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TIME-BASED PRICING AND LEADTIME POLICIES FOR A BUILD-TO-ORDER MANUFACTURER*

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We studied time-based policies on pricing and leadtime for a build-to-order and direct sales manufacturer. It is assumed that the utility of the product varies among potential customers and decreases over time, and that a potential customer will place an order if his or her utility is higher than the manufacturer's posted price. Once an order is placed, it will be delivered to the customer after a length of time called "leadtime." Because of the decrease in a customer's utility during leadtime, a customer will cancel the order if the utility falls below the ordering price before the order is received. The manufacturer may choose to offer discounted prices to customers who would otherwise cancel their orders. We discuss two price policies: common discounted price and customized discounted price. In the common discounted price policy, the manufacturer offers a single lower price to the customers; in the customized discounted price policy, the manufacturer offers the customers separately for individual new prices. Our analytical and numerical studies show that the discounted price policies results in higher revenue and that the customized discounted price policy significantly outperforms the common discounted price policy when product utility decreases rapidly. We also study two leadtime policies when production cost decreases over time. The first uses a fixed leadtime, and the second allows the leadtime to vary dynamically over time. We find that the dynamic leadtime policy significantly outperforms the fixed leadtime policy when the product cost decreases rapidly. (BUILD-TO-ORDER; PRICING; LEADTIME; E-COMMERCE)

1. Introduction

The integration of build-to-order (BTO) and direct sales (DS) is the business model championed by Dell Computer Corporation, which has become the largest computer manufacturer in the world. This revolutionary concept of manufacturing and marketing personal computers has been widely adopted, and successful applications have been found in the personal computer, consumer electronics, furniture, and farm equipment industries (Siekman 1999). The most notable recent development is use of this business model in the automotive industry, an initiative that will have even more profound impacts on future business practice (Kelly 1999).

In this thriving manufacturing-retailing field, products such as high-tech and fashion items have relatively short life cycles, and their values to customers decrease rapidly over time. Moreover, the production cost structures of these products often change rapidly because of

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technological advances and global competition. Consequently, successful time-based strategies[1] are vital for a BTO-DS manufacturer.

In this paper, we discuss two critical time-based policies on pricing and leadtime for a BTO-DS manufacturer. In traditional business systems, the practice of varying short-term prices (dynamic pricing) is not practical because of the time, cost, and confusion caused by filtering the information down the distribution channel to retailers. In today's e-commerce environment, however, online price adjustments can be made instantaneously at very low cost. In addition to dynamic pricing, flexible pricing is another practice that has drawn increased attention. Under flexible pricing, product prices may be determined by "an implicit one-to-one negotiation between buyers and sellers," and compared with fixed pricing, flexible pricing results in prices that more closely reflect the value (utility) of the product to individual consumers (Green 1998). This practice has been commonly used by institutional buyers and sellers in determining transfer prices (Luft and Libby 1997). It has been anticipated that flexible pricing will be attractive in retailing because of the increase in direct interaction and the closer relationship between a manufacturer and its customers through the e-commerce environment (Hof and Himelstein 1999).

The second policy is related to leadtime. In general, leadtime for a manufacturer includes queue time, fabrication time, inspection time, and transportation time. It is known that a well-designed BTO-DS model can eliminate the need for distributors and result in low inventory levels and production costs. However, a delay (leadtime) exists between the time a customer places an order and the time the customer receives the order. The leadtime may cause a loss in sales or a decrease in price because the product's utility to the customer falls during the leadtime, caused, for example, by competition, technology advances, and seasonal effects. Therefore, continuous leadtime compression has been a major goal for manufacturers. A new aspect of this situation, however, has been observed and discussed recently; that is, a manufacturer may benefit from a longer leadtime if the production cost decreases over time. This has been an important phenomenon in the high-tech industry mainly because of the declining costs of components.

