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Economic design of multi-characteristic models for a three-class screening procedure

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In this paper, we present different inspection methods to determine the acceptability of produced items. In the first method, items are classified into two grades. An item belongs to a grade when it meets the requirements of that grade. In the second method, a joint screening rule based on an aggregation of the characteristics is used. The two methods are then compared.

1. Introduction

Traditionally, sampling is favoured over screening (100% inspection) because it provides an economical and efficient method for obtaining information about an unknown population, such as a production lot or a production process. However, modern manufacturing systems, such as the just-in-time (JIT) system and the flexible manufacturing system (FMS), have a trend toward smaller production lot sizes to reduce inventory costs. Sampling becomes inefficient in these environments, while screening becomes more cost effective owing to the rapid development in automated inspection instruments and computerized manufacturing.

In a typical screening procedure, all the outgoing items are subject to acceptance inspection. If an item does not conform to the predetermined screening specifications, it is rejected and subjected to some corrective actions. Since items are separated into two classes (acceptance and rejection), this type of screening procedure can be named as 'two-class screening'. Basic models of two-class screening were discussed by Taguchi (1984) and Tang (1988) and a brief literature review was given in Tang and Tang (1989).

Most studies consider only one quality characteristic. However, it is very common for a product to have many important quality characteristics. For example, tensile strength and compressibility are two important quality characteristics for an alloy. Tang and Tang (1989) proposed two multi-characteristic models for two-class screening. In the first model, each characteristic has its own screening specifications, and acceptance inspection is used to determine the conformance of an item to the specifications. An item is accepted only when it simultaneously conforms to the specifications of all the quality characteristics. In the second model, a joint screening rule based on an aggregation of the quality characteristics is used. To implement the second model, it is necessary to obtain the exact values of all the quality characteristics.

In many situations, items produced by the same production process may be sorted into different product grades and sold in different prices (England and Leenders 1975). Tang (1989) considered a situation where the outgoing items are either sorted into one

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of two product grades or scrapped. The quality of the product is determined by only one quality characteristic. It is assumed that Grade 1 has tighter product specifications and thus commands a higher price than Grade 2. A quadratic function suggested by Taguchi (1984) is used to measure the loss due to consumer dissatisfaction with product quality. A 'three-class' screening model is proposed to determine the optimal screening specification limits by maximizing the expected per-item profit.

In this paper, we extend the single-characteristic model for three-class screening proposed by Tang (1989) to two multi-characteristic models. Similar to Tang and Tang (1989), in Model 1 each characteristic has distinct screening specifications, and inspection results of individual characteristics are used to determine the disposition of an item. In Model 2, a joint screening rule based on an aggregation of all the characteristics is used. In order to implement this model, it is necessary to use the exact measured values of all the characteristics to determine the disposition of an item.

In the next section, we briefly discuss the single-characteristic three-class screening model proposed by Tang (1989). Then, two multi-characteristic models, Models 1 and 2, are presented in sections 2 and 3. In section 4, a numerical example is given and a numerical study is used to compare the two models.

2. Single-characteristic model

In this section, the single-characteristic screening model proposed by Tang (1989) is presented. The situation considered in the study is as follows: the items produced by a production process are to be sorted into two grades or scrapped. It is assumed that Grade 1 has tighter specifications and a higher price than Grade 2. A loss in revenue is incurred to the producer if a Grade 1 item is classified into Grade 2 or scrapped, or if a Grade 2 item is scrapped. On the other hand, a loss caused by consumer dissatisfaction is also incurred when a Grade 2 item is classified into Grade 1, or an item which should be scrapped is classified into Grade 1 or Grade 2.

The loss due to consumer dissatisfaction may include loss in goodwill, warranty, replacement cost, and handling cost. Classical concept of attribute inspection assumes that this loss is a constant when an item does not conform to product specifications and is zero otherwise. Let r denote this constant cost. However, Taguchi (1984) argued that this cost concept was incorrect. Instead, he suggested that a quadratic function of the deviation from the product target (ideal) value could better measure the true loss. In this study, this quadratic loss function is used.

