*. Notice that this effect is stronger with a smaller λ, and it vanishes when $\lambda = 1$.

Second, the parameter $1 > \phi > 0$ reflects a (positive) knowledge spillover effect due to the fact that richer existing ideas are helpful to the development of new ideas. A higher ϕ means that the spillover effect is greater. In his pioneering article, Romer (1990) specifies $\phi = 1$; however, Jones (1995) argues that $\phi = 1$ exhibits a scale effect which is inconsistent with the empirical evidence. We thus follow Jones (1995) to assume that $\phi < 1$ to escape from the scale effect. The knowledge spillover effect is dubbed by Jones and Williams (1998) as the *standing on shoulders effect*.

Finally, the parameter $\psi \geq 0$ measures the size of innovation clusters, which is associated with the concept of creative destruction pointed out by Grossman and Helpman (1991) and Aghion and Howitt (1992). The basic idea is that innovations must come together in clusters, some of which are new, while others simply build on old fashions. More specifically, suppose that an innovation cluster, which contains $(1 + \psi)$ varieties, has been invented. Out of these $(1 + \psi)$ varieties, only one unit of variety is entirely new and thus increases the mass of the variety of intermediate goods. The remaining portion, of size ψ, simply replaces the old versions. This portion captures the spirit of creative destruction since new versions are created with the elimination of old versions. However this part does not contribute to the increase of existing varieties. In other words, for $(1 + \psi)$ intermediate goods invented, the actual augmented variety is 1, while the repackaged varieties are ψ.

As we have mentioned earlier, it is possible that the markup of the monopolistic intermediate firms is constrained by the size of innovation clusters. The intuition underlying this result can be understood as follows. Consider that the current number of varieties is A_t. Now an innovation cluster with size $(1+\psi)$ is developed. This increases the mass of varieties to A_t+1; at the same time it also replaces old-version varieties by ψ units. Subsequently, the final-good firm faces two choices. It can either adopt the new innovation cluster and then use A_t+1 intermediate goods priced at a markup, or part with the new innovation cluster and still use A_t intermediate goods to produce. If the final-good firm chooses the latter, since now ψ varieties have been displaced, the final-good firm needs only to purchase $A_t - \psi$ units of intermediate goods at a markup price, and purchase ψ units of displaced intermediate goods at a lower (competitive) price. When the size of innovation cluster is high (a larger value of ψ), the final-good firm tends not to adopt the new innovation cluster because sticking to old clusters is cheaper. As a result, the intermediate-good firms have to decrease the markup so as to attract the final-good firm to adopt the new innovation cluster. This adoption constraint explains why an increase in the size of innovation clusters reduces the markup.

Jones and Williams (2000) show that the constrained markup is negatively related to both the size of innovation clusters and the elasticity of substitution between capital goods. Specifically, they demonstrate that the constrained markup is limited not to exceed the value $[(1+\psi)/\psi]^{1/\rho\alpha-1}$. Together with the unconstrained markup we discussed in subsection 3.2.3., the finally realized markup is:

$$\eta = \min\left\{ \frac{1}{\rho\alpha} \, , \, \left(1+\frac{1}{\psi}\right)^{\frac{1}{\rho\alpha}-1} \right\}, \tag{16}$$

which is independent of i and t. Combining eqs (10) and (16) implies that all intermediate-good firms are symmetric. Therefore, notation i in subsection 3.2.3 can be dropped from here.

Given $\tilde{\varsigma}_t$, the R&D sector hires $L_{A,t}$ to create $(1+\psi)$ varieties. Thus, the profit function is $\pi_{A,t} = P_{A,t}(1+\psi)\dot{A}_t - w_t L_{A,t}$. By assuming free entry in the R&D sector, we can obtain:

$$P_{A,t} = \frac{s_t}{1-s_t}\frac{(1-\alpha)Y_t}{(1+\psi)\dot{A}_t}, \tag{17}$$

where $s_t \equiv L_{A,t}/L_t$ is the ratio of research labor to total labor supply L_t. Moreover, the no-arbitrage condition for the value of a variety is:

$$r_t P_{A,t} = \pi_{x,t} + \dot{P}_{A,t} - \psi\frac{\dot{A}_t}{A_t}P_{A,t}. \tag{18a}$$

Without creative destruction ($\psi = 0$), the familiar no-arbitrage condition reports that, for each variety, the return of the equity shares $r_t P_{A,t}$ will be equal to the sum of the flow of the monopolistic profit $\pi_{x,t}$ plus the capital gain or loss $\dot{P}_{A,t}$. When creative destruction is present, existing goods are replaced. Accompanied with new varieties $\dot{A}_t$ being invented, the amount of $\psi\dot{A}_t$ existing varieties will be replaced. Therefore, for each variety, the expected probability of being replaced is $\psi\dot{A}_t/A_t$, which gives rise to the expected capital loss expressed by the last term in eq. (18a).

3.2.5 *The government and aggregation*

The government collects capital income taxes and labor income tax to finance its public spending. The balanced budget constraint faced by the government is:

$$N_t(\tau_K r_{K,t} k_t + \tau_{L,t} w_t l_t) = G_t, \tag{18b}$$

where G_t is the total government spending. We assume that government spending is a fixed proportion of final output, i.e., $G_t = \zeta Y_t$, where ζ is the government size and $1 > \zeta > 0$. Now let us define the aggregate capital stock as $K_t = N_t k_t$, aggregate consumption $C_t = N_t c_t$, and total labor supply $L_t = N_t l_t$. After some algebra, we can obtain the resource constraint in the economy $\dot{K}_t = Y_t - C_t - G_t$.

3.2.6 *The decentralized equilibrium*

The decentralized equilibrium in this economy is an infinite sequence of allocations $\{C_t, K_t, A_t, Y_t, L_t, L_{Y,t}, L_{A,t}, x_t, v_t\}_{t=0}^{\infty}$, prices $\{w_t, r_{K,t}, r_t, p_t, P_{A,t}\}_{t=0}^{\infty}$, and policies $\{\tau_{K,t}, \tau_{L,t}\}$, such that at each instant of time:

a. households choose $\{c_t, k_t, e_t, l_t\}$ to maximize lifetime utility eq. (1) taking prices and policies as given;

b. competitive final-good firms choose $\{x_t, L_{Y,t}\}$ to maximize profit taking prices as given;

c. monopolistic intermediate firms $i \in [0, A_t]$ choose $\{v_t, p_t\}$ to maximize profit taking $r_{K,t}$ as given;

d. the R&D sector chooses $L_{A,t}$ to maximize profit taking $\{P_{A,t}, w_t\}$ and the productivity $\tilde{\zeta}_t$ as given;

e. the labor market clears, i.e., $N_t l_t = L_{A,t} + L_{Y,t}$;

f. the capital market clears, i.e., $N_t k_t = A_t v_t$;

g. the stock market for variety clears, i.e., $N_t e_t = P_{A,t} A_t$

h. the resource constraint is satisfied, i.e., $\dot{K}_t = Y_t - C_t - G_t - \delta K_t$;

i. the government budget constraint is balanced, i.e., $N_t(\tau_{K,t} r_{K,t} k_t + \tau_{L,t} w_t l_t) = G_t$.

3.3 Steady-state properties

We focus our analysis on the steady state along the balanced growth path where all variables grow constantly. We denote by g_Z the growth rate of any generic variable

Z, and drop the time subscript to denote for any variables in the steady state. The steady-state growth rates of varieties and output are given by (see Appendix 3.A):

$$g_A = \frac{\phi}{1-\lambda} n, \quad g_Y = \frac{1}{1-\alpha} \left(\frac{1}{\rho} - \alpha \right) g_A + n. \tag{19a}$$

Moreover, in order to obtain stationary endogenous variables, it is necessary to define the following transformed variables:

$$\hat{k}_t \equiv \frac{K_t}{N_t^\sigma}, \quad \hat{c}_t \equiv \frac{C_t}{N_t^\sigma}, \quad \hat{y}_t \equiv \frac{Y_t}{N_t^\sigma}, \quad \hat{a}_t \equiv \frac{A_t}{N_t^{\lambda/(1-\phi)}}, \tag{19b}$$

where $\sigma \equiv 1 + \frac{(1/\rho - \alpha)\lambda}{(1-\alpha)(1-\phi)} > 0$ is a composite parameter. For ease of exposition, in line with Eicher and Turnovsky (2001), $\hat{k}$, $\hat{c}$, $\hat{y}$, and $\hat{a}$ are dubbed scale-adjusted capital, consumption, output, R&D varieties, respectively. Based on the transformed variables and the macro equilibrium defined in subsection 3.2.6, the macro economy in the steady state can be described by the following set of equations:

$$\begin{aligned}
r &= (1-\tau_K) r_K - \delta = \beta + g_Y, && (20a) \\
s &= \frac{\frac{\eta-1}{\eta} \frac{\alpha}{1-\alpha} (1+\psi) g_A}{r - g_Y + \left(1 + \frac{\eta-1}{\eta} \frac{\alpha}{1-\alpha}\right)(1+\psi) g_A}, && (20b) \\
\frac{\hat{k}}{\hat{y}} &= \frac{\alpha}{\eta r_K}, && (20c) \\
(1-\zeta) \frac{\hat{y}}{\hat{k}} &= \frac{\hat{c}}{\hat{k}} + g_Y + \delta, && (20d) \\
\hat{y} &= \hat{a}^{1/\rho - \alpha} \hat{k}^\alpha \left((1-s) l\right)^{1-\alpha}, && (20e) \\
g_A &= \frac{1}{1+\psi} \frac{\varsigma (sl)^\lambda}{\hat{a}^{1-\phi}}, && (20f) \\
\frac{\chi l}{(1-l)} &= \frac{(1-\tau_L)(1-\alpha)}{(1-s)} \frac{\hat{y}}{\hat{c}}, && (20g) \\
\tau_L &= \frac{1-s}{1-\alpha} \left(\zeta - \tau_K \frac{\alpha}{\eta} \right), && (20h)
\end{aligned}$$

in which eight endogenous variables r, s, $\hat{c}$, $\hat{k}$, $\hat{a}$, $\hat{y}$, l, τ_L are determined.

Of particular note, our main focus is on the examination of the capital tax. By holding the proportion of the government spending constant, an increase in the capital income tax must be coupled with a reduction in the labor income tax. Therefore, we follow the literature on the Chamley-Judd result to assume that the labor income tax endogenously adjusts to balance the government budget. This approach has been dubbed "tax shifting" or "tax swap" in the literature.

3.3.1 *Comparative static analysis*

In this section, we analyze the effects of the capital taxation on the R&D share, the endogenous labor income tax rate, labour supply, and scale-adjusted variables: $\hat{a}, \hat{k}, \hat{c}$, and $\hat{y}$.[2]

The long run R&D labour share, s, is given by

$$s = \frac{\frac{\eta-1}{\eta}\frac{\alpha}{1-\alpha}(1+\psi)g_A}{r - g_Y + \left(1 + \frac{\eta-1}{\eta}\frac{\alpha}{1-\alpha}\right)(1+\psi)g_A}. \tag{21a}$$

It follows from the above equation that, in the steady state, a change in the capital income tax rate (21a) does not affect the R&D labor share (i,e.,$\partial s/\partial \tau_K = 0$). The intuition underlying $\partial s/\partial \tau_K = 0$ can be grasped as follows. The non-arbitrage condition between physical capital and R&D equity reported in (20a) requires that the return of physical capital should be equal to the return of R&D equity. Given that the return of R&D equity, $r = \beta + \frac{1}{1-\alpha}\left(\frac{1}{\rho} - \alpha\right) g_A + n$, is independent of the capital tax rate, it is clear that the capital income tax rate is impotent to affect the return of R&D equity and hence the R&D labor share.

[2]We solve the dynamic system in Appendix 3.B, and a detailed derivation of the comparative static analysis is presented in Appendix 3.C.

From (20h), we have:

$$\tau_L = \frac{1-s}{1-\alpha}\left(\zeta - \tau_K \frac{\alpha}{\eta}\right), \tag{21b}$$

Based on (21a), we have:

$$\frac{\partial \tau_L}{\partial \tau_K} = -(1-s)\frac{\frac{\alpha}{\eta}}{1-\alpha} < 0. \tag{21c}$$

Under the tax-shifting scheme, an increase in the capital income tax rate must be coupled with a reduction in the labor income tax rate.

Given a constant capital income tax rate τ_K, labor supply in the steady state is given by:

$$l = \begin{cases} 1 - \frac{\chi}{\chi + \frac{1}{[(1-\zeta)-(\delta+g_Y)\frac{\alpha(1-\tau_K)}{\eta(\beta+\delta+g_Y)}]}\frac{(1-\tau_L)(1-\alpha)}{(1-s)}} & ; \quad \chi > 0 \\ 1 & ; \quad \chi = 0 \end{cases} \tag{22a}$$

It is straightforward from eq.(22a) to infer the following result:

$$\frac{\partial l}{\partial \tau_K} = \begin{cases} \frac{\alpha\beta(\frac{1-s}{1-\alpha})[1-\zeta+\frac{\eta-1}{\eta}\frac{\alpha(\delta+g_Y)}{\beta+(1+\psi)g_A}](1-l)l}{\eta(\beta+\delta+g_Y)(1-\tau_L)[1-\zeta-(\delta+g_Y)\frac{\alpha(1-\tau_K)}{\eta(\beta+\delta+g_Y)}]} > 0 & ; \quad \chi > 0 \\ 0 & ; \quad \chi = 0 \end{cases} \tag{22b}$$

Equation (22b) indicates that, when the tax shifts form a labor income tax to a capital income tax, a rise in the capital income tax rate leads to an increase in labor supply. The rationale for this result can be understood intuitively. In response to a rise in the capital income tax rate, two conflicting effects would emerge. First, raising the capital tax rate induces the households to lower the investment-output ratio and increase the consumption-out ratio, which in turn reduces labor supply. Second, it reduces the labor income tax rate (see eq. (21b)) and raises the after-tax wage income, thereby leading to an increase in labor supply. The latter positive effect dominates the former negative effect, and hence a rise in the capital income tax rate is accompanied with an increase in labor supply.

Moreover, scale-adjusted R&D varieties $\hat{a}$ is given by:

$$\hat{a} = [\frac{\varsigma}{(1+\psi)g_A}]^{1/(1-\phi)}(sl)^{\lambda/(1-\phi)}, \tag{23a}$$

where s and l are reported in eqs (21a) and (22a). With $\partial s/\partial \tau_K = 0$, it is quite easy to derive from eq. (23a) that:

$$\frac{\partial \hat{a}}{\partial \tau_K} = \frac{\lambda}{(1-\phi)}\hat{a}\frac{\partial l}{l\partial \tau_K} > 0. \tag{23b}$$

Equation (23b) indicates that a rise in the capital income tax rate tends to boost scale-adjusted R&D varieties. The intuition behind this result is not hard to understand. Following a rise in the capital income tax rate coupled with a decline in the labor income tax rate, the household is motivated to raise its labor supply. This in turn increases labor input allocated to the R&D sector ($L_A = Nsl$). Then, as reported in eq. (23a), given that scale-adjusted R&D varieties $\hat{a}$ is positively with R&D labor input sNl, $\hat{a}$ will increase in response following a rise in τ_K.

From eqs (20a), (20c), (20d),(23a), and (20e), we can infer that:

$$\hat{y} = [\frac{\varsigma}{(1+\psi)g_A}]^{\frac{1/\rho-\alpha}{(1-\alpha)(1-\phi)}}(sl)^{\frac{1/\rho-\alpha}{1-\alpha}\frac{\lambda}{(1-\phi)}}[\frac{\alpha(1-\tau_K)}{\eta(\beta+\delta+g_Y)}]^{\frac{\alpha}{1-\alpha}}(1-s)l, \tag{24a}$$

where

$$\frac{\partial \hat{y}}{\partial \tau_K} = [-\frac{\alpha}{(1-\alpha)(1-\tau_K)} + \sigma\frac{\partial l}{l\partial \tau_K}]\hat{y} \gtrless 0. \tag{24b}$$

Equation (24b) indicates that a rise in the capital income tax rate has ambiguous effect on scale-adjusted output $\hat{y}$. As exhibited in eq. (24b), two conflicting effects emerge following a rise in the capital income tax rate. First, a rise in the capital income tax rate shrinks capital investment, which in turn generates a negative impact on output. Second, a rise in the capital income tax rate is accompanied with a fall in the labor income tax rate, which motivates the household to provide more labor supply. This leads more labor input to be allocated to the R&D sector and in turn boosts R&D varieties, thereby contributing to a positive effect on output. If labor supply is exogenous ($\chi = 0$), the second positive effect is absent ($\partial l / \partial \tau_K = 0$), and a higher capital income tax rate lowers output. However, if labor supply is endogenous ($\chi > 0$), both conflicting effects are present, and the output effect of capital income taxation depends upon the relative strength between these two effects.

From eqs (20a), (20c), and (20d), we have:

$$\hat{k} = \frac{(1-\tau_K)\Phi}{(\delta + g_Y)}\hat{y}, \tag{25a}$$

$$\hat{c} = [(1-\zeta) - (1-\tau_K)\Phi]\hat{y}, \tag{25b}$$

where $\Phi \equiv \frac{\alpha(\delta+g_Y)}{\eta(\beta+\delta+g_Y)}$ is a composite parameter.