Dynamic pricing and leadtime management have been studied extensively, but separately. The literature on dynamic pricing policy is divided into two categories: long-term policy and short-term policy (Rajen, Rakesh, and Steinberg 1992). The objective of a long-term dynamic pricing policy is to maximize the expected profit in a given planning horizon or product cycle. The two main factors considered in studies of long-term policies are learning effects on production cost and diffusion and saturation effects on product demand (Kalish 1983). Studies on short-term dynamic pricing policies, on the other hand, are motivated mainly by product perishability. The issue, therefore, is usually addressed jointly with inventory ordering policies. Furthermore, dynamic pricing has been used recently in the yield management area (Bitran and Mondschein 1997; Zhao and Zheng 2000). Typical examples are airline and hotel pricing and booking policies, where the inventory has a fixed life (such as airline tickets and hotel rooms, which do not have salvage values at the end of the selling horizon). Comprehensive literature reviews on joint production and marketing decisions, inventory models for perishable products, long-term dynamic pricing models, and yield management have been made by Kalish (1983), Eliashberg and Steinberg (1987), Rajen, Rakesh, and Steinberg (1992), and McGill and Van Ryzin (1999).

Leadtime management is an important and active area of research in manufacturing and operations management. Most of the existing studies tie leadtime to process capacity, logistics, and job scheduling. Duenyas and Hopp (1995) classify recent studies on leadtime management into several categories, including manufacturing flow time reduction, estimation and prediction of manufacturing leadtime, the setting of due dates, and the relationship with other decision parameters such as product price, inventory lot size, and dispatching rule. In

[1] More discussions on time-based strategies can be found in Stalk and Hout (1990).

addition, studies have been conducted on leadtime reduction through buyer and supplier strategic alliances for better coordination of production schedule and inventory control (Iyer and Bergen 1997). Note that it has been recognized that product price and demand are affected by leadtime. In most studies, product price and leadtime, however, are implicitly or explicitly assumed fixed over time. Because of the vast volume of the literature, readers are referred to the papers by Karmarkar (1987) and Duenyas and Hopp (1995) for reviews of related studies.

In this paper, we first discuss dynamic pricing policies, assuming that leadtime is fixed and will not be changed by the manufacturer. It is assumed that the utility of the product of interest varies among potential customers and decreases over time, and that a customer will cancel the order without penalty if his or her utility of the product falls below the ordering price before the order is delivered. The manufacturer may choose to offer discounted prices to customers who would otherwise cancel their orders. We discuss two price discount policies: common discounted price and customized discounted price. In the common discounted price policy, the manufacturer offers one lower price to the customers; in the customized discounted price policy, the manufacturer offers the customers individual new prices. The manufacturer's decision is to find the posted and discounted prices as functions of time, which maximize the expected revenue in a given planning horizon. Our analytical and numerical studies show that price discounts increase the manufacturer's revenue and that the customized discounted price policy significantly outperforms the common discounted price policy when the product utility decreases rapidly.

With regard to leadtime policy, we consider a situation where the production cost function is decreasing in time. Two leadtime policies, fixed leadtime and dynamic (time-varying) leadtime, are studied jointly with a dynamic pricing policy. Implementing a dynamic leadtime policy may be more difficult than implementing a fixed leadtime policy because it requires more effort in production planning, tracking the orders, etc. Economic gains may be significant, however, especially when the production cost decreases over time. The optimal leadtime and posted price as functions of time are found by maximizing the manufacturer's expected profit in a given planning horizon. We find that the dynamic leadtime policy significantly outperforms the fixed leadtime policy, especially when the product cost decreases at a fast rate. Mathematical proofs of all the results are given in the Appendix.