Let Y be a random variable which denotes the deviation of the quality characteristic from the target value and let y denote the value of Y of an item. Let $l(y)$ denote the quadratic loss function, which has a form

$$l(y) = ky^2 \tag{1}$$

where k is a positive constant. Taguchi suggested that k is determined by equating $l(y)$ and r at product specifications limits. Let $[-d_1, d_1]$ be the product specifications for Grade 1, then k associated with Grade 1 is given by

$$k_1 = r/d_1^2 \tag{2}$$

For simplicity, we let r be the same for Grade 2. Let $[-d_2, d_2]$ be the product specifications of Grade 2, then k for Grade 2 is determined by

$$k_2 = r/d_2^2 \tag{3}$$

Let p_1 and p_2 denote the per-item selling prices of Grade 1 and Grade 2 items, respectively. Let $\Delta p > 0$ be the price difference $p_1 - p_2$, and δ_1 and δ_2 be the screening specification limits in the following screening rules:

If $y \in [-\delta_1, \delta_1]$, the item is classified into Grade 1.
If $y \in [-\delta_2, -\delta_1] \cup [\delta_1, \delta_2]$, the item is classified into Grade 2.

Let $Pr(\mathrm{G1})$ and $Pr(\mathrm{G2})$ be the proportion of items sorted into Grades 1 and 2, respectively. Let $f(y)$ denote the probability density function of y, then

$$Pr(\mathrm{G1}) = \int_{-\delta_1}^{\delta_1} f(y)\,\mathrm{d}y \tag{4}$$

and

$$Pr(\mathrm{G2}) = \int_{-\delta_2}^{-\delta_1} f(y)\,\mathrm{d}y + \int_{\delta_1}^{\delta_2} f(y)\,\mathrm{d}y \tag{5}$$

Consequently, the per-item expected revenue is given by

$$ER = p_1 Pr(\mathrm{G1}) + p_2 Pr(\mathrm{G2}) \tag{6}$$

We define the per-item expected loss due to imperfect quality as the expected acceptance cost (EAC), which is given by

$$EAC = \int_{-\delta_1}^{\delta_1} k_1 y^2 f(y)\,\mathrm{d}y + \int_{-\delta_2}^{-\delta_1} k_2 y^2 f(y)\,\mathrm{d}y + \int_{\delta_1}^{\delta_2} k_2 y^2 f(y)\,\mathrm{d}y \tag{7}$$

The expected per-item profit EPR is determined by:

$$EPR = ER - EAC \tag{8}$$

It is easy to show that the optimal solution δ_1^* and δ_2^* are determined by

$$\delta_1^* = (\Delta p/(k_1 - k_2))^{1/2} \tag{9}$$

$$\delta_2^* = (p_2/k_2)^{1/2} \tag{10}$$

Note that for a given y, the difference betweeen Grades 1 and 2 in acceptance cost is $(k_1 - k_2)y^2$. If this difference is less than Δp, the item should be assigned to Grade 1. This is actually implied by equation (9). Similarly, equation (10) suggests that an item is classified into Grade 2 rather than being scrapped if the acceptance cost $k_2 y^2$ is smaller than p_2.

3. Multi-characteristic model 1

Consider the situation where a product has n distinct quality characteristics. Let $Y_1, Y_2, \ldots, Y_n$ denote their deviations from the corresponding target values. In Model 1, it is assumed that each quality characteristic has distinct screening specifications. An item is classified into one of the two grades only when it simultaneously conforms to all the screening specifications of that grade. The items that cannot meet all the specifications of either grade are scrapped. Furthermore, it is also assumed that Y_j follows a normal distribution with a mean zero and a standard deviation σ_j and that $Y_1, Y_2, \ldots, Y_n$ are statistically independent. Furthermore, the total loss due to the measured quality deviations $y_1, y_2 \ldots, y_n$ of an item is assumed to be additive, i.e., the sum of the single-characteristic loss function. Let $l_{1n}(y_1, y_2 \ldots, y_n)$ and $l_{2n}(y_1, y_2 \ldots, y_n)$