Based on eqs (25a) and (25b), the effects of τ_K on $\hat{k}$ and $\hat{c}$ can be expressed as:

$$\begin{aligned}
\frac{\partial \hat{k}}{\partial \tau_K} &= -\frac{\Phi}{(\delta+g_Y)}\hat{y} + \frac{(1-\tau_K)\Phi}{(\delta+g_Y)}\frac{\hat{y}}{\partial \tau_K} \qquad (26a)\\
&= [\sigma\frac{\partial l}{l\partial \tau_K} - \frac{1}{(1-\alpha)(1-\tau_K)}]\frac{(1-\tau_K)\Phi}{(\delta+g_Y)}\hat{y}\gtrless 0,\\
\frac{\partial \hat{c}}{\partial \tau_K} &= \Phi\hat{y} + [(1-\zeta)-(1-\tau_K)\Phi]\frac{\partial \hat{y}}{\partial \tau_K} \qquad (26b)\\
&= \{\Phi + [(1-\zeta)-(1-\tau_K)\Phi][\sigma\frac{\partial l}{l\partial \tau_K} - \frac{\alpha}{(1-\alpha)(1-\tau_K)}]\}\hat{y}\gtrless 0.
\end{aligned}$$

The economic intuition behind eqs (26a) and (26b) can be explained as follows. It is clear in eq. (25a) that capital income taxation affects scale-adjusted capital $\hat{k}$ through two channels. The first channel is the capital-output ratio ($\hat{k}/\hat{y} = \frac{(1-\tau_K)\Phi}{(\delta+g_Y)}$), and the second channel is the level of scale-adjusted output $\hat{y}$. The first term after the first equality in eq. (26a) indicates the first channel definitely lowers the level of $\hat{k}$. Moreover, as shown in eq. (24b), the second channel may either raise or lower the level of $\hat{k}$ since capital taxation leads to an ambiguous effect on $\hat{y}$. As a consequence, the net effect of capital taxation on the sale-adjusted capital stock $\hat{k}$ is still uncertain. Similarly, as indicated in eq. (25b), capital income taxation affects $\hat{c}$ also through two channels. The first channel is the consumption-output ratio ($\hat{c}/\hat{y} = [(1-\zeta)-(1-\tau_K)\Phi]$), and the second channel is the level of scale-adjusted output $\hat{y}$. As exhibited in eq. (24b), the first channel definitely boosts the level of $\hat{c}$, while the second channel may either raise or lower the level of $\hat{c}$ since capital taxation leads to an ambiguous effect on $\hat{y}$. As a consequence, the net effect of capital taxation on scale-adjusted consumption $\hat{c}$ remains ambiguous.

3.3.2 *Optimal capital income tax*

In this section, we analyze the optimal capital income tax that maximizes the steady-state level of social welfare. Using (1) and the previously defined transformed variables, the steady-state level of the life-time utility, denoted by U^{ss}, can be expressed as:

$$U^{ss} = \frac{\ln \hat{c} + \chi \ln(1-l)}{\beta} + \frac{g_Y - n}{\beta^2}. \tag{27}$$

By differentiating U^{ss} with respect to τ_K, we derive:

$$\frac{\partial U^{ss}}{\partial \tau_K} = \frac{1}{\beta}\left\{\left[\frac{\Phi}{(1-\zeta)-(1-\tau_K)\Phi} - \frac{\alpha}{(1-\alpha)(1-\tau_K)}\right] + \left[\sigma - \frac{\chi l}{(1-l)}\right]\frac{\partial l}{l\partial \tau_K}\right\}. \tag{28}$$

To clearly understand the intuition, we first consider the case of exogenous labor supply, which corresponds to $\chi = 0$ and $\frac{\partial l}{\partial \tau_K} = 0$. Then, by setting $\frac{\partial U^{ss}}{\partial \tau_K} = 0$, we can obtain the optimal capital income tax rate in the case of exogenous labor supply, which we denote as τ_K^*, given by:

$$\tau_K^* = 1 - (1-\zeta)\eta\left(1 + \frac{\beta}{\delta + g_Y}\right). \tag{29}$$

The following proposition is established from eq. (29):

Proposition 3 *In the case of exogenous labor supply, if $\zeta = 0$, the optimal capital income tax is always negative; if $\zeta > 0$, the sign of the optimal capital income tax is ambiguous.*

Proof. Directly inferred by using eq. (29) and the condition $\eta > 1$. ■

Many existing studies supporting a positive optimal capital tax rely on the assumption of endogenous choice of labor supply; see, e.g., Domeij (2005), Aghion et al. (2013), and Chen and Lu, (2013). The intuition is that, to have taxing capital more favorable than taxing labor, an important premise is that taxing labor results in large distortion. This premise can be true only in the case of an endogenous labor supply. In the case of an exogenous labor supply, by contrast, a labor income tax is equivalent to a lump-sum tax that will not distort any households' decision. In this case, the government should tax labor income as much as possible while leaving capital income untaxed. Therefore, it is unlikely to derive a positive optimal capital tax. Nonetheless, Proposition 3 surprisingly shows that a positive capital income tax could be optimal even when households supply labor inelastically. The intuition underlying Proposition 3 can be explained as follows. In the model where the intermediate goods sector is imperfectly competitive, the production level is too low. If there is no need for government spending ($\zeta = 0$), the government tends to subsidize capital (the input of intermediate goods) to correct this distortion by inducing a higher level of production. Thus, the optimal capital tax is negative. However, if the need for government spending is present ($\zeta > 0$), such wasteful government spending crowds out consumption, causing the level of consumption too low. In this case, subsidizing capital worsens the suboptimally low level of consumption because the subsidy encourages the accumulation of capital and further reduces consumption. Accordingly, the government tends to tax capital to restore the level of consumption. If the wasteful government spending is large, the motivation to restore consumption outweighs the motivation to correct the low level of production. As a consequence, the optimal capital tax turns to be positive.

In the case of an endogenous labor supply, a closed-form solution of the optimal capital income tax is not available. Intuitively, when the households supply labor elastically, labor income should be taxed less because the labor tax distorts the choice between labor and leisure. This gives a stronger rationale for taxing capital. Therefore, in an economy of endogenous labor supply, the optimal capital tax will be higher than that in the economy of exogenous labor supply. To carry out this intuition, let us denote τ_K^{**} as the optimal capital tax in the case of endogenous labor supply. Then, by inserting τ_K^* and eqs (22a), (22b) into eq. (28), we can demonstrate that:

$$\frac{\partial U^{ss}}{\partial \tau_K}\Big|_{\tau_K=\tau_K^*} = \frac{1}{\beta}[\sigma - \frac{\chi l}{(1-l)}]\frac{\partial l}{l\partial \tau_K} > 0, \tag{30}$$

which proves that $\tau_K^{**} > \tau_K^*$. Accordingly, we can establish the following proposition:

Proposition 4 *In the case of endogenous labor supply, the optimal capital income tax is higher than that in the case of exogenous labor supply, which implies that the optimal capital income tax is more likely to be positive.*

Proof. Proven in the text. ■

3.4 Quantitative results

In Section 3.3, we only focus on the long-run welfare effect of capital taxation. In this section, we take into account the welfare effects including transitional dynamics of capital taxation by performing a quantitative analysis to quantify the optimal capital tax. We calibrate the parameters of our theoretical model based on the US data. In particular, we explore how the optimal capital tax responds to important parameters that feature R&D externalities and the government size.

The life-time utility of a representative household reported in eq. (1) can be expressed as:

$$U = \int_0^\infty e^{-\beta t}\left[\ln \hat{c}_t + \chi \ln(1 - l_t)\right] dt + \frac{g_Y - n}{\beta^2}, \tag{31}$$

in which $\hat{c}_t$ and l_t are functions of τ_K. The optimal capital tax that takes into account the welfare effects including transitional dynamics is the one maximizes eq. (31).

3.4.1 *Calibration*

To carry out a numerical analysis, we first need to choose a baseline parameterization, reported in Table 3.1. Our model has eleven parameter values to be assigned. These parameters are either tied to a commonly used value in the existing literature or calibrated to match the empirical evidence in the US economy. We now describe each of them in detail. In line with Andolfatto et. al. (2008) and Acemoglu and Akcigit (2012), the labor income share $1-\alpha$ and the discount rate β are set to the standard values 0.4 and 0.05, respectively. The population growth rate n is set to 0.011 as used by Conesa et al. (2009). Based on Lucas (1990), the physical capital depreciation rate is given as 0.0318 such that the initial capital-output ratio of 2.5. The initial capital tax rate τ_K is set to 0.3 based on the average US effective tax rate estimated by Carey and Tchilingurian (2000). A similar value of the capital income tax rate has been adopted in Domeij (2005) and Chen and Lu (2013). As for the government size (the ratio of government spending to output), data of US exhibits around 20 percent (Gali, 1994), and has slightly increased in recent years. We therefore set ζ to be 0.22, which is the average level during 2001-2013, to reflect its increasing trend. The parameter for leisure preference χ is chosen as 1.5901 to make hours worked to be around one third.

Table 3.1. Benchmark Parameterization

Definition	Parameter	Value	Source/Target
Labour income share	$1-\alpha$	0.6	Andolfatto et. al. (2008)
Discount rate	β	0.05	Acemoglu and Akcigit (2012)
Population growth rate	n	0.011	Conesa et al. (2009)
Initial capital tax rate	τ_K	0.3	Carey and Tchilingurian (2000)
Government size	ζ	0.22	Data
Leisure preference	χ	1.5901	Total hours worked = 1/3
R&D productivity	ς	1	Normalized
Standing on toes effect	λ	0.5	Assumption
Substitution parameter	ρ	2.2727	Monopolistic markup = 1.1
Standing on shoulders effect	ϕ	0.9593	Output growth rate = 2%
Size of innovation cluster	ψ	0.25	Comin (2004)
Physical capital depreciation rate	δ	0.0318	Capital-output ratio = 2.5

Our parameterization regarding the R&D process basically follows the approach in Jones and Williams (2000). First, we normalize the R&D productivity ς to unity. The value of the parameter for standing on toes effect λ is somewhat difficult to calibrate because, as argued by Stokey (1995), the empirical literature does not provide much guidance on such a parameter. In our analysis, thus, we choose a middle value $\lambda = 0.5$ as a benchmark, but we will allow it to vary over the whole interval from 0 to 0.564.[3] The substitution parameter ρ relates closely to the markup of the intermediate firms. We set ρ to be 2.2727 such that, given $1-\alpha$, the (unconstrained) markup in our economy is 1.1, which lies within the reasonable range estimated for US industries (e.g., Norrbin, 1993 and Laitner and

[3]If the value of λ is over 0.564, the second-order condition of the government's maximization with respect to τ_K would not be satisfied.

Stolyarov, 2004). Moreover, we use the output growth rate to calibrate the extent of the standing on shoulders effect ϕ. in our model we have:

$$g_Y = (\frac{1}{\rho} - \alpha)g_A + n.$$

Given that $g_A = \phi n/(1-\lambda)$ and that we have already assigned values to $1-\alpha$, ρ, n and λ, we can then choose ϕ to target the empirical level of the output growth rate in the US, which is around 2%. This results in $\phi = 0.9593$ as our baseline value. Finally, as a benchmark we choose the size of innovation cluster $\psi = 0.25$ by following Comin (2004). In this case the markup is not bound by the adoption constraint. If the value of ψ is relatively large, the markup will be constrained (determined) by this parameter. Later in subsection 3.4.3 we will run ψ from 0 to 0.515 for a robustness check.

3.4.2 *The optimal capital tax with transitional dynamics*

Under our benchmark parameterization, Figure 3.1 plots the relationship between the level of welfare and the rate of capital income tax, which exhibits an inverse-U shaped relationship. Noticeably, the optimal capital tax is positive, and its value is around 11.9%. Thus, the Chamley-Judd result of zero capital tax does not hold in our R&D-based growth model.

The intuition underlying this result can be explained as follows. Given that the government is constrained to capital and labor taxation, to finance a fix amount of the government expenditure, not taxing capital income implies that the labor income must be taxed at a higher rate. Although a zero capital tax efficiently leaves the capital market undistorted, a high labor tax distorts the labor market severely by decreasing the after-tax wage income and thus reduces total labor supply. As

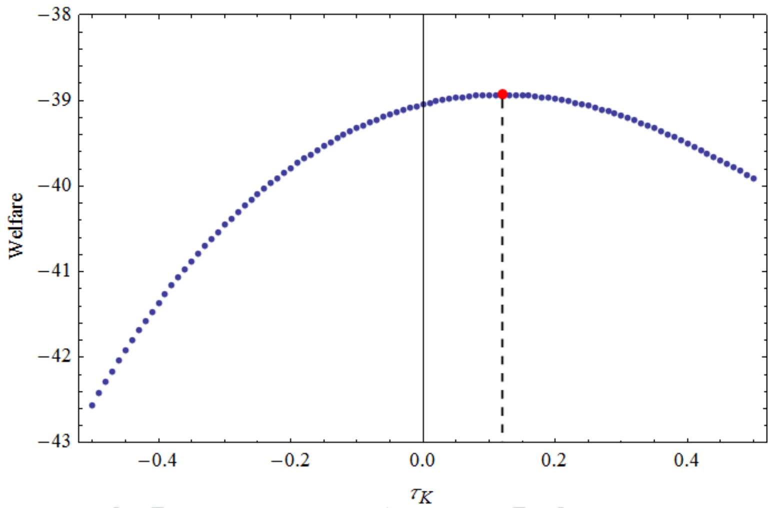


Figure 3.1: The level of welfare and the rate of capital income tax

a consequence, there is less labor devoted to the production in the R&D sector, which then results in fewer equilibrium varieties for the final-good production, and ultimately depresses the level of consumption and welfare. In summary, to achieve the social optimum, it is necessary to balance both distortions in capital and labor market. Accordingly, an extreme case of the zero capital tax is unlikely to be optimal.

3.4.3 *Policy implications of R&D externalities*

This subsection investigates how the optimal capital tax responds to relevant parameters, in particular those related to the features of innovation. More importantly, we shed some light on the roles of R&D externalities in the design of optimal tax policies. To this end, we provide a robustness check for whether the positive optimal capital tax still survives under various scenarios. In what follows, we propose some relevant parameters that need to be considered by the policymakers. The results are depicted in Figures 3.2-3.6, and several important results emerge from our robustness analysis.

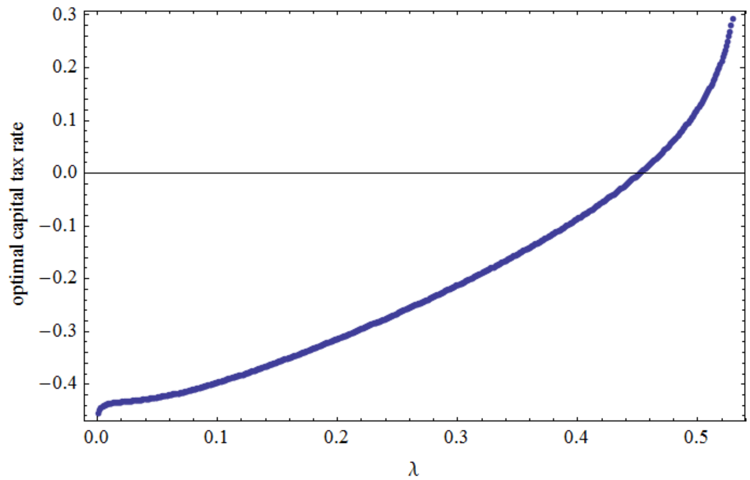


Figure 3.2: The optimal capital tax rate and the stepping on toes effect

First, the optimal capital tax is increasing in λ (the stepping on toes effect) and ϕ (the standing on shoulders effect). With sufficiently small values of λ and ϕ, the optimal capital income tax can be negative (see Figures 3.2 and 3.3). The underlying intuition behind the result can be explained as follows. Notice that a higher λ implies that the negative duplication externality is small, and a higher ϕ means that the positive spillover effect of R&D is relatively strong. Both cases indicate a similar circumstance in which the innovation process is more productive, and in which underinvestment in R&D is more likely. Under such a situation, the welfare cost of depressing innovation by raising the labor income tax is larger. Therefore, the government should increase the capital tax while reducing the labor tax.

Second, the optimal capital income tax and the substitution parameter ρ display an inverse-U shaped relationship (see figure 3.4). A lower ρ is associated with a higher monopolistic markup η, regardless of whether the adoption constraint is binding or not. The markup mainly affects the optimal capital tax in two opposite ways. The first effect (the monopoly effect) is that, when η is large (when

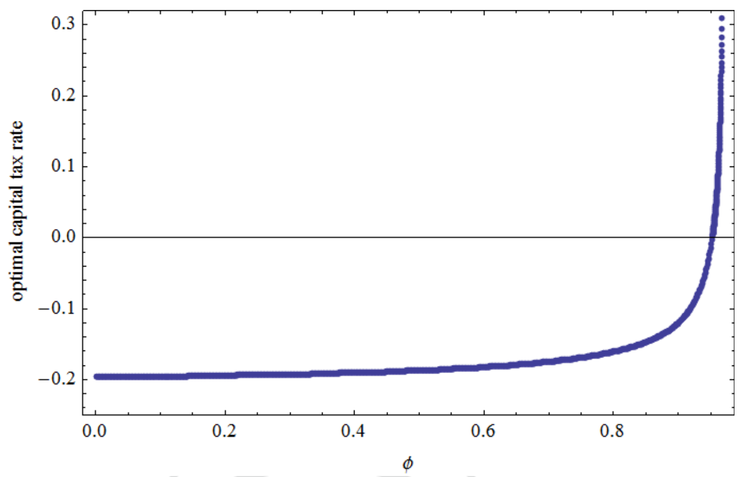


Figure 3.3: The optimal capital tax rate and the standing on shoulders effect

ρ is small), the degree of the intermediate firms' monopoly power is strong. To correct this distortion, the government tends to subsidize capital to offset the gaps between price and marginal costs; see Judd (1997, 2002). The second effect (the R&D effect) is that, a large η implies that the profits of intermediate firms are high, so will be the value of a successful innovation. This means that the R&D sector is crucial, and the welfare cost of slowing down innovation by raising the labor income tax is bigger. Thus, the government tends to tax capital income instead of taxing labor income. It is illustrated in Figure 3.2 that, with an initially very large η (a very small ρ), the monopoly effect dominants, such that the optimal capital tax is negative. As η becomes smaller (i.e., as ρ goes up), both effects decline. However the monopoly effect diminishes more rapidly than the R&D effect. The incentives to subsidize capital falls sharply, and thus the optimal capital income tax begins to increase with a rise in ρ. Finally, when η is very small (a sufficiently high value of ρ), there are few rents flowing to the R&D sector, rendering the R&D effect to be irrelevant. As a result, the government turns to prefer taxing labor again. Thus, the optimal capital income tax decreases with a rise of ρ when ρ is sufficiently high.