2. Common and Customized Price Discounts

In this section, we study the optimal pricing policy that maximizes the expected revenue of a BTO-DS manufacturer. It is assumed that the manufacturer acts as its own retailer and is targeting a large group of potential customers who are heterogeneous in their preferences (utilities) for the product. The manufacturer is to formulate a pricing strategy to maximize the total expected revenue in a fixed planning horizon. Assume that initially the manufacturer posts a price, and a potential customer considers the price to decide whether to place an order. Once an order is placed, it will be delivered to the customer after a time length called "leadtime." It is known that leadtime is affected by product demand, manufacturing capacity and utilization, scheduling, logistics, and other factors. In this section, we assume that the manufacturer has a fixed leadtime and that a customer may cancel his/her order without penalty before the order is delivered. For simplicity, we assume that the shipping time can be ignored.[2]

We consider three cases in terms of price discounts. In Subsection 2.1, we develop a basic model with no price discounts, and in Subsections 2.2 and 2.3, we discuss the common and customized discounted price policies, respectively.

[2] Tatsioplolous and Kingsman (1983) reported that approximately 10% and 2% of total leadtime are caused by actual processing and shipping times, respectively.

2.1. *Basic Model*

Assume that the utility of the product to the customer is a function of time, t:

$$U(t) = Ke^{-\lambda t}, \qquad t \geq 0, \tag{1}$$

where $\lambda > 0$ is the rate of decrease in utility, and K is the utility at time 0 and may vary among the potential customers. It is assumed that K follows a normal distribution with mean $\mu > 0$ and variance σ^2, and that μ is much larger than σ so that $\Pr\{K < 0\}$ is negligible. It is also assumed that λ, μ, and σ are finite constants. As pointed out by Bitran and Mondschein (1997), the variation in customers' utilities is caused mainly by heterogeneity in the market segment and a lack of information about customers' tastes and needs. The decrease in the utility function may be caused by competition, technology advances, and seasonal effects. For seasonal products, λ, K, and σ can be estimated by using past price and demand data, and economical, technological, and subjective information. If historical data are scarce or not available, marketing studies and subjective assessment may be necessary for developing estimates for these parameters.

We assume an asymmetric information structure, where the customers individually do not possess the information for forming their expectations for immediate future values of utilities (Narasimhan 1989). This assumption is valid when the decrease in the utility function is driven mainly by competitors' prices or technology advances, and individual customers do not have information on short-term price trends or technology development. In contrast, the manufacturer is capable of assessing the customers' utility function, using market research, forecasts of competitors' prices and technology advances, and other subjective and empirical information. Under the assumption, individual customers compare their current utilities of the product with the posted price to decide whether to place orders. Let $p(t)$ denote the posted price at time t. We consider a situation discussed by Kalish (1983), where the demand rate at time t is not affected by "diffusion/saturation effect" and is solely a function of $p(t)$. Therefore, a customer will place an order at time t if $U(t) \geq p(t)$, and the expected demand rate at time t is given by:

$$D(p(t), t) = \beta \Pr\{U(t) \geq p(t)\}, \tag{2}$$

where $\beta > 0$ is the maximum market potential; i.e., the demand rate when $p(t) = 0$. Equation (2) can be written as:

$$D(p(t), t) = \beta[1 - F(p(t)e^{\lambda t})],$$

where $F(\cdot)$ is the cumulative distribution function (CDF) of K.

For the orders placed at time t, the scheduled delivery time is $t + \tau$, where the leadtime τ is assumed known and fixed in this section. We further assume that the customers are allowed to cancel their orders before the scheduled delivery time, and that a customer who placed an order at time t will cancel the order if his/her utility falls below $p(t)$ at any time between t and $t + \tau$. Therefore, the actual (realized) expected demand rate at time t is $D(p(t), t + \tau)$. Assuming the manufacturer has sufficient capacity and that the costs associated with handling order cancellations are negligible, the expected revenue rate at time t, denoted by $r(p(t), t)$, is

$$r(p(t), t) = p(t)\, D(p(t), t + \tau).$$

The manufacturer introduces the product to the market at time 0 and plans the end of the product cycle at time T. In other words, the manufacturer's planning horizon is $[0, T]$. For a given pricing strategy, $p(\cdot)$, the expected total revenue over $[0, T]$ is given by

$$ER(p(\cdot)) = \int_0^T \beta p(t)\left[1 - \Phi\left(\frac{p(t) - \alpha(t)\mu}{\alpha(t)\sigma}\right)\right]dt,$$

where $\alpha(t) = e^{-\lambda(t+\tau)}$, and $\Phi(\cdot)$ is the standard normal CDF. Note that a discount rate is not considered because products of interest tend to have short lives.