denote the loss functions for Grades 1 and 2, respectively. The function forms of these two functions are given by

$$l_{1n}(y_1, y_2 \ldots, y_n) = k_{11}y_1^2 + k_{12}y_2^2 + \ldots + k_{1n}y_n^2 \quad \text{for Grade 1} \tag{11}$$

$$l_{2n}(y_1, y_2 \ldots, y_n) = k_{21}y_1^2 + k_{22}y_2^2 + \ldots + k_{2n}y_n^2 \quad \text{for Grade 2} \tag{12}$$

where k_{ij} is a constant associated with Grade i and characteristic j.

3.1. *Model formulation*

Let δ_{ij} denote the screening specification limit associated with Grade i and the jth quality characteristic. Let p_{ij} denote the probability that y_j is in $[-\delta_{ij}, \delta_{ij}]$. Since Y_j are statistically independent, the proportion of items sorted into Grade 1 is given by

$$Pr(\mathrm{G1}) = \prod_{j=1}^{n} p_{1j} \tag{13}$$

It can also be shown that the proportion of items sorted into Grade 2 is given by

$$Pr(\mathrm{G2}) = \prod_{j=1}^{n} p_{2j} - \prod_{j=1}^{n} p_{1j} \tag{14}$$

As a result, the expected revenue from selling the item is

$$ER = p_1 Pr(\mathrm{G1}) + p_2 Pr(\mathrm{G2}) \tag{15}$$

The expected acceptance cost due to the imperfect quality is obtained by averaging the quadratic loss functions over the acceptance regions. Let us define

$$m_{ij} = \int_{-\delta_{ij}}^{\delta_{ij}} y_j^2 f_j(y_j)\,\mathrm{d}y_j \tag{16}$$

which is equal to $\sigma_j^2[2\Phi(\delta_{ij}/\sigma_j) - 1] - 2\sigma_j\delta_{ij}\phi(\delta_{ij}/\sigma_j)$, where $\Phi(\cdot)$ and $\phi(\cdot)$ are the standard normal distribution and density functions, respectively. It can be verified that the expected acceptance cost can be expressed as

$$EAC = \sum_{j=1}^{n} k_{1j}m_{1j}\left(\prod_{i \neq j}^{n} p_{1i}\right) + \left[\sum_{j=1}^{n} k_{2j}m_{2j}\left(\prod_{i \neq j}^{n} p_{2i}\right) - \sum_{j=1}^{n} k_{2j}m_{1j}\left(\prod_{i \neq j}^{n} p_{1i}\right)\right] \tag{17}$$

Consequently, EAC can be evaluated by using the standard normal distribution and density functions and some simple computation. The expected per-item profit is the difference of ER and ECA, which can be simplified as

$$EPR = \left\{\Delta p \prod_{j=1}^{n} p_{1j} - \sum_{j=1}^{n}\left[(k_{1j} - k_{2j})m_{1j}\prod_{i \neq j}^{n} p_{1i}\right]\right\} + \left[p_2 \prod_{j=1}^{n} p_{2j} - \sum_{j=1}^{n}\left(k_{2j}m_{2j}\prod_{i \neq j}^{n} p_{2i}\right)\right] \tag{18}$$