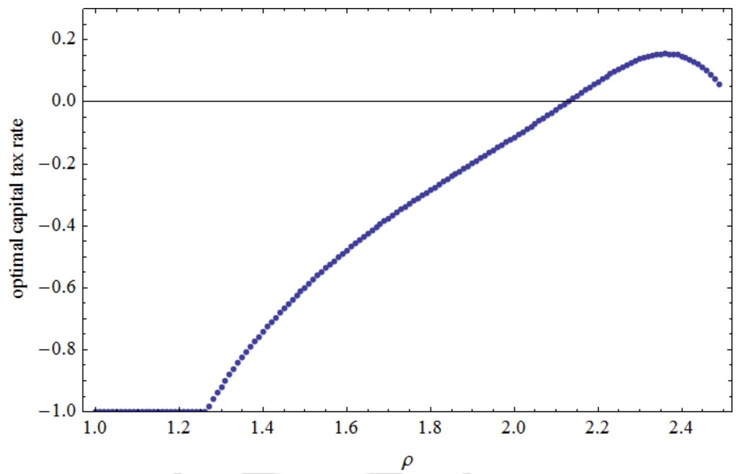


Figure 3.4: The optimal capital tax rate and the substitution parameter

Third, the optimal capital tax increases in response to a rise in the size of innovation cluster (creative destruction). The intuition is as follows. Given our baseline parameterization, the markup is not limited by the adoption constraint. In this case, ψ simply functions as a negative R&D externality, like the stepping on toes effect λ does. A higher ψ means that the negative externality is larger, thereby decreasing the importance of the R&D sector. Therefore, a higher ψ makes taxing labor more favorable than taxing capital (see figure 3.5).

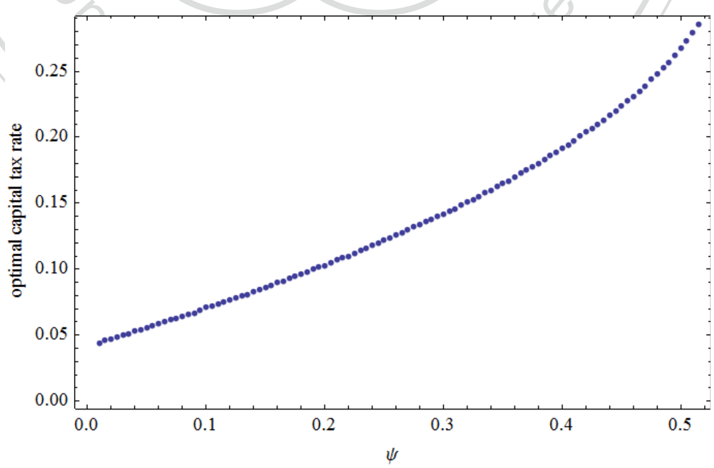


Figure 3.5: The optimal capital tax rate and creative destruction

Finally, the optimal capital tax is increasing in the government spending ratio ζ

(see figure 3.6). This result is in consistence with the Aghion et al. (2013) finding. When the need for public expenditure is sufficiently small, the government can collect labor tax revenues to finance the government spending and also to subsidize capital. Note that in this case the monopoly effect dominants the R&D effect so that the optimal capital tax is negative. As the size of government expenditure increases, it is not promising to count solely on raising the labor tax, because the distortion to the R&D sector would be sufficiently strong. Moreover, as we have discussed in Section 3.3, when the wasteful government increases, the government has an incentive to restore consumption by raising the capital tax. These effects turn the optimal capital income tax rate to gradually become positive.

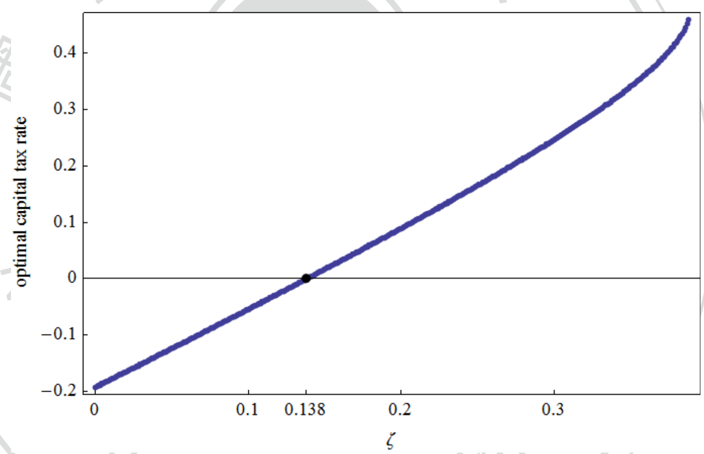


Figure 3.6: The optimal capital tax and the government size

3.5 Conclusion

In this paper, we have set up a non-scale innovation-based growth model, and used it to examine whether the the Chamley-Judd result of a zero optimal is vailid. By calibrating our model to the US economy, we have found that the optimal capital income tax is positive, at a rate of around 11.9 percent. We have also found that the result of a positive optimal capital income tax is robust with respect to varying the degrees of various types of R&D externalities.

Some extensions for future research are worth noting. First, to reflect the empirical reality, it would be useful to consider more complex optimal tax structures. Second, since R&D investment usually has liquidity problems (see, e.g., Lach, 2002), it would be relevant to introduce credit constraint on R&D investment into our model. Third, our model has assumed infinitely-lived agents. In the vein of optimal capital taxation, however, an important issue concerns the intergenerational consideration. Thus, it would be important to analyze the implications of innovation in a model with finitely-lived individuals and bequests. These future directions would generate new insights to the debate on the Chamley-Judd result.

Appendix 3.A. Deriving the steady-state growth rate

To solve for the steady-state growth rate of the economy, from eqs (14) and (15) we have:

$$\frac{\dot{A}_t}{A_t} = \frac{\varsigma}{1+\psi}\frac{L_{A,t}^{\lambda}}{A_t^{1-\phi}}. \tag{A1}$$

where $g_{A,t} = \dot{A}_t/A_t$. Let g_Z denote $g_{Z,t} = \frac{\dot{Z}}{Z}$ the growth rate of any generic variable Z, and drop the time subscript to denote for any variables in the steady state. The steady-state growth rate of varieties is given by:

$$g_A = \frac{\varsigma}{1+\psi}\frac{L_A^{\lambda}}{A^{1-\phi}}. \tag{A2}$$

Moreover, The R&D labor share is $s_t = L_{A,t}/(N_t l_t)$. By doing so, eq. (A2) can alternatively written as:

$$g_A = \frac{\varsigma}{1+\psi}\frac{(sNl)^{\lambda}}{A^{1-\phi}}. \tag{A3}$$

By taking logarithms of eq. (A3) and differentiating the resulting equation with respect to time, we have the following steady-state expression:

$$g_A = \frac{\lambda}{1-\phi}n. \tag{A4}$$

Equipped with the symmetric feature $x(i) = x$, the equilibrium condition for the capital market $K = Av$, and the production in the intermediate-good sector $x = v$, the aggregate production function can be rewritten as:

$$Y_t = A_t^{\frac{1}{\rho}-\alpha} L_t^{\alpha} K_t^{1-\alpha}. \tag{A5}$$

Taking logarithms of eq. (A5) and differentiating the resulting equation with respect to time, we can infer the following result:

$$g_Y = \frac{(\frac{1}{\rho}-\alpha)}{1-\alpha} g_A + n. \tag{A6}$$

Inserting eq. (A4) into eq. (A6) yields:

$$g_Y = \sigma n, \tag{A7}$$

where $\sigma \equiv 1 + \frac{(\frac{1}{\rho}-\alpha)}{1-\alpha}\frac{\lambda}{1-\phi}$ is a composite parameter.

We now turn to solve the steady-state R&D labor share. In the long run substituting $\dot{A}_t = g_A A_t$ and differentiating the resulting equation with respect to time give rist to:

$$\dot{P}_A / P_A = g_Y - g_A \tag{A8}$$

From eqs (13), (17), (18a), in the steady state we have:

$$\begin{aligned} \pi_x &= \frac{\eta - 1}{\eta} \alpha \frac{Y}{A} && \text{(A9)} \\ P_A &= \frac{s}{1-s} \frac{(1-\alpha)Y/A}{(1+\psi)g_A} && \text{(A10)} \\ r &= \frac{\pi_x}{P_A} + \frac{\dot{P}_A}{P_A} - \psi g_A && \text{(A11)} \end{aligned}$$

Subsituting eqs (A8), (A9), and (A10) into eq. (A11) yields the result:

$$r = \frac{\frac{\eta-1}{\eta}\alpha Y/A}{\frac{s}{1-s}\frac{(1-\alpha)Y/A}{(1+\psi)g_A}} + g_Y - (1+\psi)g_A \tag{A12}$$

Based on eq. (A12), we have the staionary R&D labor share s as follows:

$$s = \frac{\frac{\eta-1}{\eta}\frac{\alpha}{1-\alpha}(1+\psi)g_A}{r - g_Y + (1+\frac{\eta-1}{\eta}\frac{\alpha}{1-\alpha})(1+\psi)g_A} \tag{A13}$$

Appendix 3.B. Transition dynamics

This appendix solves the dynamic system of the model under *tax shifting from labor income taxes to capital income taxes*. The set of equations under the model is expressed by:

$$\frac{1}{c_t} = q_t, \tag{B1}$$

$$\chi = q_t(1-\tau_{L,t})w_t(1-l_t), \tag{B2}$$

$$r_t = (1-\tau_K)r_{K,t} - \delta, \tag{B3}$$

$$\frac{\dot{c}_t}{c_t} = r_t - n - \beta, \tag{B4}$$

$$w_t = (1-\alpha)\frac{Y_t}{L_{Y,t}}, \tag{B5}$$

$$\eta r_{K,t} = \alpha A_t^{\frac{1}{\rho}-1} L_{Y,t}^{1-\alpha} x_t^{\alpha-1}, \tag{B6}$$

$$r_{K,t}K_t = \frac{\alpha}{\eta}Y_t, \tag{B7}$$

$$\pi_{x,t} = \frac{\eta-1}{\eta}\alpha\frac{Y_t}{A_t}, \tag{B8}$$

$$r_t P_{A,t} = \pi_{x,t} + \dot{P}_{A,t} - \psi\frac{\dot{A}_t}{A_t}P_{A,t}, \tag{B9}$$

$$G_t = \zeta Y_t, \tag{B10}$$

$$G_t = N_t(\tau_K r_{K,t} k_t + \tau_{L,t} w_t l_t), \tag{B11}$$

$$Y_t = A_t^{1/\rho-\alpha} L_{Y,t}^{1-\alpha} K_t^{\alpha}, \tag{B12}$$

$$\dot{K}_t = Y_t - C_t - G_t - \delta K_t, \tag{B13}$$

$$\frac{\dot{A}_t}{A_t} = \frac{\varsigma}{1+\psi}\frac{L_{A,t}^{\lambda}}{A_t^{1-\phi}}, \tag{B14}$$

$$P_{A,t} = \frac{s_t}{1-s_t}\frac{(1-\alpha)Y_t}{(1+\psi)\dot{A}_t}, \tag{B15}$$

$$N_t l_t = L_{Y,t} + L_{A,t}. \tag{B16}$$

The above 16 equations determine 16 unknown $\{c_t, l_t,\ A_t,\ K_t,\ L_{Y,t}, x_t,\ r_{K,t},\ \pi_{x,t},\ r_t,$ $G_t,\ \tau_{L,t},\ Y_t,\ q_t,\ L_{A,t},\ P_{A,t},\ w_t\}$, where q_t is the Hamiltonian multiplier, $C_t = N_t c_t$, $K_t \equiv N_t k_t = A_t x_t$, and $s_t = L_{A,t}/N_t l_t$. Based on $K_t = N_t k_t = A_t x_t$, eqs (B1), (B2), (B5), and (B12), we can obtain:

$$\chi = \frac{1}{c_t}(1-\tau_{L,t})(1-\alpha)\frac{Y_t}{L_{Y,t}}(1-l_t). \tag{B17a}$$

From eqs (B5), (B7), and (B11), we have:

$$\tau_{L,t} = \frac{\zeta - \frac{\alpha}{\eta}\tau_K}{1-\alpha}(\frac{L_{Y,t}}{N_t l_t}). \tag{B17b}$$

Moreover, to solve the balanced growth rate, we define the following transformed variables:

$$\hat{k}_t \equiv \frac{K_t}{N_t^{\sigma}},\quad \hat{c}_t \equiv \frac{C_t}{N_t^{\sigma}},\quad \hat{y}_t \equiv \frac{Y_t}{N_t^{\sigma}},\quad \hat{a}_t \equiv \frac{A_t}{N_t^{\lambda/(1-\phi)}},\quad l_{Y,t} \equiv (1-s_t)l_t,\quad s_t \equiv L_{A,t}/N_t l_t. \tag{B18}$$

Based on eqs (B16), (17), (18a), and the above definitions, we can obtain:

$$\frac{\chi}{(1-l_t)} = \frac{1}{\hat{c}_t}(1 - \frac{\zeta - \frac{\alpha}{\eta}\tau_K}{1-\alpha}\frac{l_{Y,t}}{l_t})(1-\alpha)\hat{a}_t^{1/\rho-\alpha}(\hat{k}_t/l_{Y,t})^{\alpha}. \tag{B19a}$$

From eq. (B19a), we can infer the following expression:

$$l_t = l_t(\hat{k}_t, \hat{a}_t, \hat{c}_t, l_{Y,t}; \tau_K), \tag{B19b}$$

where

$$\frac{\partial l_t}{\partial \hat{k}_t} = \frac{\alpha}{\hat{k}_t(\frac{1}{1-l_t} - \frac{\tau_{L,t}}{(1-\tau_{L,t})l_t})}, \tag{B20a}$$

$$\frac{\partial l_t}{\partial \hat{a}_t} = \frac{(1/\rho - \alpha)}{\hat{a}_t(\frac{1}{1-l_t} - \frac{\tau_{L,t}}{(1-\tau_{L,t})l_t})}, \tag{B20b}$$

$$\frac{\partial l_t}{\partial \hat{c}_t} = -\frac{1}{\hat{c}_t(\frac{1}{1-l_t} - \frac{\tau_{L,t}}{(1-\tau_{L,t})l_t})}, \tag{B20c}$$

$$\frac{\partial l_t}{\partial l_{Y,t}} = -\frac{\frac{\tau_{L,t}}{(1-\tau_{L,t})l} + \alpha}{l_{Y,t}(\frac{1}{1-l_t} - \frac{\tau_{L,t}}{(1-\tau_{L,t})l_t})}, \tag{B20d}$$

$$\frac{\partial l_t}{\partial \tau_K} = \frac{\frac{\alpha l_{Y,t}}{\eta(1-\alpha)l}}{(1-\tau_{L,t})(\frac{1}{1-l_t} - \frac{\tau_{L,t}}{(1-\tau_{L,t})l_t})}. \tag{B20e}$$

Based on (B3), (B4), (B7), (B12), (B18), and $C_t = N_t c_t$, we have:

$$g_{\hat{c},t} \equiv \frac{d\hat{c}_t/dt}{\hat{c}_t} = (1-\tau_K)\frac{\alpha}{\eta}(\hat{a}_t)^{1/\rho-\alpha}(\frac{l_{Y,t}}{\hat{k}_t})^{1-\alpha} - \delta - \beta - g_Y. \tag{B21}$$

From eqs (B10), (B12), (B13), and (B18), we can directly infer:

$$g_{\hat{k},t} \equiv \frac{d\hat{k}_t/dt}{\hat{k}_t} = (1-\zeta)(\hat{a}_t)^{1/\rho-\alpha}(\frac{l_{Y,t}}{\hat{k}_t})^{1-\alpha} - \frac{\hat{c}_t}{\hat{k}_t} - \delta - g_Y. \tag{B22}$$

According to eqs (B14) and (B18), we can futher obtain:

$$g_{\hat{a},t} \equiv \frac{d\hat{a}_t/dt}{\hat{a}_t} = \frac{\varsigma}{1+\psi}\frac{[l_t(\hat{k}_t, \hat{a}_t, \hat{c}_t, l_{Y,t}; \tau_K) - l_{Y,t}]^{\lambda}}{\hat{a}_t^{1-\phi}} - g_A. \tag{B23}$$

In what follows, to simplify the notation we suppress those arguments of the laobr supply function. From eq. (B18), taking logarithms of eqs (B19a) and (B12) and differentiating the resulting equations with respect to time, we have:

$$g_{\hat{y},t} = (1/\rho - \alpha)g_{\hat{a},t} + \alpha g_{\hat{k},t} + (1-\alpha)(\dot{l}_Y/l_Y), \tag{B24}$$

$$\dot{l}_t/l_t = 1/[l_t/(1-l_t)-\tau_{L,t}/(1-\tau_{L,t})]\{(1/\rho-\alpha)g_{\hat{a},t}+\alpha g_{\hat{k},t}-g_{\hat{c},t}-[\alpha+\tau_{L,t}/(1-\tau_{L,t})](\dot{l}_{Y,t}/l_{Y,t})\}. \tag{B25}$$

Taking logarithms of eq. (B15) differentiating the resulting equation with respect to time, we obtain:

$$\frac{\dot{P}_{A,t}}{P_{A,t}} = (g_{\hat{y},t}+g_Y) - \lambda n + (1-\lambda)\frac{\dot{l}_t}{1-l_{Y,t}} - [1+(1-\lambda)\frac{l_{Y,t}}{l_t-l_{Y,t}}](\dot{l}_{Y,t}/l_{Y,t}). \tag{B26}$$

Combinning eqs (B9), (B15), (B18), (B21), (B24), (B25), and (B26) together, we obtain:

$$\begin{aligned} \frac{dl_{Y,t}/dt}{l_{Y,t}} \equiv \frac{\dot{l}_{Y,t}}{l_{Y,t}} = & -\frac{r_t - g_Y - g_{\hat{c},t} + \lambda n + [\psi + \phi - \frac{(\eta-1)\alpha(1+\psi)l_{Y,t}}{(1-\alpha)\eta(l_t-l_{Y,t})}](g_{\hat{a},t}+g_A)}{\{\frac{(1-\lambda)l_t/(l_t-l_{Y,t})[\alpha+\tau_{L,t}/(1-\tau_{L,t})]}{l_t/(1-l_t)-\tau_{L,t}/(1-\tau_{L,t})} + \alpha + (1-\lambda)\frac{l_{Y,t}}{l_t-l_{Y,t}}\}} \\ & + \frac{[1+\frac{(1-\lambda)l_t/(l_t-l_{Y,t})}{l_t/(1-l_t)-\tau_{L,t}/(1-\tau_{L,t})}][(1/\rho-\alpha)g_{\hat{a},t}+\alpha g_{\hat{k},t}-g_{\hat{c},t}]}{\{\frac{(1-\lambda)l_t/(l_t-l_{Y,t})[\alpha+\tau_{L,t}/(1-\tau_{L,t})]}{l_t/(1-l_t)-\tau_{L,t}/(1-\tau_{L,t})} + \alpha + (1-\lambda)\frac{l_{Y,t}}{l_t-l_{Y,t}}\}}. \end{aligned} \tag{B27}$$

Note that $r_t - g_Y - g_{\hat{c},t} = \beta$. As a result, In the steady state we have $r - g_Y = \beta$.