LEMMA 1. *Let a and b be two constants, where* $0 < a \leq 1$. *Define*

$$g(z, a, b) = 1 - \Phi(z) - a(z + b)\phi(z), \tag{3}$$

where $\phi(\cdot)$ *is the standard normal probability density function* (PDF). *Let* z^* *denote the solution to* $g(z, a, b) = 0$. *If* z^* *exists, it is a decreasing function of a and b.*

Lemma 1 is an important result used throughout the paper in analysis and discussion. The following proposition gives the optimal solution, which maximizes $ER(p(\cdot))$ and its properties.

PROPOSITION 1. *Let* $p^*(t)$ *denote the optimal price function of time t, which maximizes* $ER(p(\cdot))$.

(i) $p^*(t) = \alpha(t)(\mu + z^*\sigma)$, *where* z^* *is a constant that satisfies* $g(z^*, 1, \mu/\sigma) = 0$.

(ii) *The realized demand rate associated with the optimal price is given by*

$$D(p^*(t), t + \tau) = \beta[1 - \Phi(z^*)].$$

(iii) *The expected revenue rate associated with the optimal price is given by*

$$r(p(t), t) = \beta\alpha(t)(\mu + z^*\sigma)[1 - \Phi(z^*)].$$

The proposition indicates that the optimal strategy is to decrease the price over time to target a fixed proportion of potential customers. Because z^* is a constant for given μ and σ, the optimal price is a decreasing, exponential function of time, and its pattern is similar to that of (1). The optimal price at time t is, however, a function of $t + \tau$. This result is intuitive because a customer makes the decision on whether to cancel the order at the delivery time. Furthermore, because $\alpha(t)$ is a decreasing function of λ and τ, as λ or τ becomes larger, the optimal price and expected revenue become smaller while the realized demand rate remains unchanged. Consequently, the proposition gives a simple structure for evaluating possible options when the related model parameters change. For example, a smaller leadtime results in an increase in the optimal price and expected revenue, but it does not change the realized demand rate. Furthermore, if λ becomes larger, a shorter leadtime is necessary to maintain the same level of expected revenue.

2.2. *Common Discounted Price*

Under a given $p(t)$, it is possible that a customer, who places an order at time t, cancels his/her order before the scheduled delivery time $t + \tau$. To avoid possible order cancellation, the manufacturer may offer the customer a discounted price. Suppose the manufacturer does not have to make the new price known to all the customers and offers the discounted price only to the customers whose utilities at time $t + \tau$ are smaller than $p(t)$. In this subsection, we assume that a common discounted price is offered by the manufacturer. Because of the variation in the customers' utilities, order cancellation attempts may occur at different times before the scheduled delivery time. To simplify the implementation of the pricing policy and avoid multiple cancellation attempts at different times by a customer, the manufacturer may offer a customer the discounted price immediately after the first cancellation attempt is made.

A customer who places an order at time t has a utility of $U(t) \geq p(t)$, or $Ke^{-\lambda t} \geq p(t)$. Multiplying $e^{-\lambda\tau}$ to both sides of the last inequality, we obtain

$$U(t + \tau) \geq p(t)e^{-\lambda\tau}$$

which suggests that those who place orders at time t will not cancel their orders if the product price is adjusted to $p(t)e^{-\lambda\tau}$ or lower.