3.2. *Analysis of the model*

Let $EPR1$ denote the expression in the braces { } of (18). Note that $EPR1$ is determined only by the screening specification limits associated with Grade 1. Similarly, let $EPR2$ denote the expression in the second brackets [ ] of (18), which is determined by the screening specification limits associated with Grade 2. Consequently, we can easily obtain the first-derivative conditions for optimal solution associated with the jth characteristic:

$$(k_{1j} - k_{2j})\delta_{1j}^2 + \sum_{i \neq j}^{n} (k_{1i} - k_{2i})m_{1i}/p_{1i} = \Delta p \tag{19}$$

$$k_{2j}\delta_{2j}^2+\sum_{i\neq j}^{n}k_{2i}m_{2i}/p_{2i}=p_2 \tag{20}$$

Comparing (19) with the corresponding expression (9) in the single characteristic model, it is found that (19) has an additional term

$$\sum_{i\neq j}^{n}(k_{1i}-k_{2i})m_{1i}/p_{1i}$$

which is actually the sum of expected conditional acceptance costs of all the other characteristics. Since $(k_{1i}-k_{2i})m_{1i}/p_{1i}$ is non-negative, it is obvious that introducing additional characteristics into the model results in the optimal screening specification limits of the jth characteristic becoming more stringent. In fact $[\Delta p/(k_{1j}-k_{2j})]^{1/2}$ provides an upper bound for the optimal specification limit δ_{1j}^*

Note that in this model it is assumed that the exact values of the characteristics are not available. Instead, only the results of acceptance inspection are used to decide the disposition of an item. Based on this information structure, the interaction effects of other characteristics on a given characteristic can be only in terms of expected values rather than the exact values of the characteristics. Similar to the single characteristic model, the optimal screening rule is to classify an item into Grade 1 when the difference in acceptance costs is smaller than Δp:

$$(k_{1j}-k_{2j})\delta_{1j}^2+\sum_{i\neq j}^{n}[(k_{1i}-k_{2i})m_{1i}/p_{1i}]\leqslant\Delta p \tag{21}$$

Similarly, the solution to the specification limits in Grade 2 is to classify an item into Grade 2 when

$$k_{2j}\delta_{2j}^2+\sum_{i\neq j}^{n}(k_{2i}m_{2i}/p_{2i})\leqslant p_2 \tag{22}$$

and $(p_2/k_{2j})^{1/2}$ is an upper bound for the optimal specification limit δ_{2j}^*.

3.3. *Solution procedure*

Based on the discussion in the last section, the solution obtained by the following iterative search algorithm satisfies the first-derivative conditions for the optimal solution.

Step 1. Obtain the initial solutions δ_{1j} and δ_{2j} by solving their respective single-characteristic models independently.

Step 2. Iteratively solve and update (19) and (20) for each characteristic until the solution converges.

This algorithm, as previously mentioned, provides approximate solutions which satisfy the first-derivative conditions for the optimal solution. Our computation experience shows that this algorithm did provide the optimal solutions and converged quickly for the problems tested.

4. Multi-characteristic model 2

In Model 1, direct trade-offs among the quality of characteristics are not possible. For instance, an item is classified into Grade 2 if it fails to meet the screening specifications of any one of the characteristics for Grade 1 even though the values of other characteristics may be very close to the target values. In this section, we propose Model 2 based on a joint screening rule. Notice that to implement this model, the exact measured values of all the characteristics have to be obtained. Therefore, the inspection cost of using Model 2 is usually higher than that of using Model 1.

4.1. *Optimal screening rule*

Recall that in the single characteristic model, the optimal specifications are determined by comparing the difference in acceptance costs of two grades and the difference in prices. Therefore, the optimal solution is independent of the distribution of the quality characteristic. In a multi-characteristic case, if the characteristics are aggregated, the problem actually becomes a single-characteristic problem. Therefore a reasonable joint screening rule is to classify an item into Grade 1 if

$$\sum_{j=1}^{n} (k_{1j}-k_{2j})y_j^2 \leqslant \Delta p \tag{23}$$

to classify an item into Grade 2 if

$$\sum_{j=1}^{n} (k_{1j}-k_{2j})y_j^2 > \Delta p \qquad \text{and} \qquad \sum_{j=1}^{n} k_{2j}y_j^2 \leqslant p_2 \tag{24}$$