Inserting eq. (B18) into eq. (B17b) yields:

$$\tau_{L,t} = \frac{\zeta - \frac{\alpha}{\eta}\tau_K}{1-\alpha}\frac{l_{Y,t}}{l_t}. \tag{B28}$$

Based on eqs (B21), (B22), (B23),(B27), and (B28), the dynamic system can be expressed as:

$$\frac{d\hat{k}_t/dt}{\hat{k}_t} = (1-\zeta)(\hat{a}_t)^{1/\rho-\alpha}(\frac{l_{Y,t}}{\hat{k}_t})^{1-\alpha} - \frac{\hat{c}_t}{\hat{k}_t} - \delta - g_Y, \tag{B29a}$$

$$\frac{d\hat{a}_t/dt}{\hat{a}_t} = \frac{\varsigma}{1+\psi}\frac{(l_t-l_{Y,t})^{\lambda}}{\hat{a}_t^{1-\phi}} - g_A, \tag{B29b}$$

$$\frac{d\hat{c}_t/dt}{\hat{c}_t} = (1-\tau_K)\frac{\alpha}{\eta}(\hat{a}_t)^{1/\rho-\alpha}(\frac{l_{Y,t}}{\hat{k}_t})^{1-\alpha} - \delta - \beta - g_Y, \tag{B29c}$$

$$\frac{dl_{Y,t}/dt}{l_{Y,t}} = -\frac{\beta+\lambda n+[\psi+\phi-\frac{(\eta-1)\alpha(1+\psi)l_{Y,t}}{(1-\alpha)\eta(l_t-l_{Y,t})}](g_{\hat{a},t}+g_A)}{\{\frac{(1-\lambda)l_t/(l_t-l_{Y,t})[\alpha+\tau_{L,t}/(1-\tau_{L,t})]}{l_t/(1-l_t)-\tau_{L,t}/(1-\tau_{L,t})}+\alpha+(1-\lambda)\frac{l_{Y,t}}{l_t-l_{Y,t}}\}} + \frac{[1+\frac{(1-\lambda)l_t/(l_t-l_{Y,t})}{l_t/(1-l_t)-\tau_{L,t}/(1-\tau_{L,t})}][(1/\rho-\alpha)g_{\hat{a},t}+\alpha g_{\hat{k},t}-g_{\hat{c},t}]}{\{\frac{(1-\lambda)l_t/(l_t-l_{Y,t})[\alpha+\tau_{L,t}/(1-\tau_{L,t})]}{l_t/(1-l_t)-\tau_{L,t}/(1-\tau_{L,t})}+\alpha+(1-\lambda)\frac{l_{Y,t}}{l_t-l_{Y,t}}\}}. \tag{B29d}$$

Linearizing eqs (B29a), (B29b), (B29c), and (B29d) around the steady-state equilibrium yields:

$$\begin{pmatrix} d\hat{k}_t/dt \\ d\hat{a}_t/dt \\ d\hat{c}_t/dt \\ dl_{Y,t}/dt \end{pmatrix} = \begin{pmatrix} b_{11} & b_{12} & b_{13} & b_{14} \\ b_{21} & b_{22} & b_{23} & b_{24} \\ b_{31} & b_{32} & b_{33} & b_{34} \\ b_{41} & b_{42} & b_{43} & b_{44} \end{pmatrix} \begin{pmatrix} \hat{k}_t - \hat{k} \\ \hat{a}_t - \hat{a} \\ \hat{c}_t - \hat{c} \\ \hat{l}_{Y,t} - l_Y \end{pmatrix} + \begin{pmatrix} b_{15} \\ b_{25} \\ b_{35} \\ b_{45} \end{pmatrix} d\tau_K, \tag{B30}$$

where

$b_{11} = \frac{\partial(d\hat{k}_t/dt)}{\partial\hat{k}_t}$, $b_{12} = \frac{\partial(d\hat{k}_t/dt)}{\partial\hat{a}_t}$, $b_{13} = \frac{\partial(d\hat{k}_t/dt)}{\partial\hat{c}_t}$, $b_{14} = \frac{\partial(d\hat{k}_t/dt)}{\partial l_{Y,t}}$, $b_{15} = \frac{\partial(d\hat{k}_t/dt)}{\partial\tau_K}$,

$b_{21} = \frac{\partial(d\hat{a}_t/dt)}{\partial\hat{k}_t}$, $b_{22} = \frac{\partial(d\hat{a}_t/dt)}{\partial\hat{a}_t}$, $b_{23} = \frac{\partial(d\hat{a}_t/dt)}{\partial\hat{c}_t}$, $b_{24} = \frac{\partial(d\hat{a}_t/dt)}{\partial l_{Y,t}}$, $b_{25} = \frac{\partial(d\hat{a}_t/dt)}{\partial\tau_K}$,

$b_{31} = \frac{\partial(d\hat{c}_t/dt)}{\partial\hat{k}_t}$, $b_{32} = \frac{\partial(d\hat{c}_t/dt)}{\partial\hat{a}_t}$, $b_{33} = \frac{\partial(d\hat{c}_t/dt)}{\partial\hat{c}_t}$, $b_{34} = \frac{\partial(d\hat{c}_t/dt)}{\partial l_{Y,t}}$, $b_{35} = \frac{\partial(d\hat{c}_t/dt)}{\partial\tau_K}$,

$b_{41} = \frac{\partial(dl_{Y,t}/dt)}{\partial\hat{k}_t}$, $b_{42} = \frac{\partial(dl_{Y,t}/dt)}{\partial\hat{a}_t}$, $b_{43} = \frac{\partial(dl_{Y,t}/dt)}{\partial\hat{c}_t}$, $b_{44} = \frac{\partial(dl_{Y,t}/dt)}{\partial l_{Y,t}}$, $b_{45} = \frac{\partial(dl_{Y,t}/dt)}{\partial\tau_K}$.

Due to the complicated calculations, we do not list the analytical results for b_{ij}, where $i \in \{1, 2, 3, 4, 5\}$ and $j \in \{1, 2, 3, 4, 5\}$.

Let ℓ_1, ℓ_2, ℓ_3, and ℓ_4 be the four characteristic roots of the dynamic system. Due to the complexity calculations of the four characteristic roots, we do not try to prove the saddle-point stability analytically. Instead, we show that the dynamic system exists two positive and two negative characteristic roots via a numerical simulation. For expository convenience, in what follows let ℓ_1 and ℓ_2 be the negative root as well as ℓ_3 and ℓ_4 be the positive roots. The general general solution is given by:

$$\begin{pmatrix} \hat{k}_t \\ \hat{a}_t \\ \hat{c}_t \\ l_{Y,t} \end{pmatrix} = \begin{pmatrix} \hat{k}(\tau_K) \\ \hat{a}(\tau_K) \\ \hat{c}(\tau_K) \\ l_Y(\tau_K) \end{pmatrix} + \begin{pmatrix} 1 & 1 & 1 & 1 \\ h_{21} & h_{22} & h_{23} & h_{24} \\ h_{31} & h_{32} & h_{33} & h_{34} \\ h_{41} & h_{42} & h_{43} & h_{44} \end{pmatrix} \begin{pmatrix} D_1 e^{\ell_1 t} \\ D_2 e^{\ell_2 t} \\ D_3 e^{\ell_3 t} \\ D_4 e^{\ell_4 t} \end{pmatrix}. \qquad \text{(B31a)}$$

where D_1, D_2, D_3, and D_4 are undetermined coefficients and

$$\triangle_j = \begin{vmatrix} b_{12} & b_{13} & b_{14} \\ b_{22}-\ell_j & b_{23} & b_{24} \\ b_{32} & b_{33}-\ell_j & b_{34} \end{vmatrix} ;\ j \in \{1,\ 2,\ 3,\ 4\}, \tag{B31b}$$

$$h_{2j} = \begin{vmatrix} \ell_j - b_{11} & b_{13} & b_{14} \\ -b_{21} & b_{23} & b_{24} \\ -b_{31} & b_{33}-\ell_j & b_{34} \end{vmatrix} / \triangle_j \ ;\ j \in \{1,\ 2,\ 3,\ 4\}, \tag{B31c}$$

$$h_{3j} = \begin{vmatrix} b_{12} & -b_{11} & b_{14} \\ b_{22}-\ell_j & -b_{21} & b_{24} \\ b_{32} & -b_{31} & b_{34} \end{vmatrix} / \triangle_j \ ;\ j \in \{1,\ 2,\ 3,\ 4\}, \tag{B31d}$$

$$h_{4j} = \begin{vmatrix} b_{12} & b_{13} & \ell_j - b_{11} \\ b_{22}-\ell_j & b_{23} & -b_{21} \\ b_{32} & b_{33}-\ell_j & -b_{31} \end{vmatrix} / \triangle_j \ ;\ j \in \{1,\ 2,\ 3,\ 4\}. \tag{B31e}$$

The government changes the capital tax rate τ_K from τ_{K0} to τ_{K1} at t=0, based on eqs (B31a)-(B31e), we employ the following equations to discribe the dynamic adjustment of $\hat{k}_t$, $\hat{a}_t$, $\hat{c}_t$ and $l_{Y,t}$:

$$\hat{k}_t = \begin{cases} \hat{k}(\tau_{K0}); & t = 0^- \\ \hat{k}(\tau_{K1}) + D_1 e^{\ell_1 t} + D_2 e^{\ell_2 t} + D_3 e^{\ell_3 t} + D_4 e^{\ell_4 t}; & t \geq 0^+ \end{cases} \tag{B32a}$$

$$\hat{a}_t = \begin{cases} \hat{a}(\tau_{K0}); & t = 0^- \\ \hat{a}(\tau_{K1}) + h_{21} D_1 e^{\ell_1 t} + h_{22} D_2 e^{\ell_2 t} + h_{23} D_3 e^{\ell_3 t} + h_{24} D_4 e^{\ell_4 t}; & t \geq 0^+ \end{cases} \tag{B32b}$$

$$\hat{c}_t = \begin{cases} \hat{c}(\tau_{K0}); & t = 0^- \\ \hat{c}(\tau_{K1}) + h_{31} D_1 e^{\ell_1 t} + h_{32} D_2 e^{\ell_2 t} + h_{33} D_3 e^{\ell_3 t} + h_{34} D_4 e^{\ell_4 t}; & t \geq 0^+ \end{cases} \tag{B32c}$$

$$l_{Y,t} = \begin{cases} l_Y(\tau_{K0}); & t = 0^- \\ l_Y(\tau_{K1}) + h_{41} D_1 e^{\ell_1 t} + h_{42} D_2 e^{\ell_2 t} + h_{43} D_3 e^{\ell_3 t} + h_{44} D_4 e^{\ell_4 t}; & t \geq 0^+ \end{cases} \tag{B32d}$$

where 0^- and 0^+ denote the instant before and after the policy implementation, respectively. The values for D_1, D_2, D_3 and D_4 are determined by:

$$\hat{k}_{0^-} = \hat{k}_{0^+}, \tag{B33a}$$

$$\hat{a}_{0^-} = \hat{a}_{0^+}, \tag{B33b}$$

$$D_3 = D_4 = 0. \tag{B33c}$$

Equations (B33a) and (B33b) indicate that both $\hat{k}_t$ $(= \frac{K_t}{N_t^{\sigma}})$ and $\hat{a}_t$ $(= \frac{A_t}{N_t^{\lambda/(1-\phi)}}$) remain intact at the instant of policy implementation since K_t, A_t, and N_t are predetermined variables. Equation (B33c) is the stability condition which ensures that all $\hat{k}_t$, $\hat{a}_t$, $\hat{c}_t$ and $l_{Y,t}$ converge to their new steady-state equilibrium. By using eqs (B33a) and (B33b), we can obtain:

$$D_1 = \frac{[\hat{k}(\tau_{K0}) - \hat{k}(\tau_{K1})]h_{22} - [\hat{a}(\tau_{K0}) - \hat{a}(\tau_{K1})]}{h_{22} - h_{21}}, \tag{B34a}$$

$$D_2 = \frac{[\hat{a}(\tau_{K0}) - \hat{a}(\tau_{K1})] - [\hat{k}(\tau_{K0}) - \hat{k}(\tau_{K1})]h_{21}}{h_{22} - h_{21}}. \tag{B34b}$$

Inserting eqs (B33c), (B34a), and (B34b) into eqs (B32a)-(B32d) yields:

$$\hat{k}_t = \begin{cases} \hat{k}(\tau_{K0}); & t = 0^- \\ \hat{k}(\tau_{K1}) + \frac{[\hat{k}(\tau_{K0}) - \hat{k}(\tau_{K1})]h_{22} - [\hat{a}(\tau_{K0}) - \hat{a}(\tau_{K1})]}{h_{22} - h_{21}} e^{\ell_1 t} & t \geq 0^+ \\ \quad + \frac{[\hat{a}(\tau_{K0}) - \hat{a}(\tau_{K1})] - [\hat{k}(\tau_{K0}) - \hat{k}(\tau_{K1})]h_{21}}{h_{22} - h_{21}} e^{\ell_2 t}; & \end{cases}$$

$$\hat{a}_t = \begin{cases} \hat{a}(\tau_{K0}); & t = 0^- \\ \hat{a}(\tau_{K1}) + \frac{\{[\hat{k}(\tau_{K0}) - \hat{k}(\tau_{K1})]h_{22} - [\hat{a}(\tau_{K0}) - \hat{a}(\tau_{K1})]\}h_{21} e^{\ell_1 t}}{h_{22} - h_{21}} & t \geq 0^+ \\ \quad + \frac{\{[\hat{a}(\tau_{K0}) - \hat{a}(\tau_{K1})] - [\hat{k}(\tau_{K0}) - \hat{k}(\tau_{K1})]h_{21}\}h_{22} e^{\ell_2 t}}{h_{22} - h_{21}}; & \end{cases}$$

$$\hat{c}_t = \begin{cases} \hat{c}(\tau_{K0}); & t = 0^- \\ \hat{c}(\tau_{K1}) + \frac{\{[\hat{k}(\tau_{K0}) - \hat{k}(\tau_{K1})]h_{22} - [\hat{a}(\tau_{K0}) - \hat{a}(\tau_{K1})]\}h_{31} e^{\ell_1 t}}{h_{22} - h_{21}} & t \geq 0^+ \\ \quad + \frac{\{[\hat{a}(\tau_{K0}) - \hat{a}(\tau_{K1})] - [\hat{k}(\tau_{K0}) - \hat{k}(\tau_{K1})]h_{21}\}h_{32} e^{\ell_2 t}}{h_{22} - h_{21}}; & \end{cases}$$

$$l_{Y,t} = \begin{cases} l_Y(\tau_{K0}); & t = 0^- \\ l_Y(\tau_{K1}) + \frac{\{[\hat{k}(\tau_{K0}) - \hat{k}(\tau_{K1})]h_{22} - [\hat{a}(\tau_{K0}) - \hat{a}(\tau_{K1})]\}h_{41} e^{\ell_1 t}}{h_{22} - h_{21}} & t \geq 0^+ \\ \quad + \frac{\{[\hat{a}(\tau_{K0}) - \hat{a}(\tau_{K1})] - [\hat{k}(\tau_{K0}) - \hat{k}(\tau_{K1})]h_{21}\}h_{42} e^{\ell_2 t}}{h_{22} - h_{21}}; & \end{cases}$$