Let $p_1(t)$ denote the discounted price. From the above discussion, $p_1(t)$ should not be lower than $p(t)e^{-\lambda\tau}$. On the other hand, the optimal $p_1(t)$ may be higher than $p(t)e^{-\lambda\tau}$, if the additional revenue from a higher price can offset the loss incurred by order cancellation. Because a customer with $U(t+\tau) > p_1(t)$ will not cancel his or her order, the total realized demand rate is given by $\beta[1 - F(p_1(t)e^{\lambda(t+\tau)})]$. Similarly, the realized demand rate associated with the original posted price is $\beta[1 - F(p(t)e^{\lambda(t+\tau)})]$. Consequently, the realized demand rate associated with the discounted price is $\beta[F(p(t)e^{\lambda(t+\tau)}) - F(p_1(t)e^{\lambda(t+\tau)})]$, and the expected revenue rate at time t from the customers who keep the orders under the discounted price is given by

$$r_1(p(t), p_1(t), t) = \beta p_1(t)[F(p(t)e^{\lambda(t+\tau)}) - F(p_1(t)e^{\lambda(t+\tau)})].$$

The total expected revenue at time t is the sum of $r(p(t), t)$ and $r_1(p(t), p_1(t), t)$:

$$r(p(t), p_1(t), t) = \beta p(t)\left[1 - \Phi\left(\frac{p(t) - \alpha(t)\mu}{\alpha(t)\sigma}\right)\right] + \beta p_1(t)\left[\Phi\left(\frac{p(t) - \alpha(t)\mu}{\alpha(t)\sigma}\right) - \Phi\left(\frac{p_1(t) - \alpha(t)\mu}{\alpha(t)\sigma}\right)\right],$$

where

$$p_1(t) \geq p(t)e^{-\lambda\tau}. \tag{4}$$

The total expected revenue over $[0, T]$ is given by

$$ER(p(\cdot), p_1(\cdot)) = \int_0^T r(p(t), p_1(t), t)dt. \tag{5}$$

The optimal pricing policy is obtained by maximizing (5), subject to (4). We consider two cases based on the Karush-Kuhn-Tucker conditions for optimality. In case A, (4) is relaxed. If the solution found in case A violates (4), we consider case B, where (4) is binding. We obtain the first-order conditions for optimality under these two cases.

Case A. The first order conditions for optimality are

$$1 - \Phi(z) - (z - z_1)\phi(z) = 0, \tag{6}$$

$$\Phi(z) - \Phi(z_1) - (z_1 + \mu/\sigma)\phi(z_1) = 0, \tag{7}$$

where $z = (p(t) - \alpha(t)\mu)/\alpha(t)\sigma$, and $z_1 = (p_1(t) - \alpha(t)\mu)/\alpha(t)\sigma$.

Let z^* and z_1^* be the solutions to (6) and (7). Then, the "optimal" prices are given by

$$p^*(t) = \alpha(t)(\mu + z^*\sigma),$$

and

$$p_1^*(t) = \alpha(t)(\mu + z_1^*\sigma).$$

The expected revenue rate at t, $r(p(t), p_1(t), t)$, can be expressed as:

$$r(p(t), p_1(t), t) = p^*(t)\beta[1 - \Phi(z^*)] + p_1^*(t)\beta[\Phi(z^*) - \Phi(z_1^*)], \tag{8}$$

where $\beta[1 - \Phi(z^*)]$ is the realized demand rate for the customers who pay the original posted price, and $\beta[\Phi(z^*) - \Phi(z_1^*)]$ is the realized demand rate for the customers who pay the discounted price. The total realized demand rate is $\beta[1 - \Phi(z_1^*)]$.