and to scrap an item if

$$\sum_{j=1}^{n} k_{2j}y_j^2 > p_2 \tag{25}$$

To illustrate this screening rule, we now consider a two-characteristic problem with the parameters given in Table 1. We assume that p_1 is \$12·0, p_2 is \$7·0, and r is \$15·0. Then the optimal screening rule is to classify an item to Grade 1 if

$$1{\cdot}39y_1^2 + 1{\cdot}67y_2^2 \leqslant 5{\cdot}0 \tag{26}$$

and classify an item to Grade 2 if

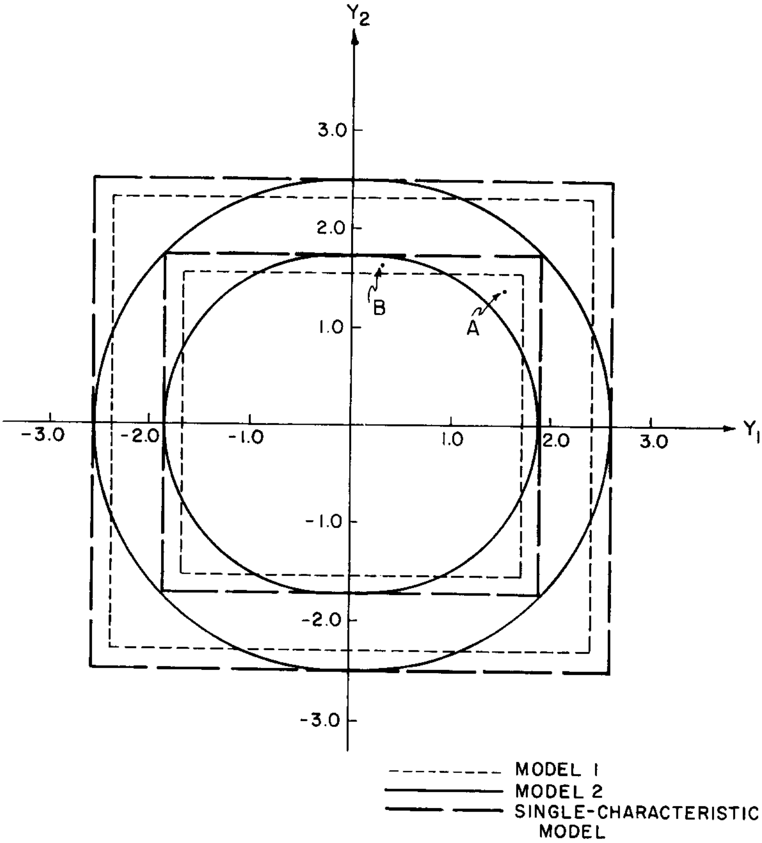
$$1{\cdot}39y_1^2 + 1{\cdot}67y_2^2 \geqslant 5{\cdot}0 \qquad \text{and} \qquad 1{\cdot}01y_1^2 + 1{\cdot}16y_2^2 \leqslant 7{\cdot}0 \tag{27}$$

Let the two ellipses in the figure represent the boundaries of (26) and (27). Then, all the items outside the larger ellipse should be scrapped. The area outside the smaller ellipse but inside the larger ellipse is the acceptance region for Grade 2 and the area inside the smaller ellipse is the acceptance region for Grade 1. For comparisons, the acceptance regions associated with Model 1 are also given in the figure. Notice that the acceptance regions of Model 2 are actually bounded by those of the single characteristics models.

Consider the rectangles associated with Model 1 in the figure. The area bounded by the smaller rectangle is the acceptance region for Grade 1 and the area outside the smaller rectangle, inside the large rectangle is the acceptance region for Grade 2, and

	Grade 1		Grade 2	
Characteristic	d_{ij}	δ_{ij}^*	d_{ij}	δ_{ij}^*
1	2·50	1·70	3·85	2·44
2	2·30	1·57	3·60	2·29

Table 1. A two-characteristic example. d_{ij} is the product specification limit of Grade i for characteristic j; δ_{ij}^* is the optimal screening specification limit of Grade i for characteristic j in Model 1.