Appendix 3.C. Proof of comparative statics

From eqs (B29a)-(B29d), we have:

$$\frac{d\hat{k}_t/dt}{\hat{k}_t} = (1-\zeta)(\hat{a}_t)^{1/\rho-\alpha}(\frac{l_{Y,t}}{\hat{k}_t})^{1-\alpha} - \frac{\hat{c}_t}{\hat{k}_t} - \delta - g_Y, \tag{C1a}$$

$$\frac{d\hat{a}_t/dt}{\hat{a}_t} = \frac{\varsigma}{1+\psi}\frac{[l_t(\hat{k}_t,\hat{a}_t,\hat{c}_t,l_{Y,t};\tau_K) - l_{Y,t}]^{\lambda}}{\hat{a}_t^{1-\phi}} - g_A, \tag{C1b}$$

$$\frac{d\hat{c}_t/dt}{\hat{c}_t} = (1-\tau_K)\frac{\alpha}{\eta}(\hat{a}_t)^{1/\rho-\alpha}(\frac{l_{Y,t}}{\hat{k}_t})^{1-\alpha} - \delta - \beta - g_Y, \tag{C1c}$$

$$\frac{dl_{Y,t}/dt}{l_{Y,t}} = -\frac{\beta+\lambda n+[\psi+\phi-\frac{(\eta-1)\alpha(1+\psi)l_{Y,t}}{(1-\alpha)\eta(l_t-l_{Y,t})}](g_{\hat{a},t}+g_A)}{\{\frac{(1-\lambda)l_t/(l_t-l_{Y,t})[\alpha+\tau_{L,t}/(1-\tau_{L,t})]}{l_t/(1-l_t)-\tau_{L,t}/(1-\tau_{L,t})}+\alpha+(1-\lambda)\frac{l_{Y,t}}{l_t-l_{Y,t}}\}} + \frac{[1+\frac{(1-\lambda)l_t/(l_t-l_{Y,t})}{l_t/(1-l_t)-\tau_{L,t}/(1-\tau_{L,t})}][(1/\rho-\alpha)g_{\hat{a},t}+\alpha g_{\hat{k},t}-g_{\hat{c},t}]}{\{\frac{(1-\lambda)l_t/(l_t-l_{Y,t})[\alpha+\tau_{L,t}/(1-\tau_{L,t})]}{l_t/(1-l_t)-\tau_{L,t}/(1-\tau_{L,t})}+\alpha+(1-\lambda)\frac{l_{Y,t}}{l_t-l_{Y,t}}\}}. \tag{C1d}$$

In the steady state $\frac{d\hat{k}_t/dt}{\hat{k}_t} = \frac{d\hat{a}_t/dt}{\hat{a}_t} = \frac{d\hat{c}_t/dt}{\hat{c}_t} = \frac{dl_{Y,t}/dt}{l_{Y,t}} = 0$, we then have the following steady-state results:

$$\frac{\hat{c}}{\hat{k}} = (1-\zeta)(\hat{a})^{1/\rho-\alpha}(\frac{l_Y}{\hat{k}})^{1-\alpha} - \delta - g_Y, \tag{C1e}$$

$$g_A = \frac{\varsigma}{1+\psi}\frac{(l-l_Y)^{\lambda}}{\hat{a}^{1-\phi}}, \tag{C1f}$$

$$\beta = (1-\tau_K)\frac{\alpha}{\eta}(\hat{a})^{1/\rho-\alpha}(\frac{l_Y}{\hat{k}})^{1-\alpha} - \delta - g_Y, \tag{C1g}$$

$$0 = \beta + \lambda n + [\psi+\phi-\frac{(\eta-1)\alpha(1+\psi)l_Y}{(1-\alpha)\eta(l-l_Y)}]g_A. \tag{C1h}$$

Based on $l_Y \equiv (1-s)l$, we have:

$$\frac{l_Y}{l-l_Y} = \frac{(1-s)l}{l-(1-s)l} = \frac{(1-s)}{s}. \tag{C2a}$$

Inserting eq. (C2a) into eq. (C1h) yields:

$$s = \frac{\frac{\eta-1}{\eta}\frac{\alpha}{1-\alpha}(1+\psi)g_A}{\beta + \left(1 + \frac{\eta-1}{\eta}\frac{\alpha}{1-\alpha}\right)(1+\psi)g_A}. \tag{C2b}$$

From eqs (B3) and (C1g), we can obtain

$$r - g_Y = \beta. \tag{C3}$$

Equation eq. (C1g) can be rearranged as:

$$\hat{y}/\hat{k} = (\hat{a})^{1/\rho-\alpha}(\frac{l_Y}{\hat{k}})^{1-\alpha} = \frac{\eta(\beta+\delta+g_Y)}{\alpha(1-\tau_K)}. \tag{C4a}$$

Substituting eq. (C4a) into eq. (C1e) gives rise to:

$$\frac{\hat{c}}{\hat{y}} = \{(1-\zeta)\frac{\eta(\beta+\delta+g_Y)}{\alpha(1-\tau_K)} - \delta - g_Y\}\frac{\hat{k}}{\hat{y}} = (1-\zeta) - (\delta+g_Y)\frac{\alpha(1-\tau_K)}{\eta(\beta+\delta+g_Y)}. \tag{C5a}$$

To ensure the steady-state consumption-output ratio $\hat{c}/\hat{y}$ is positive, we impose the restriction $(1-\zeta) - (\delta+g_Y)\frac{\alpha(1-\tau_K)}{\eta(\beta+\delta+g_Y)} > 0$ for all values of the time preference rate β. As a consequence, $\lim_{\beta\to 0}\ \hat{c}/\hat{y}>0$ implies:

$$(1-\zeta) - \frac{\alpha(1-\tau_K)}{\eta} > 0. \tag{C5b}$$

From $l_Y \equiv (1-s)l$ and eq. (C1f), We can derive:

$$\hat{a} = [\frac{\varsigma}{(1+\psi)g_A}]^{1/(1-\phi)}(sl)^{\lambda/(1-\phi)}. \tag{C6}$$

Based on eq. (B28) and $l_Y \equiv (1-s)l$, we can infer the following expression:

$$\tau_L = (1-s)\frac{\zeta - \frac{\alpha}{\eta}\tau_K}{1-\alpha}, \tag{C7a}$$

where

$$\frac{\partial \tau_L}{\partial \tau_K} = -(1-s)\frac{\frac{\alpha}{\eta}}{1-\alpha} < 0. \tag{C7b}$$

Equipped with eqs (B1), (B2), (B5), and $L_Y = N(1-s)l$, we can obtain:

$$\frac{l}{1-l}\chi = \frac{\hat{y}}{\hat{c}}\frac{(1-\tau_L)(1-\alpha)}{(1-s)}. \tag{C8}$$

Inserting eqs (C5a) and (C7a) into eq. (C8) yields:

$$l = \begin{cases} 1 - \dfrac{\chi}{\chi + \frac{1}{[(1-\zeta)-(\delta+g_Y)\frac{\alpha(1-\tau_K)}{\eta(\beta+\delta+g_Y)}]}\frac{(1-\tau_L)(1-\alpha)}{(1-s)}} & ; \quad \chi > 0 \\ 1 & ; \quad \chi = 0 \end{cases}, \tag{C9a}$$

where

$$\frac{\partial l}{\partial \tau_K} = \begin{cases} \dfrac{\alpha\beta(\frac{1-s}{1-\alpha})[1-\zeta+\frac{\eta-1}{\eta}\frac{\alpha(\delta+g_Y)}{\beta+(1+\psi)g_A}](1-l)l}{\eta(\beta+\delta+g_Y)(1-\tau_L)[1-\zeta-(\delta+g_Y)\frac{\alpha(1-\tau_K)}{\eta(\beta+\delta+g_Y)}]} > 0 & ; \quad \chi > 0 \\ 0 & ; \quad \chi = 0 \end{cases}. \tag{C9b}$$

Combinning eqs (C2b), (C6), and (C9b) together, we can derive

$$\hat{a} = [\frac{\varsigma}{(1+\psi)g_A}]^{1/(1-\phi)}(sl)^{\lambda/(1-\phi)}, \tag{C10a}$$

where

$$\frac{\partial \hat{a}}{\partial \tau_K} = \frac{\lambda}{(1-\phi)}\hat{a}\frac{\partial l}{l\partial \tau_K} > 0. \tag{C10b}$$

Based on eqs (C4a), (C9b), (B12), and (B18), we have:

$$\hat{y} = \hat{a}^{\frac{1/\rho - \alpha}{1-\alpha}} [\frac{\alpha(1-\tau_K)}{\eta(\beta+\delta+g_Y)}]^{\frac{\alpha}{1-\alpha}} (1-s)l, \tag{C11a}$$

where

$$\frac{\partial \hat{y}}{\partial \tau_K} = [\sigma \frac{\partial l}{l \partial \tau_K} - \frac{\alpha}{(1-\alpha)(1-\tau_K)}] \hat{y} \gtrless 0, \quad \sigma \equiv 1 + \frac{1/\rho - \alpha}{1-\alpha} \frac{\lambda}{1-\phi}. \tag{C11b}$$

According to eqs (C4a), (C5a), and (C11b) yields:

$$\hat{k} = \frac{\alpha(1-\tau_K)}{\eta(\beta+\delta+g_Y)} \hat{y}, \tag{C12a}$$

$$\hat{c} = [(1-\zeta) - (\delta+g_Y) \frac{\alpha(1-\tau_K)}{\eta(\beta+\delta+g_Y)}] \hat{y}, \tag{C12b}$$

Inserting eq. (C11a) into (C12a) and (C12b), we can derive the following comparative statics:

$$\frac{\partial \hat{k}}{\partial \tau_K} = \frac{\alpha(1-\tau_K)\hat{y}}{\eta(\beta+\delta+g_Y)} \{\sigma \frac{\partial l}{l \partial \tau_K} - \frac{1}{(1-\alpha)(1-\tau_K)}\} \gtrless 0, \tag{C12c}$$

$$\frac{\partial \hat{c}}{\partial \tau_K} = \{\frac{\alpha(\delta+g_Y)}{\eta(\beta+\delta+g_Y)} + [(1-\zeta) - \frac{\alpha(1-\tau_K)(\delta+g_Y)}{\eta(\beta+\delta+g_Y)}][\sigma \frac{\partial l}{l \partial \tau_K} - \frac{\alpha}{(1-\alpha)(1-\tau_K)}]\} \hat{y} \gtrless 0. \tag{C12d}$$

CHAPTER 4

SHORT-RUN AND LONG-RUN EFFECTS OF CAPITAL TAXATION ON ECONOMIC GROWTH IN A R&D-BASED MODEL WITH ENDOGENOUS MARKET STRUCTURE

4.1 Introduction

The linkage between capital taxation and economic growth has been studied extensively in the field of macroeconomics. In general, the existing studies on this topic can be classified into two strands of literature. The first strand emphasizes the growth engine of capital accumulation, and finds that capital taxation stifle economic growth. To be more specific, the tax imposed on capital income leads the household to reduce its accumulation of physical capital, and hence is detrimental to economic growth. The relevant literature in this strand includes Judd (1985), Chamley (1986), King and Rebelo (1990), Jones et al. (1993), Devereux and Love (1994), and Milesi-Ferretti and Roubini (1998).

The second strand instead highlights the growth engine of R&D investment, and finds that mixed relationship between capital taxation and economic growth. More specifically, a rise in the capital income tax rate motivates intermediate firms to lower its demand for physical capital, thereby causing a decline in the profit of intermediate firms. This in turn lowers R&D investment and economic growth. However, if the government adopt a tax shifting scheme to balance its budget (i.e., a rise in the capital income tax rate is coupled with a fall in the labor income tax rate), then an additional effect on the household's labor supply is present. This additional effect generates a stimulating effect on R&D investment and economic growth.[1] As a consequence, if this additional effect is taken into consideration, capital taxation may either spur or stifle economic growth. The relevant literature in this strand includes Lin and Russo (1999), Zeng and Zhang (2002), Aghion et al. (2013), and Chen et al. (2016).[2]

With regard to the empirical studies on capital taxation and economic growth, there is also lack of consensus within the existing literature. Lee and Gordon (2005), Hungerford (2010), Arnold et al. (2011), and Mertens and Ravn (2013) find that capital taxation, such as corporate profit tax, capital gains tax, dividends tax, has an adverse effect on economic growth, while Mendoza et al. (1997), Angelopoulos et al. (2007), and ten Kate and Milionis (2015) find that capital taxation may be neutral with or even beneficial to economic growth.

The main purpose of this chapter is to explain these mixed observations from the perspective of time horizon. To this end, we set up a second-generation R&D-based growth model developed by Dinopoulos and Thompson (1998) and Peretto (1998). As is well known, some salient features are exhibited in the second-generation R&D-based growth model. The first feature is that both vertical and horizontal innovations are present simultaneously. In the vertical dimension, each of incumbent firms engages in in-house R&D to improve the quality of their specific product. In the horizontal dimension, firms enter the market through the creation new products. The second feature is that the economic growth rate is crucially related to the rate of returns to the firm's in-house R&D. The third feature is that

[1]With this additional reduction in the labor income tax rate, the household is inclined to provide higher labor supply, causing the final-good firm to raise its production. Then, the final-goods firm will increase its demand for intermediate goods, thereby resulting in a rise in the profit of intermediate firms and boosting R&D investment and economic growth.

[2]Yilmaz (2013) specifies that an increase in the capital income tax rate is coupled with a rise in the subsidy on R&D to balance the government budget. Based on the fact that the additional subsidy effect is beneficial to R&D investment and economic growth, his analysis also shows that capital taxation may either boost or depress economic growth.

the returns to the firm's in-house R&D are determined by its market size rather than aggregate market size. With the second and third features, an expansion in the scale of the aggregate economy is completely fragmented by the proliferation of endogenous product varieties, causing the second-generation R&D-based growth model to be able to eliminate the undesirable scale effect.

Based on these features, our Schumpeterian growth model with endogenous market structure (EMS) finds that, in response to a change in the capital tax rate, the long-run and short-run responses of the economic growth rate exhibit distinct patterns.[3] To be more precise, in the short run where the number of firms is fixed, a higher capital income tax rate is harmful to economic growth. During the transitional process, with the number of firms adjust endogenously, economic growth keeps on rising as each of the in-house R&D firms continues to expand its market size. In the long run, with the equal counteracting strength between the short run and the transition period, capital taxation is neutral with economic growth. This provides a plausible explanation for the mixed empirical observations between capital taxation and economic growth.

Some empirical studies support the features exhibited in the second-generation R&D-based growth model. As mentioned previously, a prominent advantage in this strand of the literature is that an expansion in the scale of the aggregate economy is perfectly fragmented by the proliferation of endogenous product varieties. This makes the undesirable scale effect be eliminated. Laincz and Peretto (2006) use the US data over the period 1964-2001, and find that the empirical evidence supports this feature. Moreover, the empirical works of Cohen and Klepper (1996a; 1996b) and Adams and Jaffe (1996) support that the plant-level productivity of the R&D firms depends upon the firm's market size (R&D per plant) rather than aggregate

[3] EMS is characterized by firm's endogenous entry.

market size (total amount of R&D), which is also the main feature of this strand of the literature.

Several existing studies on taxation and economic growth in the R&D-based growth model are closely related to our paper. By using a non-scale R&D growth model developed by Howitt (1999), Zeng and Zhang (2002) show that the balanced growth rate of per capita output is independent of labor income tax and consumption tax, while it is negatively related to capital income tax. Conversely, Lin and Russo (1999) analyze how the taxes imposed on distinct sources of capital income affects the long-run growth rate, and find that a higher capital income tax rate for innovative firms could stimulate economic growth if the tax system allows tax credits for R&D spending. In departing from these two papers, this paper instead highlights that the dynamic adjustment of the firm's market size is crucial for determining the effects of capital taxation on economic growth in both the short run and the transition period. More recently, Aghion et al. (2013) and Hong (2014) develop a quality-ladder R&D-based growth model to deal with optimal capital taxation. More specifically, they turn their main focus to the normative analysis to examine the validity of the Chamley-Judd (Chamley 1986; Judd 1985) result, i.e., a zero optimal capital tax. This paper instead sets up a second-generation R&D-based growth model, and focuses on the positive analysis regarding how capital taxation affects economic growth in both the short run and the long run.

The rest of the paper is arranged as follows. Section 4.2 sets up a R&D-based growth model featuring EMS. Section 4.3 analyzes the effects of capital taxation on economic growth and market structure. Section 4.4 calibrates the parameters and provides a quantitative analysis of capital taxation. Finally, some concluding remarks are provided in Section 4.5.

4.2 The model

4.2.1 *Households*

Households are homogenous, infinitely-lived, and endowed with one unit of time which can be allocated between work and leisure. The representative household maximizes the following lifetime utility:[4]

$$U = \int_0^\infty e^{-\rho t} \left[\ln C_t + \chi(1 - L_t)\right] \, dt, \rho > 0, \chi \geq 0, \tag{1}$$

subject to:

$$\dot{K}_t + \dot{A}_t = r_{A,t} A_t + (1 - \tau_{K,t}) r_{K,t} K_t + (1 - \tau_{L,t}) w_t L_t - C_t. \tag{2}$$

In eq. (1), C_t is consumption of final goods and L_t is total labor supply so that $1 - L_t$ is leisure time. The parameter ρ is time preference, and the parameter χ reflects the preference for leisure. In eq. (2), K_t is physical capital and $r_{K,t}$ is the return on capital. A_t is the value of equity shares issued by intermediate firms and $r_{A,t}$ is the return on equity shares. Physical capital and R&D stocks are perfectly substitute, so that the returns on these two assets follow the no-arbitrage condition: $r_{A,t} = (1-\tau_{K,t}) r_{K,t}$. We assume perfectly mobile labor; accordingly a uniform wage rate, denoted by w_t, will hold across sectors. The government imposes the capital income tax $\tau_{K,t}$ and the labor income tax $\tau_{L,t}$ on the households.

From standard dynamic optimization, we can derive the usual Keynes-Ramsey rules:

$$\frac{\dot{C}_t}{C_t} = (1 - \tau_{K,t}) r_{K,t} - \rho, \tag{3}$$

[4] Our results are robust to a more general utility function given by $\ln C_t + \chi \frac{(1-L_t)^{1-\eta}}{1-\eta}$ for $\chi > 0$ and $\eta \geq 0$. However, when $\eta > 0$, the equilibrium allocations of labor do not have closed form solutions. Therefore, we are centering on the special case of η= 0 for analytical tractability.

and the optimality condition for labor supply determines the wage rate:

$$(1-\tau_{L,t})w_t = \chi C_t. \tag{4}$$

4.2.2 *The final goods sector*

There is a competitive representative firm producing a single final good Y_t (numeraire). Following Peretto (2007, 2011) and Chu and Ji (2016), the production function is specified as:[5]

$$Y_t = \int_0^{N_t} X_t^{\theta}(j)(Z_t^{\alpha}(j)Z_t^{1-\alpha}\frac{L_{Y,t}}{N_t})^{1-\theta}dj, \theta \in (0,1), \alpha \in (0,1), \tag{5}$$

where $X_t(j)$ is intermediate goods of type $j \in [0, N_t]$, N_t is the number of intermediate goods, $Z_t(j)$ is the quality of good $X_t(j)$, $Z_t \equiv \frac{1}{N_t}\int_0^{N_t} Z_t(j)dj$ is the average quality of all intermediate goods, which represents the knowledge stock in the economy. The productivity of intermediate good $X_t(j)$ is positively related to its respective quality $Z_t(j)$ and also to the existing knowledge stock Z_t because of R&D spillovers. $L_{Y,t}$ is labor used in final goods production. Notice that since the final goods producer uses total N_t types of intermediate goods, the amount of labor input collocated with each type of intermediate good is $L_{Y,t}/N_t$.