CASE B. If $p_1^*(t)$ found in case A is smaller than $p^*(t)e^{-\lambda\tau}$, the solution is infeasible, and $p_1(t)$ must be equal to $p(t)e^{-\lambda\tau}$, and the first-order condition is

$$(1 - e^{-\lambda\tau})\left[1 - \Phi(z) - \left(z + \frac{\mu}{\sigma}\right)\phi(z)\right] + e^{-\lambda\tau}[1 - \Phi(z_2) - (z_2 + \mu/\sigma)\phi(z_2)] = 0 \quad (9)$$

where $z_2 = [p(t)e^{-\lambda\tau} - \alpha\mu]/(\alpha\sigma)$. Because

$$z_2 = e^{-\lambda\tau}(z + \mu/\sigma) - \mu/\sigma$$

the left side of (9) is a function of z, from which we can use a direct search procedure to find z^*. The optimal posted and discounted prices are then determined, respectively, by

$$p^*(t) = \alpha(t)(\mu + z_2^*\sigma), \qquad \text{and}$$

$$p_1^*(t) = e^{-\lambda\tau}p^*(t)$$

Note that, in using this solution, $p_1^*(t)$ is actually the manufacturer's optimal posted price at time $t + \tau$, suggesting that the manufacturer simply offers the posted price at the scheduled delivery time as the discounted price to the customers who want to cancel their orders. Equation (8) can be used to obtain the expected revenue rate by replacing z_1^* by z_2^*, and the demand rates can also be obtained accordingly.

PROPOSITION 2. *The optimal posted price of the basic model is between the optimal discounted and posted prices of the common discounted price model. Under the optimal common discounted price policy,*

(i) *the total realized demand rate is larger than that of the basic model*;
(ii) *the realized demand rates remain unchanged over time*; *and*
(iii) *the expected revenue is larger than that of the basic model.*

The proposition indicates that, compared with the basic model, the common discounted price model takes advantage of the heterogeneity of the customers by charging those with higher utility values with a larger posted price and retains the orders of those with lower utility values by offering them a lower price. As a result, the total realized demand rate and the expected revenue are higher for the common discounted price model. Furthermore, the realized demand rates under the optimal posted and discounted prices remain constant over time, suggesting that the effort associated with implementing the price discount procedure stays steady over time.

2.3. *Customized Discounted Price*

Under the customized discounted price policy, the manufacturer offers individual discounted prices to those customers who placed orders at time t but whose utilities at time $t + \tau$ fell below the posted price $p(t)$. We assume that the manufacturer has an efficient (perfect) procedure to assess customers' utilities so that the discounted price is equal to the customer's utility at $t + \tau$. Note that this assumption can be generalized by letting the discounted price be the customer's utility at $t + \tau$, multiplied by an efficiency factor between 0 and 1 for the manufacturer's ability of assessing a customer's utility. The model can be easily modified accordingly, but the solution procedure becomes more complicated. Note that, similar to the situation in the common discounted price policy, the manufacturer may offer the customized discounted price immediately to a customer whose utility drops below the posted price to simplify the implementation of the policy. However, to determine the customized discounted price for a customer, the manufacturer has to individually assess the customer's utility at the time of order cancellation to determine the customer's utility at the scheduled delivery time. Consequently, the implementation of the customized discounted price policy requires significantly more effort than the common discounted price policy.

The expected revenue at time t from the discounted prices is determined by

$$r_2(p(t), t) = \int_{p(t)e^{-\lambda\tau}}^{p(t)} \beta U(t+\tau) f(U(t+\tau)) d(U(t+\tau)),$$

where $f(\cdot)$ is the PDF of $U(t+\tau)$, which is a normal PDF with mean $\alpha(t)\mu$ and SD $\alpha(t)\sigma$.

Using the first incomplete moment of a normal random variable (Tang 1987), we obtain

$$r_2(p(t), t) = \beta\alpha(t)\{\mu[\Phi(z) - \Phi(z_3)] - \sigma[\phi(z) - \phi(z_3)]\},$$

where $z = [p(t) - \alpha(t)\mu]/[\alpha(t)\sigma)]$ (as in the previous subsections) and $z_3 = [(p(t)e^{-\lambda\tau} - \alpha(t)\mu]/[\alpha(t)\sigma)]$. Consequently, the total expected revenue for the manufacturer at time t is given by

$$r(p(t), t) = \beta p(t)[1 - \Phi(z)] + \alpha(t)\beta\{\mu[\Phi(z) - \Phi(z_3)] - \sigma[\phi(z) - \phi(z_3)]\}.$$