The acceptance regions of a two-characteristic problem.

the area outside the larger rectangle is the rejection region. It is interesting to point out that, using Model 1, the items represented by the points A and B in Fig. 1 are classified into Grades 1 and 2, respectively. However, using Model 2, the classifications of these two items are just the opposite. In fact, point A has a higher acceptance cost than point B and should be classified as a Grade 2 item. Therefore, Model 2 has economic advantage over Model 1.

4.2. *Evaluation of* Pr(*G1*), Pr(*G2*) *and EAC*

It has been shown by Tang and Tang (1989), following Robbins (1948), that the distribution of a linear combination of several independent chi-square random variables can be written as a series of chi-square distributions with coefficients being computed iteratively. Without loss of generality, in this section we assume that the variances of all the quality characteristics are 1. Furthermore, let $(k_{11}-k_{21})$ be the smallest value among all the $(k_{1j}-k_{2j})$ in Grade 1 and let k_{21} be the smallest value among k_{2j}. Then

$$Pr(\mathrm{G1})=\mathrm{prob}\left[\sum_{j=1}^{n}(k_{1j}-k_{2j})y_j^2\leqslant \Delta p\right]$$

$$=\sum_{j=0}^{\infty} c_j F_{n+2j}(\Delta p/(k_{11}-k_{21})) \qquad (28)$$

where $F_{n+2j}(\cdot)$ is the chi-square distribution function with $n+2j$ degrees of freedom and c_j is obtained by

$$c_j=\beta_j\left\{\prod_{i=2}^{n}\left[(k_{1i}-k_{2i})/(k_{11}-k_{21})\right]^{-1/2}\right\} \qquad j=0,1,2,\dots$$

with

(i) $\alpha_h=\sum_{j=2}^{n}[1-(k_{11}-k_{21})/(k_{1j}-k_{2j})]^h/2h \qquad h=1,2,3,\dots$

(ii) $\beta_0=1$

(iii) $\beta_j=(1/j)\sum_{h=1}^{j}h\alpha_h\beta_{j-h} \qquad$ for $j=1,2,3,\dots$

It has been shown that, if the infinite series in (28) is truncated at the $(t+1)$th term, the error is bounded by

$$0\leqslant Pr(\mathrm{G1})-\sum_{j=0}^{t}c_jF_{n+2j}(\Delta p/(k_{11}-k_{21}))\leqslant 1-\sum_{j=0}^{t}c_j \tag{29}$$

Note that for a given value of error, the smaller $(k_{11}-k_{21})$ is, the larger is the number of terms t required.

Let $Pr(\mathrm{G1}\cup\mathrm{G2})=Pr(\mathrm{G2})+Pr(\mathrm{G1})$, then (28) can be used to evaluate $Pr(\mathrm{G1}\cup\mathrm{G2})$ by replacing Δp, $k_{11}-k_{21}$ and $k_{1j}-k_{2j}$ by, respectively, p_2, k_{21} and k_{2j}. Exact forms for computing $Pr(\mathrm{G1}\cup\mathrm{G2})$ were given in Lo (1989). As a result, $Pr(\mathrm{G2})$ can be obtained by $Pr(\mathrm{G1}\cup\mathrm{G2})-Pr(\mathrm{G1})$.