The first-order conditions for the profit maximization problem of the final goods producer yield conditional demand functions for labor and intermediate goods:

$$w_t = (1-\theta)\frac{Y_t}{L_{Y,t}}, \tag{6}$$

$$p_t(j) = \theta X_t^{\theta-1}(j)\left[Z_t^{\alpha}(j)Z_t^{1-\alpha}\frac{L_{Y,t}}{N_t}\right]^{1-\theta}, \tag{7}$$

[5] Peretto (2013) considers a more general specification L_Y/N_t^{σ}, where $0 < \sigma < 1$ measures the soical return to varieties. Under tax shifting from labor income taxes to capital income taxes, our neutral result of capital taxation on long-run economic growth is robust to such a more general setting.

where $p_t(j)$ is the price of $X_t(j)$.

4.2.3 *The intermediate goods sector*

The intermediate goods sector is monopolistically competitive and comprised by a continuum of mass N_t of incumbent firms, each of which produces a single intermediate good $X_t(j)$, $j \in [0, N_t]$ with a perpetually protected patent for that good. Intermediate firms produce intermediate goods using capital as inputs with the technology that one unit of capital is used to produce one unit of intermediate goods, i.e., $X_t(j) = k_t(j)$ where $k_t(j)$ is the amount of capital employed by firm j. Intermediate firms also undertake in-house R&D that improves the quality of the good they produce. In-house R&D (vertical R&D) requires labor as inputs. The innovation technology is:

$$\dot{Z}_t(j) = \varphi Z_t(j) l_{Z,t}(j), \tag{8}$$

where φ reflects the productivity of in-house R&D, and $l_{Z,t}(j)$ is research labor employed by intermediate firm j.

The profit function of incumbent intermediate firm j is given by:[6]

$$\Pi_t(j) = p_t(j) X_t(j) - r_{K,t} k_t(j) - w_t l_{Z,t}(j). \tag{9}$$

The value of the jth monopolistic firm is:

$$V_t(j) = \int_t^\infty \exp\left(-\int_t^s r_v dv\right) \Pi_s(j) ds, \tag{10}$$

where r_t is the interest rate.

By solving the firm's maximization problem, we obtain the following first-order conditions:

[6] Our long-run growth effect of capital taxation is robust if we consider a sunk cost wl_x, where l_x is a fixed labor input for intermediate firms to operate in the business.

$$p_t(j) = \frac{1}{\theta} r_{K,t}, \tag{11a}$$

$$w_t = \lambda_t(j)\varphi Z_t(j), \tag{11b}$$

$$-\dot{\lambda}_t(j) + r_t\lambda_t(j) = \alpha\left[(p_t(j) - r_{K,t})\left(\frac{\theta}{p_t(j)}\right)^{\frac{1}{1-\theta}} Z_t^{\alpha-1}(j)Z_t^{1-\alpha}\frac{L_{Y,t}}{N_t}\right] + \lambda_t(j)\varphi l_{Z,t}(j), \tag{11c}$$

where $\lambda_t(j)$ is the co-state variable of $Z_t(j)$. eq. (11a) indicates that intermediate firms are symmetric. By taking log of eq. (11b) and total differentiating with respect to time we can obtain $\dot{w}_t/w_t = \dot{\lambda}_t/\lambda_t + \dot{Z}_t/Z_t$. Then, inserting eqs (11a) and (11c) into the above expression yields:

$$r_t = \frac{\alpha\varphi Z_t}{w_t}\left[(1-\theta)p_t(j)\left(\frac{\theta}{p_t(j)}\right)^{\frac{1}{1-\theta}}\frac{L_{Y,t}}{N_t}\right] + \frac{\dot{w}_t}{w_t}, \tag{12}$$

in which we have used the symmetry condition.

4.2.4 *Entrants*

Following Peretto (1998), it is assumed that a (potential) entrepreneur can create a new firm by running an R&D project. It hires labor to develop the blueprint that creates new a type of intermediate good and simultaneously expands the number of intermediate firms. The entry technology is specified as:

$$\dot{N} = \beta L_{N,t}, \tag{13a}$$

where β is the productivity in the variety R&D sector, and $L_{N,t}$ is the total amount of labor used for variety R&D. Since the value of an incumbent is V_t and the entry cost for each potential entrant is w_t/β, the no-arbitrage condition for entry is $V_t = w_t/\beta$.

In the horizontal R&D secotr (entry), the familiar Bellman equation is:

$$r_t = \frac{\Pi_t}{V_t} + \frac{\dot{V}_t}{V_t} \tag{13b}$$

4.2.5 *Government*

The government levies capital income taxes, labor income taxes, and lump-sum taxes to finance its public spending G_t. The balanced government budget constraint can be expressed as:

$$\tau_{K,t} r_{K,t} K_t + \tau_{L,t} w_t L_t = G_t \tag{14}$$

The government spending is a fixed proportion $\zeta \in (0,1)$ of final output, namely:

$$G_t = \zeta Y_t. \tag{15}$$

4.2.6 *Markets clearing and aggregation*

Given that the intermediate firms are symmetric, the capital market clearing condition is $K_t = N_t k_t$. The stock market clearing conditions are $A_t = N_t V_t$ and $r_t = r_{A,t}$. The labor market clearing condition is $L_t = L_{Y,t} + L_{N,t} + L_{Z,t}$, in which $L_{Z,t} = \int_0^{N_t} l_{Z,t}(j)dj = N_t l_{Z,t}$ is the aggregate labor used for vertical R&D. By using the market clearing conditions and the relative first-order conditions, we combine the households' budget constraint (2) and the government budget constraint (14) to obtain the resource constraint in this economy: $\dot{K} = Y_t - C_t - G_t$. By applying the symmetric condition we can also obtain the aggregate production function for final goods:

$$Y_t = K_t^{\theta}(Z_t L_{Y,t})^{1-\theta}. \quad (16)$$

4.2.7 *Decentralized equilibrium*

The decentralized equilibrium is defined as an infinite sequence of allocations $\{C_t, K_t, A_t, Y_t, X_t, L_t, L_{Y,t}, L_{N,t}, L_{Z,t}, G_t\}_{t=0}^{\infty}$, prices $\{w_t, r_t, r_{A,t}, r_{K,t}, p_t(j), V_t(j)\}_{t=0}^{\infty}$, policies $\{\tau_{K,t}, \tau_{L,t}\}$, such that at any instant of time:

a. households choose $\{C_t, K_t, A_t, L_t\}$ to maximize lifetime utility (1) taking prices and policies as given;

b. competitive final goods firms choose $\{X_t(j), L_{Y,t}\}$ to maximize profit taking prices as given;

c. monopolistic intermediate firms $j \in [0, N_t]$ choose $\{k_t(j), p_t(j), l_{Z,t}(j)\}$ to maximize profit taking $\{r_t, r_{K,t}, w_t, \tau_{K,t}\}$ as given;

d. entrants make entry decisions taking $\{V_t, w_t\}$ as given;

e. the final goods market, capital market and labor market clear;

f. the government budget constraint is balanced: $\tau_{K,t} r_{K,t} K_t + \tau_{L,t} w_t L_t = G_t$.

4.3 Long-run effects of capital taxation

In this section, we examine the long-run effects of capital taxation on growth and entry. Before doing so, we first characterize the balanced-growth path (BGP) in this model. In the steady-state, labor allocations are stationary. Accordingly, from the resource constraint and eqs (6), (13), and (16), we can easily derive the properties of the BGP equilibrium:

$$\frac{\dot{K}_t}{K_t} = \frac{\dot{Z}_t}{Z_t} = \frac{\dot{Y}_t}{Y_t} = \frac{\dot{C}_t}{C_t} = \frac{\dot{G}_t}{G_t} = \frac{\dot{w}_t}{w_t}, \dot{N}_t = 0. \tag{17}$$

In the following analysis we denote $\gamma_{x,t}$ as the growth rate of any generic variables x and denote γ as the balanced growth rate of all growing variables.

This section focuses on the steady-state effects of capital taxation. For this purpose, we define the transformed variables $\omega = w/K$, $c = C/K$, $z = Z/K$, $y = Y/K$, and drop the time subscript to denote the stationary levels of (transformed) variables in the steady state. By using the property of eq. (17), the BGP equilibrium of the macro economy can be described by the following set of equations:

$$\gamma = r - \rho, \tag{18a}$$

$$\omega = \chi c/(1-\tau_L), \tag{18b}$$

$$\omega = (1-\theta)y/L_Y, \tag{18c}$$

$$1 = (\theta/p)^{\frac{1}{1-\theta}} zL_Y, \tag{18d}$$

$$p = r_K/\theta, \tag{18e}$$

$$(1-\tau_K)r_K = r, \tag{18f}$$

$$\gamma = \varphi l_Z, \tag{18g}$$

$$r = \frac{\alpha\varphi(1-\theta)\theta y/N}{\omega} + \gamma, \tag{18h}$$

$$r = \frac{\beta\left[(1-\theta)\,\theta y/N - \omega l_Z\right]}{\omega} + \gamma, \tag{18i}$$

$$\tau_L = \frac{\zeta - \theta^2\tau_K}{1-\theta}\frac{L_Y}{L}, \tag{18j}$$

$$\gamma = (1-\zeta)y - c, \tag{18k}$$

$$y = \left(\theta^2/r_K\right)^{\frac{\theta}{1-\theta}} zL_Y, \tag{18l}$$

$$L = L_Y + Nl_Z, \tag{18m}$$

in which we solve for thirteen unknowns $\{\gamma, r, \omega, c, \tau_L, z, p, r_K, y, L, L_Y, l_z, N\}$. The detailed derivation of eq. (18) is presented in Appendix A. It should be noted that in this model the labor income tax τ_L is treated as an endogenous variable because it will endogenously adjust in order to balance the government constraint as the capital income tax changes.

We are now ready to solve the balanced growth rate. By inserting eq. (18c) into eq. (18h) we can obtain $\frac{L_Y}{N} = \frac{\rho}{\alpha\varphi\theta}$. By inserting eq. (18c) into eq. (18i) we can obtain $l_Z = \frac{\theta L_Y}{N} - \frac{\rho}{\beta} = \frac{\rho}{\alpha\varphi} - \frac{\rho}{\beta}$. Combining these two expressions and putting into eq. (18g) yields the closed-form balanced growth rate:

$$\gamma = \frac{\rho}{\alpha}\left(1 - \frac{\alpha\varphi}{\beta}\right). \tag{19}$$

In eq. (19) we have assumed that $\beta > \alpha\varphi$ to ensure that both l_Z and γ are positive. It directly follows from eq. (19) that:

$$\frac{\partial\gamma}{\partial\tau_K} = 0. \tag{20}$$

Before explaining the result of eq. (20), it is useful to first discuss the long-run effects of the capital income tax on labor allocations and entry. By inserting $\frac{L_Y}{N} = \frac{\rho}{\alpha\varphi\theta}$ and $l_Z = \frac{\rho}{\alpha\varphi} - \frac{\rho}{\beta}$ into (18m), we can obtain the relation $L = \Theta L_Y$ where $\Theta \equiv 1 + \theta - \frac{\alpha\varphi\theta}{\beta} > 0$ is a composite parameter independent of τ_K. With $L = \Theta L_Y$, we can directly infer from (18j) that τ_L is a decreasing function of τ_K:

$$\tau_L = \frac{1}{\Theta}\frac{\zeta - \theta^2\tau_K}{(1-\theta)}. \tag{21}$$

eq. (21) states that an increase in the capital tax is coupled with a decrease in the labor tax. This result is termed as "tax shifting" in the literature.

Next, we put eqs (18c)-(18e), (18k), and (18l) into eq. (18b) to eliminate ω, c, y, z and p, and then insert eqs (18a) and (18f) to eliminate r and r_K; accordingly, we attain the following expression:

$$L = \Theta L_Y = \frac{\Theta(1-\tau_L)(1-\theta)}{\chi\left[(1-\zeta) - \gamma\theta^2(1-\tau_K)/(\gamma+\rho)\right]}. \tag{22}$$

eq. (22) is a closed form solution of equilibrium labor force given eqs (19) and (21). Differentiating L with respect to τ_K yields:

$$\frac{\partial L}{\partial\tau_K} = \frac{(1-\theta)\theta^3\frac{\alpha\varphi}{\beta}\gamma}{\chi\left[(1-\zeta) - (1-\tau_K)\theta^2\gamma/(\gamma+\rho)\right]^2(\gamma+\rho)} > 0. \tag{23}$$

To understand the intuition of eq. (23), let us first consider a hypothetical case where the labor income tax τ_L is fixed. This case could be thought of as the situation under which the government has another policy instrument such as a lump-sum tax (or transfer). The lump-sum tax adjusts to balance the government constraint as the capital tax increases, so that the labor income tax is left unchanged. In this case, eq. (21) is absent, and thus in eq. (22) τ_L is treated as an exogenous variable. As such, we can easily see from eq. (22) that an increase in the capital tax results in less labor supply. The intuition can be explained as follows. A unilateral increase in the capital tax depresses the intermediate-goods sector. Specifically, it increases the production costs of intermediate firms, and in turn results in less intermediate goods for final goods production, leading to a lower marginal product of final-goods labor. On the other hand, the profits of intermediate firms decreases, which also depresses the returns of in-house R&D labor. Both effects point to a lower labor supply because the returns on both labor decrease. Now we turn to the case of tax shifting. In this case, to hold the ratio of government spending to GDP constant, an increase in the capital tax is accompanied with a decrease in the labor tax. The decrease of the labor tax boosts the households' labor supply. Although the abovementioned negative effect is still present, the positive effect triggered by the lower labor tax is stronger. As a consequence, the total labor supply increases in response to the rise of the capital tax under the case of tax shifting.

Finally, from the expressions $L = \Theta L_Y$, $\frac{L_Y}{N} = \frac{\rho}{\alpha\varphi\theta}$, and $L_Z = Nl_Z = N(\frac{\rho}{\alpha\varphi} - \frac{\rho}{\beta})$, we can also derive the effects of the capital tax on final-goods labor, in-house R&D labor, and the number of intermediate firms:

$$\frac{\partial L_Y}{\partial \tau_K} > 0, \quad \frac{\partial L_Z}{\partial \tau_K} > 0, \quad \frac{\partial N}{\partial \tau_K} > 0, \quad \frac{\partial l_Z}{\partial \tau_K} = 0.$$

The following proposition highlights our findings in this section:

Proposition 5 *In the long run, an increase in the capital income tax has a positive effect on total labor force, final-goods labor, aggregate in-house R&D labor, and the number of intermediate firms, while it has a neutral effect on in-house R&D labor per firm and the growth rate.*

We have explained above that the rise of the capital income tax boosts total labor force by reducing the labor income tax. However, this increase of the labor force has no effect on long-run growth. This is the distinct feature of the second-generation R&D-based growth model, in which the scale effect (of labor) is removed. For example, Peretto (1998) demonstrates that the steady-state productivity growth does not depend on population size. Changes in population size have only transitory effects on economic growth. Likewise, in our model, the long-run growth rate is independent of the aggregate labor force. More important insights into the effects of capital taxation can be obtained by carrying out a quantitative analysis of the effects along the transition, which we present in the next section.

4.4 Quantitative analysis

In this section, we provide a quantitative analysis to explore the effects of capital taxation along the transition.[7] We generalize the utility function as follows:

$$U = \int_0^\infty e^{-\rho t} \left[\ln C_t + \chi \frac{(1-L_t)^{1-\eta}}{1-\eta} \right] dt, \tag{24}$$

where $\eta \geq 0$ determines the Frisch elasticity of labor supply. When $\eta = 0$, eq.(24) reduces to the special case shown in eq. (1). Our model has nine parameters$\{\rho$, θ, η, α, ζ, χ, τ_K, β, $\varphi\}$. We choose the following benchmark parameter values that

[7] We solve the dynamic system in Appendix 4.A.

are within the plausible ranges used in the literature. First, in line with Andolfatto et. al. (2008) and Acemoglu and Akcigit (2012), the capital income share θ and the discount rate ρ are set to the values 0.4 and 0.05, respectively. Second, the initial capital tax rate τ_K is set to 0.3 based on the average US effective tax rate estimated by Carey and Tchilingurian (2000). As for the government size (the ratio of government spending to output), data of US exhibits around 20 percent (Gali, 1994), and has slightly increased in recent years. We set ζ to be 0.22, which is the average level during 2001-2013. Third, the parameter for leisure preference χ is chosen as 0.9135 such that total hours worked is around one third of time endowment. Moreover, we set $\eta = 1.67$, implying a Frisch elasticity of 1.2; see Chetty et al. (2011). For the in-house R&D productivities, we choose $\varphi = 8.94$ to target the empirical level of the output growth rate in the US, which is around 2%. As for the R&D spillovers, we choose $\alpha = 0.2052$ such that the ratio of R&D to GDP is 1.97%; see King (2004). Lastly, we assume that an entrant incurs 0.5 units of labor as a setup cost, indicating the value $\beta = 2$. Table 4.1 reports our calibrated parameter values.

Table 4.1: Calibrated parameter values

θ	ρ	τ_K	ζ	χ	η	φ	α	β
0.4	0.05	0.3	0.22	0.9135	1.67	8.94	0.2052	2

We conduct a policy experiment of a small increase in the capital income tax rate from its initial value 30% to 31%. Figures 4.1-4.8 depict the effects on the growth rate and important variables along the transition path. As shown in Figure 4.1, in the short run where the number of intermediate firms is fixed, raising the capital tax has a negative impact on economic growth.