Using the first derivative of $r(p(t), t)$ with respect to $p(t)$, we find the first-order condition for the optimal price at time t to be

$$1 - \Phi(z) - e^{-\lambda\tau}\left(z_3 + \frac{\mu}{\sigma}\right)\phi(z_3) = 0 \tag{10}$$

Because

$$z = e^{\lambda\tau}\left(z_3 + \frac{\mu}{\sigma}\right) - \frac{\mu}{\sigma} \tag{11}$$

the left side of (10) is a function of z_3. Let z_3^* denote the value of z_3 that satisfies (10). Note that z_3^* does not depend on t, and that $z^* \geq z_3^*$. The optimal price at time t is then given by

$$p^*(t) = e^{-\lambda t}(\mu + z_3^*\sigma),$$

and the total realized demand rate is $1 - \Phi(z_3^*)$. The expected revenue rate at time t associated with the optimal pricing policy is given by

$$r(p^*(t), t) = \beta e^{-\lambda t}(\mu + z_3^*\sigma)[1 - \Phi(z^*)] + \beta\alpha(t)\{\mu[\Phi(z^*) - \Phi(z_3^*)] - \sigma[\phi(z^*) - \phi(z_3^*)]\},$$

where $1 - \Phi(z^*)$ and $\Phi(z^*) - \Phi(z_3^*)$ are the expected realized demand rates associated with the posted price and discounted prices, respectively.

PROPOSITION 3. *For the optimal customized discounted price model,*

(i) *the optimal posted price of the basic model is between the optimal posted price, $p^*(t)$, and the lowest discounted price, $p^*(t)e^{-\lambda\tau}$, of the customized discounted price model*;
(ii) *the total realized demand rate is larger than that of the basic model*;
(iii) *the realized demand rates remain unchanged over time*; *and*
(iv) *the expected revenue is larger than that of the common discounted price model and of the basic model.*

Because the results stated in the proposition are very similar to those in Proposition 2, the advantages of the customized discounted price model over the basic model are similar to those of the common discounted price model. The expected revenue associated with the optimal solution is higher than that of the common discounted price model mainly because the customized discounted price model fully takes advantage of the heterogeneity of the customers.

3. Leadtime and Production Cost

In this section, we extend the basic model discussed in the last section to study a joint price and leadtime policy when the production cost decreases over time. Two types of leadtime policies are considered. In the first, leadtime is fixed in the entire planning horizon; in the second, the leadtime is allowed to vary dynamically, depending on the time the order is placed.

3.1. *Fixed Leadtime*

We assume that the production cost of an item delivered at time t is a decreasing exponential function as follows:

$$C(t) = ce^{-\lambda_1 t}, \qquad \text{for} \qquad t \geq 0, \tag{12}$$

where $c > 0$ is the production cost at $t = 0$, and $\lambda_1 > 0$ is the rate of decrease in production cost. For simplicity, we assume that the times of producing and shipping a finished item to the customer are negligible and that the costs associated with handling order cancellations are also negligible. The production cost function is appropriate when the main investment is made at the end of the production process, such as installation of a major component,[3] so that the cost of producing an item is a decreasing function of its time of delivery. Under the assumptions, we keep the model tractable and focus on the effect of production cost on the pricing and leadtime determination.