Using the fact that $yf_n(y)=nf_{n+2}(y)$, where $f_n(y)=F_n'(y)$ is the probability density function of the chi-square distribution with n degrees of freedom, we can show that $EAC=EAC_1+EAC_2$, where

$$EAC_1=\sum_{j=0}^{\infty}[(k_{11}-k_{21})c_j(n+2j)]F_{n+2j+2}(\Delta p/(k_{11}-k_{21})) \tag{30}$$

$$EAC_2=\sum_{j=0}^{\infty}[k_{21}c_j(n+2j)]F_{n+2j+2}(p_2/k_{21}) \tag{31}$$

5. Numerical analyses

In this section, a five-characteristic example is presented along with sensitivity studies to compare the performance of the two multi-characteristic models. Four factors are considered in the sensitivity analyses: Δp, r, d_j and σ_j. A FORTRAN program has been developed for implementing the solution algorithms and evaluating model parameters.

5.1. *Example*

Consider a five-characteristic problem with parameter values given in Table 2. It is assumed that r is \$15·0, and p_1 and p_2 are \$12·0 and \$7·0, respectively. The optimal screening specification limits for Model 1 are also given in Table 2. The solution procedure stopped when the differences of δ_{ij} between two successive iterations of all the characteristics were smaller than 0·0001. The algorithm stopped after only six iterations. Using the optimal solution, 26·13% of the items are classified into Grade 1, 43·33% are classified into Grade 2, and 30·54% are scrapped. EAC is \$3·65 and EPR is \$2·52.

Characteristic	Grade 1 d_{ij}	Grade 1 δ^*_{ij}	Grade 2 d_{ij}	Grade 2 δ^*_{ij}
1	2·50	1·30	3·85	1·92
2	2·30	1·19	3·60	1·81
3	2·30	1·19	3·60	1·81
4	2·20	1·13	3·50	1·77
5	2·20	1·13	3·50	1·77

Table 2. A five-characteristic example. d_{ij} is the product specification limit of Grade i for characteristic j; δ^*_{ij} is the optimal screening specification limit of Grade i for characteristic j in Model 1.

For Model 2, Pr(G1) and Pr(G2) are 29·30% and 40·73%, respectively, and 29·97% of the items are scrapped. ECA associated with the optimal solution is \$3·63, and EPR is \$2·73. The difference in EPR between Models 1 and 2 is about 8%.

5.2. *Effect of* Δp

Price change of a product may be caused by change in demand, competition, or product improvement. In this section, we study the effects of such a change on the optimal solutions of the two models. Specifically, p_1 was fixed and p_2 was varied from 70 to 115% of the baseline value. The corresponding optimal solutions and several important model characteristics are reported in Table 3.

It was found in both models that when Δp decreased, Pr(G1) decreased, Pr(G2) increased, and Pr(G1$\cup$G2) increased. This indicates that some items, which were originally classified into Grade 1 or scrapped, were classified into Grade 2 when p_2 became larger. EAC and EPR increased as p_2 increased.

It was also observed that Model 1 was more sensitive to Δp. In particular, as Δp changed, the per cent changes in EAC, EPR, Pr(G1) and Pr(G2) were higher than those in Model 2.

5.3. *Effect of* r

In this section, we study the effects of the change in r from 15 (baseline value) to 19·5 (130% of the baseline value). Since k_{ij} is determined by r (equations (2) and (3)), for a given d_{ij}, k_{ij} increases as r increases. Therefore, acceptance cost for a given quality deviation increases as r increases.

The results show that, as r increased, Pr(G1), Pr(G2), EAC, and EPR decreased. The change of r had the same effect on the acceptance probabilities of both grades, i.e. the proportion in both grades decreased as r increased. This relationship was different from the one in section 5.2, where p_2 had a reverse influence on the acceptance probability in Grades 1 and 2. Furthermore, this numerical study also shows that Model 1 was more sensitive to the change of r then Model 2.

5.4. *Effects of product specifications*

Product specifications may be changed by government regulations, competition or consumer demand. To study the effect of tightening product specifications, we obtain the results of reducing 5% of d_{ij} of characteristics 1, 3 and 5 individually, two at the same time, and all three together.