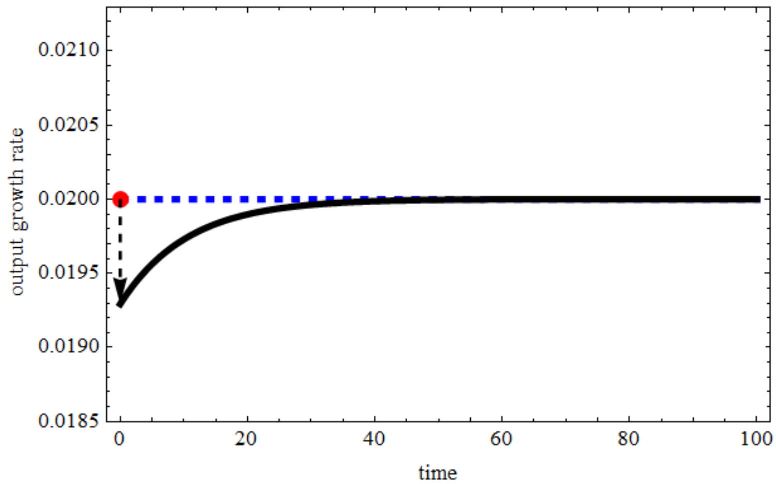


Figure 4.1: Transition path of the output growth rate

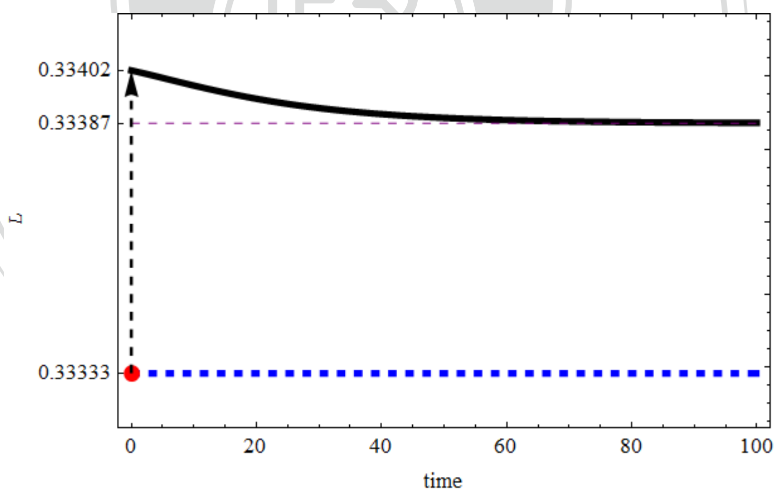


Figure 4.2: Transition path of total labor force

The intuition behind could be best understood by looking into the diverse effects of the capital tax on labor used in different sectors. Specifically, in the short run, the increase in the capital tax reduces the labor income tax (tax shifting) and wage. The decreased wage has an immediate beneficial effect on three segments that use labor as inputs: the final-goods sector, the in-house R&D sector of intermediate firms, and potential entrants. However, for the final-goods sector, the higher capital tax decreases the supply of intermediate goods; for the intermediate firms, the higher capital tax increases their production costs. Only the entry labor can enjoy the pure benefit of the lower wage without being directly (and negatively) affected by the higher capital tax. Therefore, in the final-goods sector and in-house R&D sector, the marginal product of labor is temporally less than marginal product of labor used for entry. This subsequently causes labor to flow out from these two sectors to the entrants as a sudden response. This is what we see in Figures 4.3, 4.5 and 4.6.

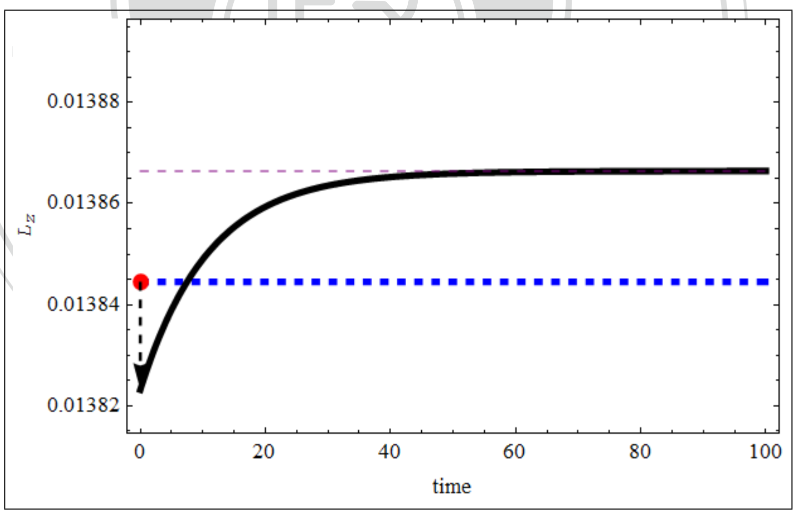


Figure 4.3: Transition path of aggregate in-house R&D labor

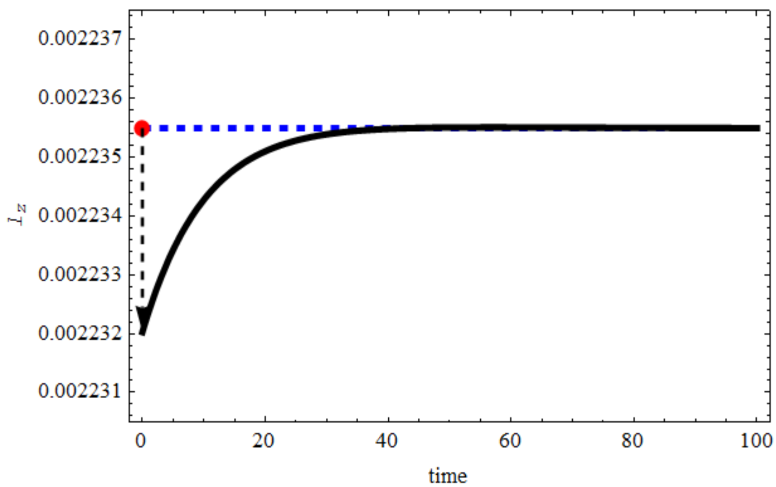


Figure 4.4: Transition path of in-house R&D labor per firm

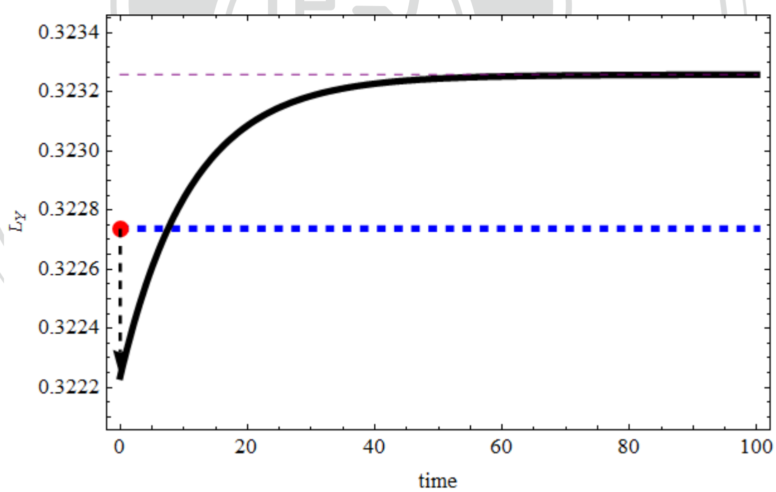


Figure 4.5: Transition path of final-goods labor

In second-generation R&D-based growth models, an important trait is that the growth rate is highly related to the market size or, more specifically, the in-house R&D researchers hired by *each* intermediate firm. As we have discussed above, at the point when the capital income tax rises, the aggregate in-house R&D labor flows out. Furthermore, in the short-run, the number of intermediate firms is fixed. This indicates that the in-house R&D labor per firm also decreases (Figure 4.4), which then is associated with a lower growth rate. Thus we see in Figure 4.1 that the growth rate immediately jumps down as the policy shock occurs.

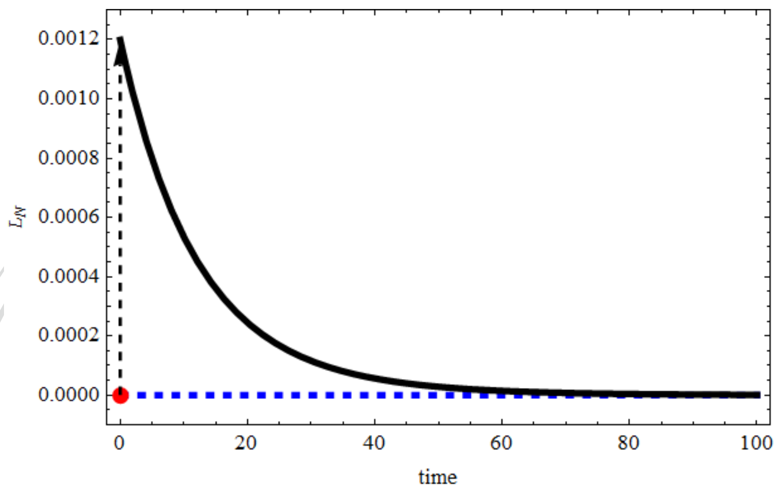
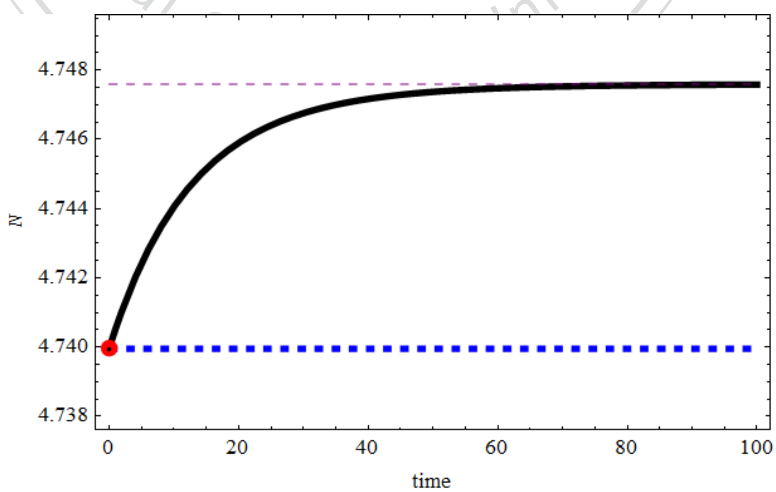


Figure 4.6: Transition path of labor used for entry



Figrue 4.7: Transition path of firm number

Along the transition process, new intermediate firms start to enter the market. Thus the number of intermediate firms rises (Figure 4.7). The profits of intermediate firms declines, which reduces the gap between the value of intermediate firms and entry cost, and therefore slows down the speed of entry. Moreover, because the benefit of entry declines, labor gradually flows back to the final-goods sector and in-house R&D sector. In particular, although during the transition process both aggregate in-house R&D labor and firm number are increasing, it turns out that the former exhibits a faster rate of growth. Therefore, the firm size gradually increases, leading the growth rate eventually to return to its original value. Our results with regard to the diverse growth effects of capital taxation in the short run and in the long run may provide a possible explanation for the mixed empirical observations between capital taxation and economic growth.

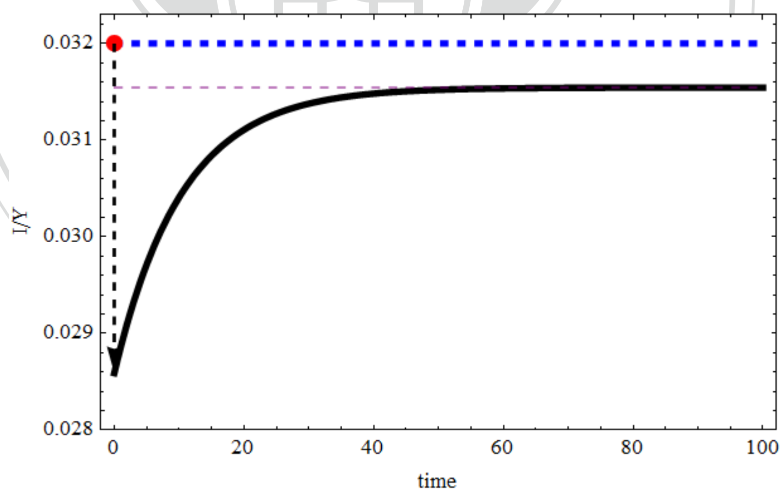


Figure 4.8: Transition path of the investment rate

4.5 Concluding remarks

In this Chapter, we examine the short-run and long-run effects of capital taxation on economic growth in an R&D-based growth model with endogenous market structure. In earlier traditional AK-type growth models, raising the capital tax has a harmful long-run effect on growth. In our analysis, however, we show that the negative growth effect sustains only in the short run. In the short run where the number of intermediate firms is fixed, raising the capital tax depresses growth because labor flows out from the in-house R&D sector. During the transitional period, with the number of firms adjust endogenously, economic growth keeps on rising as each of the in-house R&D firms continues to expand its market size. In the long run, with the equal counteracting strength between the short run and the transition period, capital taxation leads to a zero long-term effect on economic growth. Our analytical results succeed in matching some empirical observations that the negative growth effect of capital taxation may be neglectably small in the long run (Lucas, 1990; Stokey and Rebelo, 1995).

Appendix 4.A

This appendix solves the dynamic system of the model under tax shifting from labor income taxes to capital income taxes. The set of equations under the model is expressed by:

$$
\begin{aligned}
\frac{\dot{C}_t}{C_t} &= r_t - \rho, && \text{(A1)}\\
r_t &= (1-\tau_K) r_{K,t}, && \text{(A2)}\\
\chi(1-L_t)^{-\eta} &= \frac{(1-\tau_{L,t})w}{C_t}, && \text{(A3)}\\
\dot{K}_t/K_t &= (1-\zeta)Y_t/K_t - C_t/K_t, && \text{(A4)}\\
w_t L_{Y_t} &= (1-\theta)Y_t, && \text{(A5)}\\
r_{K,t}K_t &= \theta^2 Y_t, && \text{(A6)}\\
r_t &= \frac{\alpha\varphi\left[(1-\theta)\theta\left(\frac{\theta^2}{r_{K,t}}\right)^{\frac{\theta}{1-\theta}}(Z_t L_{Y,t}/N_t)\right]}{w_t} + \frac{\dot{w}_t}{w_t}, && \text{(A7)}\\
\frac{\dot{Z}_t}{Z_t} &= \varphi l_{Z,t}, && \text{(A8)}\\
r_t &= \frac{\dot{w}_t}{w_t} + \frac{\beta\left((1-\theta)\,\theta\frac{Y_t}{N_t} - w_t l_{Z,t}\right)}{w_t}, && \text{(A9)}\\
{}_{L,t} &= \frac{\zeta-\theta^2\tau_K}{1-\theta}\frac{L_{Y,t}}{L_t}, && \text{(A10)}\\
Y_t &= K_t^{\theta}(Z_t L_{Y,t})^{1-\theta}, && \text{(A11)}\\
L_t &= L_{Y,t} + N l_{Z,t} + L_{N,t}, && \text{(A12)}\\
\dot{N}_t &= L_{N,t}/\beta. && \text{(A13)}
\end{aligned}
$$

The above 13 equations determine 13 unknown $\{C_t,\ K_t,\ L_t,\ r_{K,t},\ L_{Y,t},\ Z_t,\ r,\ l_{Z,t}, N_t,$ $\tau_{L,t},\ Y_t,\ w_t,\ L_{N,t}\}$. Moreover, to solve the balanced growth rate, we deonte two transformed variables$z_t \equiv Z_t/K_t$ and $c_t \equiv C_t/K_t$, respectively. From eqs (A3), (A5), (A11), $c_t \equiv C_t/K_t$, and $z_t \equiv Z_t/K_t$,we have:

$$\chi(1-L_t)^{-\eta} = \frac{(1-\frac{\zeta-\theta^2\tau_K}{1-\theta}\frac{L_{Y,t}}{L_t})(1-\theta)(z_t L_{Y,t})^{1-\theta}}{c_t L_{Y,t}}. \tag{A14}$$

From eq. (A14), we can infer the folowing expression:

$$L_t = L_t(z_t, c_t, L_{Y,t}; \tau_K), \tag{A15}$$

where

$$\begin{aligned}
\frac{\partial L_t}{\partial z_t} &= \frac{(1-\theta)L_t}{z_t\left(\eta L_t/(1-L_t) - \tau_{L,t}/(1-\tau_{L,t})\right)}, \\
\frac{\partial L_t}{\partial c_t} &= -\frac{L_t}{c_t\left(\eta L_t/(1-L_t) - \tau_{L,t}/(1-\tau_{L,t})\right)}, \\
\frac{\partial L_t}{\partial L_{Y,t}} &= -\frac{\left(\theta + \tau_{L,t}/(1-\tau_{L,t})\right)L_t}{L_{Y,t}\left(\eta L_t/(1-L_t) - \tau_{L,t}/(1-\tau_{L,t})\right)}, \\
\frac{\partial L_t}{\partial \tau_K} &= \frac{\theta^2 L_{Y,t}}{(1-\theta)(1-\tau_{L,t})\left(\eta L_t/(1-L_t) - \tau_{L,t}/(1-\tau_{L,t})\right)}.
\end{aligned}$$

We now turn to deal with the transitional dynamics of the model. From eqs (A12) and (A13) we can infer:

$$\dot{N} = \beta N_t \left(\frac{L_t}{N_t} - \frac{L_{Y,t}}{N_t} - l_{Z,t}\right). \tag{A16}$$

Based on eqs (A5), (A6), (A7), (A9), and (A11), we can obtain:

$$l_{Z,t} = \left(1 - \frac{\alpha\varphi}{\beta}\right)\theta\frac{L_{Y,t}}{N_t}. \tag{A17}$$