	Model 1				Model 2			
Δp	Pr(G1)	Pr(G2)	ΔEAC	ΔEPR	Pr(G1)	Pr(G2)	ΔEAC	ΔEPR
7·10	44·9	0·8	−7·1	−18·9	47·8	0·8	−4·0	−16·4
6·75	41·8	8·4	−6·8	−18·3	44·8	7·9	−3·8	−15·8
6·40	38·7	15·8	−6·1	−16·6	41·8	14·8	−3·5	−14·4
6·05	35·5	23·0	−5·1	−13·9	38·8	21·6	−2·9	−12·0
5·70	32·4	30·1	−3·7	−10·2	35·6	28·1	−2·1	−8·8
5·35	29·2	36·9	−2·0	−5·6	32·5	34·5	−1·1	−4·8
5·00*	26·1	43·3	0·0	0·0	29·3	40·7	0·0	0·0
4·65	23·0	49·6	2·1	6·5	26·1	46·7	1·3	5·6
4·30	20·1	55·5	4·5	13·8	23·0	52·4	2·7	11·9
3·95	17·1	61·1	7·0	21·9	19·9	57·9	4·4	19·0

Table 3. The effect of Δp (%). ΔEAC is the relative change in EAC from the baseline value; ΔEPR is the relative change in EPR from the baseline value; the asterisk marks the baseline.

Reducing d_{ij} effectively results in an increase of k_{ij}, implying a larger acceptance cost. Consequently, δ^*_{ij} of the characteristics with tightened specifications were significantly smaller. At the same time, this also slightly reduced δ^*_{ij} of other characteristics with unchanged specifications. Moreover, δ^*_{ij} decreased further if there is an additional characteristic with reduced specification limit. The larger number of tightened characteristics, the greater the reduction on EAC, Pr(G1), Pr(G2), and EPR.

5.5. *Effect of characteristic variance*

Reducing the variance of the process implies an improvement of the quality of items produced by the process (Taguchi *et al.* 1984, 1989, Jessup 1985). In many manufacturing processes, the characteristics of the product are determined at different stages. Consequently, reducing the process variance of a production stage results in a reduction of the variance of a characteristic.

To study the effect of reducing characteristics variances, we obtain the results of changing the variances of characteristic 1, 3 and 5 from 1·0 to 0·81, individually, two at the same time, and all three together. The effects on the optimal screening specification limits of Models 1 and 2 were investigated.

When there was only one characteristic having a smaller variance, δ^*_{ij} of this characteristic were significantly larger and δ^*_{ij} of other characteristics were slightly larger than their corresponding baseline values. This also resulted in an increase in Pr(G1), Pr(G2), EAC and EPR. When there were more than one characteristic having smaller variance, the reduction in individual δ^*_{ij} was relatively larger than that when only one characteristic variance was reduced. In addition, the per cent improvement in EPR associated with a simultaneous reduction in several characteristic variances was larger than the sum of the per cent improvements in EPR associated with individual characteristics.

6. Conclusion

In this paper, we considered a situation where items produced in a manufacturing process are sorted into two grades or scrapped. Two product grades have different selling prices and product specifications. The product quality is determined by more than one characteristic. Two models have been proposed using different inspection methods. Optimal solution procedures were proposed and demonstrated. Numerical analyses showed the sensitivities of the solutions with respect to several important model parameters. The models suggested in this paper can be extended to the situation where more than two product grades are available.

The function used in this paper to measure the loss due to imperfect quality is an additive function. It is known that there are several important conditions for using an additive loss function (Keeney and Raiffa 1976). When these conditions are not met, other forms of loss function should be considered. The optimal screening rule of Model 2 can be easily obtained for other loss function. However, depending on the loss function, the solution procedure for Model 1 may be difficult to develop. One important reason that Model 1 performed reasonably well relative to Model 2 is that the difference in acceptance regions between two models was not significant. However, it is expected that the difference in acceptance regions would be significant when other forms of loss functions, such as a multiplicative one, were used. In this case, Model 1 would not perform as well as Model 2.

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