Combining eqs (A15), (A16), and (A17) together yields:

$$\dot{N} = \beta \left[L_t(z_t, c_t, L_{Y,t}; \tau_K) - \left(1 + (1 - \frac{\alpha\varphi}{\beta})\theta \right) L_{Y,t} \right]. \tag{A18}$$

To simplify the notation, in what follows we suppress those arguments of the laobr supply function. Substituting eq. (A17) into eq. (A8) yields:

$$\frac{\dot{Z}_t}{Z_t} = \varphi \left(1 - \frac{\alpha\varphi}{\beta} \right) \theta \frac{L_{Y,t}}{N_t}. \tag{A19}$$

From eqs (A1), (A2), (A6), (A11), $c_t \equiv C_t/K_t$, and $z_t \equiv Z_t/K_t$, we can infer:

$$\frac{\dot{C}_t}{C_t} = (1 - \tau_K)\theta^2 (z_t L_{Y,t})^{1-\theta} - \rho \tag{A20}$$

Based on eqs (A2), (A4), (A6), (A11), $c_t \equiv C_t/K_t$, and $z_t \equiv Z_t/K_t$, we can obtain:

$$\frac{\dot{K}_t}{K_t} = (1 - \zeta)\theta^2 (z_t L_{Y,t})^{1-\theta} - c_t \tag{A21}$$

From eqs (A5) and (A11), we have:

$$w_t = \frac{(1-\theta) K_t^{\theta} (Z_t L_{Y,t})^{1-\theta}}{L_{Y_t}} \tag{A22}$$

Taking logarithms of eq. (A22) and differentiating the resulting equation with respect to time yields:

$$\frac{\dot{w}_t}{w_t} = \theta \frac{\dot{K}_t}{K_t} + (1-\theta) \frac{\dot{Z}_t}{Z_t} - \theta \frac{L_{Y,t}}{L_{Y,t}} \tag{A23}$$

From eqs (A2), (A5), (A6), (A7), (A11), and $z_t \equiv Z_t/K_t$, we have:

$$\frac{\dot{w}_t}{w_t} = (1-\tau_K)\theta^2 (z_t L_{Y,t})^{1-\theta} - \frac{\alpha\varphi\theta L_Y}{N_t} \tag{A24}$$

Substituting eqs (A19), (A20), and (A21) into eq. (A23) and then combining the resulting equation with eq. (A24) together, we thus infer the following expression:

$$\frac{\dot{L}_{Y,t}}{L_{Y,t}} = \left[1-\zeta-\theta(1-\tau_K)\right](z_t L_{Y,t})^{1-\theta} - c_t + (1-\theta)\varphi\left[\left(1-\frac{\alpha\varphi}{\beta}+\frac{\alpha}{1-\theta}\right)\frac{L_{Y,t}}{N_t}\right] \tag{A25}$$

Equipped with the definition $z_t \equiv Z_t/K_t$ and $c_t \equiv C_t/K_t$, we have:

$$\dot{z}_t/z_t \equiv \dot{Z}_t/Z_t - \dot{K}_t/K_t \tag{A26}$$

$$\dot{c}_t/c_t \equiv \dot{C}_t/C_t - \dot{K}_t/K_t \tag{A27}$$

From eqs (A18), (A19),(A20), (A21), (A25), (A26), and (A27), the dynamic system can be expressed as:

$$\dot{z}/z = \varphi\left(1-\frac{\alpha\varphi}{\beta}\right)\theta\frac{L_{Y,t}}{N_t} - (1-\zeta)(z_t L_{Y,t})^{1-\theta} + c_t, \tag{A28a}$$

$$\dot{N} = \beta\left[L_t(z_t, c_t, L_{Y,t};\tau_K) - \left(1+(1-\frac{\alpha\varphi}{\beta})\theta\right)L_{Y,t}\right], \tag{A28b}$$

$$\dot{c}_t/c_t = \left[(1-\tau_K)\theta^2-(1-\zeta)\right](z_t L_{Y,t})^{1-\theta} - \rho + c_t, \tag{A28c}$$

$$\frac{\dot{L}_{Y,t}}{L_{Y,t}} = \left[1-\zeta-\theta(1-\tau_K)\right](z_t L_{Y,t})^{1-\theta} - c_t + (1-\theta)\varphi\left(1-\frac{\alpha\varphi}{\beta}+\frac{\alpha}{1-\theta}\right)\frac{L_{Y,t}}{N_t}. \tag{A28d}$$

Linearizing eqs (A28a), (A28b), (A28c), and (A28d) around the steady-state equilibrium yields:

$$\begin{bmatrix} \dot{z}_t \\ \dot{N}_t \\ \dot{c}_t \\ \dot{L}_{Y,t} \end{bmatrix} = \begin{bmatrix} b_{11} & b_{12} & b_{13} & b_{14} \\ b_{21} & b_{22} & b_{23} & b_{24} \\ b_{31} & b_{32} & b_{33} & b_{34} \\ b_{41} & b_{42} & b_{43} & b_{44} \end{bmatrix} \begin{bmatrix} z_t - z \\ N_t - N \\ c_t - c \\ L_{Y,t} - L_Y \end{bmatrix} + \begin{bmatrix} b_{15} \\ b_{25} \\ b_{35} \\ b_{45} \end{bmatrix} d\tau_K, \tag{A29}$$

where

$$\frac{\partial \dot{z}_t}{\partial z_t} = b_{11}, \frac{\partial \dot{z}_t}{\partial N_t} = b_{21}, \frac{\partial \dot{z}_t}{\partial c_t} = b_{31}, \frac{\partial \dot{z}_t}{\partial L_{Y,t}} = b_{14}, \frac{\partial \dot{z}_t}{\partial \tau_K} = b_{15}, \tag{A30a}$$

$$\frac{\partial \dot{N}_t}{\partial z_t} = b_{21}, \frac{\partial \dot{N}_t}{\partial N_t} = b_{22}, \frac{\partial \dot{N}_t}{\partial c_t} = b_{23}, \frac{\partial \dot{N}_t}{\partial L_{Y,t}} = b_{24}, \frac{\partial \dot{N}_t}{\partial \tau_K} = b_{25}, \tag{A30b}$$

$$\frac{\partial \dot{c}_t}{\partial z_t} = b_{31}, \frac{\partial \dot{c}_t}{\partial N_t} = b_{32}, \frac{\partial \dot{c}_t}{\partial c_t} = b_{33}, \frac{\partial \dot{c}_t}{\partial L_{Y,t}} = b_{34}, \frac{\partial \dot{c}_t}{\partial \tau_K} = b_{35}, \tag{A30c}$$

$$\frac{\partial \dot{L}_{Y,t}}{\partial z_t} = b_{41}, \frac{\partial \dot{L}_{Y,t}}{\partial N_t} = b_{42}, \frac{\partial \dot{L}_{Y,t}}{\partial c_t} = b_{43}, \frac{\partial \dot{L}_{Y,t}}{\partial L_{Y,t}} = b_{44}, \frac{\partial \dot{L}_{Y,t}}{\partial \tau_K} = b_{45}. \tag{A30d}$$

Due to the complicated calculations, we do not list the analytical results for b_{ij}, where $i \in \{1, 2, 3, 4, 5\}$ and $j \in \{1, 2, 3, 4, 5\}$.

Let μ_1, μ_2, μ_3, and μ_4 be the four characteristic roots of the dynamic system. Due to the complexity calculations of the four characteristic roots, we do not try to prove the saddle-point stability analytically. Instead, we show that the dynamic system exists two positive and two negative characteristic roots via a numerical simulation. For expository convenience, in what follows let μ_1 and μ_2 be the negative root as well as μ_3 and μ_4 be the positive roots.

The general general solution is given by:

$$\begin{bmatrix} z_t \\ N_t \\ c_t \\ L_{Y,t} \end{bmatrix} = \begin{bmatrix} z(\tau_K) \\ N(\tau_K) \\ c(\tau_K) \\ L_Y(\tau_K) \end{bmatrix} + \begin{bmatrix} 1 & 1 & 1 & 1 \\ h_{21} & h_{22} & h_{23} & h_{24} \\ h_{31} & h_{32} & h_{33} & h_{34} \\ h_{41} & h_{42} & h_{43} & h_{44} \end{bmatrix} \begin{bmatrix} D_1 e^{\mu_1 t} \\ D_2 e^{\mu_2 t} \\ D_3 e^{\mu_3 t} \\ D_4 e^{\mu_4 t} \end{bmatrix} \tag{A31a}$$

where D_1, D_2, D_3, and D_4 are undetermined coefficients and

$$\Delta_j \equiv \begin{vmatrix} b_{12} & b_{13} & b_{14} \\ b_{22}-\mu_j & b_{23} & b_{24} \\ b_{32} & b_{33}-\mu_j & b_{34} \end{vmatrix} ;\ j \in \{1,\ 2,\ 3,\ 4\} \tag{A31b}$$

$$h_{2j} = \begin{vmatrix} \mu_j - b_{11} & b_{13} & b_{14} \\ -b_{21} & b_{23} & b_{24} \\ -b_{31} & b_{33}-\mu_j & b_{34} \end{vmatrix} /\Delta_j ;\ j \in \{1,\ 2,\ 3,\ 4\} \tag{A31b}$$

$$h_{3j} = \begin{vmatrix} b_{12} & \mu_j - b_{11} & b_{14} \\ b_{22}-\mu_j & -b_{21} & b_{24} \\ b_{32} & -b_{31} & b_{34} \end{vmatrix} /\Delta_j ;\ j \in \{1,\ 2,\ 3,\ 4\} \tag{A31c}$$

$$h_{4j} = \begin{vmatrix} b_{12} & u_{13} & \mu_j - b_{11} \\ b_{22}-\mu_j & b_{23} & -b_{21} \\ b_{32} & b_{33}-\mu_j & -b_{31} \end{vmatrix} /\Delta_j ;\ j \in \{1,\ 2,\ 3,\ 4\} \tag{A31e}$$

The government changes the capital tax rate τ_K from τ_{K0} to τ_{K1} at $t = 0$, based on eqs (A31a)-(A31e), we can employ the following equations to discribe the dynamic adjustment of z_t, N_t, c_t, and $L_{Y,t}$:

$$z_t = \begin{cases} z(\tau_{K0}) & ; t = 0^- \\ z(\tau_{K1}) + D_1 e^{\mu_1 t} + D_2 e^{\mu_2 t} + D_3 e^{\mu_3 t} + D_4 e^{\mu_4 t} & ; t \geq 0^+ \end{cases} \tag{A32a}$$

$$N_t = \begin{cases} N(\tau_{K0}) & ; t = 0^- \\ N(\tau_{K1}) + D_1 h_{21} e^{\mu_1 t} + D_2 h_{22} e^{\mu_2 t} + D_3 h_{23} e^{\mu_3 t} + D_4 h_{24} e^{\mu_4 t} & ; t \geq 0^+ \end{cases} \tag{A32b}$$

$$c_t = \begin{cases} c(\tau_{K0}) & ; t = 0^- \\ c(\tau_{K1}) + h_{31} D_1 e^{\mu_1 t} + h_{32} D_2 e^{\mu_2 t} + h_{33} D_3 e^{\mu_3 t} + h_{34} D_4 e^{\mu_4 t} & ; t \geq 0^+ \end{cases} \tag{A32c}$$

$$L_{Y,t} = \begin{cases} L_Y(\tau_{K0}) & ; t = 0^- \\ L_Y(\tau_{K1}) + h_{41} D_1 e^{\mu_1 t} + h_{42} D_2 e^{\mu_2 t} + h_{43} D_3 e^{\mu_3 t} + h_{44} D_4 e^{\mu_4 t} & ; t \geq 0^+ \end{cases} \tag{A32d}$$

where $0^-$and 0^+ denote the instant before and after the policy implementation, respectively.The values for D_1, D_2, D_3 and D_4 are determined by:

$$z_{0^-} = z_{0^+}, \tag{A33a}$$

$$N_{0^-} = N_{0^+}, \tag{A33b}$$

$$D_3 = 0, \tag{A33c}$$

$$D_4 = 0. \tag{A33d}$$

Equation (A33a) and eq. (A33b) indicate that both $z_t (= Z_t/K_t)$ and N_t intact at the instant of policy implementation since Z_t, K_t, and N_t are predetermined variables. Equation (A33c) and eq. (A33d) are the stability conditions which ensures that all z_t, N_t, c_t, and $L_{Y,t}$ converge to their new steady-state equilibrium. By using eqs (A33a)-(A33d), we can obtain:

$$D_1 = \frac{[z(\tau_{K0}) - z(\tau_{K1})]h_{22} - [N(\tau_{K0}) - N(\tau_{K1})]}{h_{22} - h_{21}}, \tag{A34a}$$

$$D_2 = \frac{[N(\tau_{K0}) - N(\tau_{K1})] - [z(\tau_{K0}) - z(\tau_{K1})]h_{21}}{h_{22} - h_{21}}. \tag{A34b}$$

Substituting eqs (A33c), (A33d), (A34a), and (A34b) into eqs (A32a)-(A32d), the time path for z_t, N_t, c_t and $L_{Y,t}$ can then be described as:

$$z_t = \begin{cases} z(\tau_{K0}) & ; t = 0^- \\ z(\tau_{K1}) + \frac{[z(\tau_{K0}) - z(\tau_{K1})]h_{22} - [N(\tau_{K0}) - N(\tau_{K1})]}{h_{22} - h_{21}} e^{\mu_1 t} & \\ \qquad + \frac{[N(\tau_{K0}) - N(\tau_{K1})] - [z(\tau_{K0}) - z(\tau_{K1})]h_{21}}{h_{22} - h_{21}} e^{\mu_2 t} & ; t \geq 0^+ \end{cases}$$

$$N_t = \begin{cases} N(\tau_{K0}) & ; t = 0^- \\ N(\tau_{K1}) + \frac{[z(\tau_{K0}) - z(\tau_{K1})]h_{22} - [N(\tau_{K0}) - N(\tau_{K1})]}{h_{22} - h_{21}} h_{21} e^{\mu_1 t} & \\ \qquad + \frac{[N(\tau_{K0}) - N(\tau_{K1})] - [z(\tau_{K0}) - z(\tau_{K1})]h_{21}}{h_{22} - h_{21}} h_{22} e^{\mu_2 t} & ; t \geq 0^+ \end{cases}$$

$$c_t = \begin{cases} c(\tau_{K0}) & ; t = 0^- \\ c(\tau_{K1}) + h_{31} \frac{[z(\tau_{K0}) - z(\tau_{K1})]h_{22} - [N(\tau_{K0}) - N(\tau_{K1})]}{h_{22} - h_{21}} e^{\mu_1 t} & \\ \qquad + h_{32} \frac{[N(\tau_{K0}) - N(\tau_{K1})] - [z(\tau_{K0}) - z(\tau_{K1})]h_{21}}{h_{22} - h_{21}} e^{\mu_2 t} & ; t \geq 0^+ \end{cases}$$

$$L_{Y,t} = \begin{cases} L_Y(\tau_{K0}) & ; t = 0^- \\ L_Y(\tau_{K1}) + h_{41} \frac{[z(\tau_{K0}) - z(\tau_{K1})]h_{22} - [N(\tau_{K0}) - N(\tau_{K1})]}{h_{22} - h_{21}} e^{\mu_1 t} & \\ \qquad + h_{42} \frac{[N(\tau_{K0}) - N(\tau_{K1})] - [z(\tau_{K0}) - z(\tau_{K1})]h_{21}}{h_{22} - h_{21}} e^{\mu_2 t} & ; t \geq 0^+ \end{cases}$$

CHAPTER 5
CONCLUSION

This dissertation has provided a systematic analysis regarding the growth and welfare effects of capital taxation with distinct R&D-based growth models. More specifically, we have dealt with the growth and welfare effects of capital income taxation in three different types of R&D models, namely, the first-generation R&D-based growth model developed by Romer (1990), the semi-endogenous growth model developed by Jones and Williams (2000), and the second-generation R&D-based growth model developed by Dinopoulos and Thompson (1998) and Peretto (1998). The main findings of each chapter can be summarized as follows.

In Chapter 2, we have constructed a first-generation R&D-based growth model to examine the effcts of capital taxation on innovation and economic growth. We have found that capital taxation has drastically diffrent effcts in the short run and in the long run. An increase in the capital income tax rate has both a consumption effct and a tax-shifting effct on the equilibrium growth rates of technology and output. In the long run, the tax-shifting effct dominates the consumption effect, yielding an overall positive effct of capital taxation on steady-state economic growth. However, in the short run the consumption effect becomes the dominant force, causing an initial negative effct of capital taxation on the equilibrium growth rates. These contrasting effcts of capital taxation at diffrent time horizons may provide a plausible explanation for the mixed evidence in the empirical literature on capital taxation and economic growth.

In Chapter 3, we have set up an innovation-based growth model, and examined whether the Chamley-Judd result of a zero optimal capital income tax is valid in it. By calibrating our model to the US economy, we have found that the optimal capital income tax is positive, at a rate of around 11.9 percent. We have also found that the result of a positive optimal capital income tax is robust with respect to varying the degrees of various types of R&D externalities.

In Chapter 4, we have built up a second-generation R&D-based growth model which features endogenous market structure, and examined the short-run and long-run effects of capital taxation on economic growth. In this chapter, we have shown that the negative growth effect sustains only working in the short run. Specifically, in the short run the number of intermediate firms is fixed, a rise in the capital tax rate tends to lower the growth rate because labor flows out from the in-house R&D sector. During the transitional period, with the number of firms adjust endogenously, economic growth keeps on rising as each of the in-house R&D firms continues to expand its market size. In the long run, with the equal counteracting strength between the short run and the transition period, capital taxation leads to a zero long-term effect on economic growth. Our analytical results have succeeded in matching some empirical observations that the negative growth effect of capital taxation may be neglectably small in the long run (Lucas, 1990; Stokey and Rebelo, 1995).

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