Because the profit of the item ordered at time t and delivered at time $t + \tau$ is the difference of $p(t)$ and $C(t + \tau)$, the expected profit for given $p(\cdot)$ and τ in the planning horizon is obtained by

$$EPR(\tau, p(\cdot)) = \int_0^T \beta[p(t) - ce^{-\lambda_1(t+\tau)}]\left[1 - \Phi\left(\frac{p(t) - \alpha(t)\mu}{\alpha(t)\sigma}\right)\right]dt \tag{13}$$

The following necessary condition for the optimal price is obtained from $\partial EPR/\partial p(t)$:

$$1 - \Phi\left(\frac{p(t) - \alpha(t)\mu}{\alpha(t)\sigma}\right) - \frac{(p(t) - ce^{-\lambda_1(t+\tau)})e^{\lambda(t+\tau)}}{\sigma}\phi\left(\frac{p(t) - \alpha(t)\mu}{\alpha(t)\sigma}\right) = 0 \tag{14}$$

For a given τ, (14) leads to the following result for the optimal price at time t:

$$p^*(t) = \alpha(t)(\mu + z^*\sigma) \tag{15}$$

where z^* is the solution to

$$1 - \Phi(z^*) - \left(z^* + \frac{\mu - ce^{(\lambda-\lambda_1)(t+\tau)}}{\sigma}\right)\phi(z^*) = 0 \tag{16}$$

Equation (16) indicates that z^* changes over time. Consequently, the demand rate changes over time even under a fixed leadtime, which is different from the results in the last section.

Because $EPR(\tau, p^*(\cdot))$ is now a function of τ, the problem of finding the optimal fixed leadtime is as follows:

$$\operatorname*{Max}_{\tau} EPR(\tau, p^*(\cdot)) = \int_0^T \beta[p^*(t) - ce^{-\lambda_1(t+\tau)}]\left[1 - \Phi\left(\frac{p^*(t) - \alpha(t)\mu}{\alpha(t)\sigma}\right)\right]dt$$

subject to (15) and (16).

Because z^* is a function of both τ and t, as indicated by (16), a numerical procedure is required to find the optimal price and leadtime policy in the planning horizon. Although it is difficult to derive analytical properties of the optimal policy, from (16) we know that, for a given τ and λ, a large λ_1 will result in a smaller z^* and, thus, a lower optimal price.

[3] This is a practice commonly seen in the BTO business model (Siekman 1999).

Furthermore, if $\lambda < \lambda_1$, then a larger τ implies a smaller optimal z^* and a smaller optimal price. The opposite is true when $\lambda > \lambda_1$. When $\lambda = \lambda_1$, we have a constant z^*.

3.2. *Dynamic Leadtime*

When the leadtime is allowed to vary over time, we can consider, at each time point within the planning horizon, the future decrease in the production cost to determine the leadtime and price. Because the leadtime may be a function of time, we use $\tau(t)$ to denote the leadtime for the orders placed at time t.

PROPOSITION 4. *The optimal leadtime at time t, $0 \le t \le T$, is given by*

$$\tau^*(t) = \max\left\{0, \frac{1}{\lambda_1}\ln\left(\frac{\lambda_1 c}{\lambda p^*(t)}\right) - t\right\}, \tag{17}$$

where

$$p^*(t) = [(\mu + z^*\sigma)/h]^{\lambda_1/(\lambda_1-\lambda)}, \tag{18}$$

$h = (\lambda_1 c/\lambda)^{(\lambda/\lambda_1)}$, *and z^* is a constant that satisfies $g(z^*, 1 - \lambda/\lambda_1, \mu/\sigma) = 0$.*

In practice, one would expect that the production cost is smaller than the product price (i.e., $ce^{-\lambda_1 t} < p^*(t)$). Consequently, (17) suggests that the optimal leadtime will be 0, if λ_1 is not larger than λ. This leads to a general principle that when the decreasing rate of production cost (λ_1) is smaller than that of the product's utility (λ), there is no economic advantage for the manufacturer to delay the delivery time of the product. It should be pointed out, however, that this is not the only situation where the optimal leadtime is 0. As indicated by (17), the optimal leadtime is 0 as long as $\lambda_1 c$ is not larger than $\lambda p^*(t)$. Furthermore, although the optimal leadtime may not be 0 at a smaller t, it will become 0 because it decreases linearly over time.

We define t_0 as the time when the optimal leadtime becomes 0. Using (17), t_0 is given by

$$t_0 = \frac{1}{\lambda_1}\ln\left(\frac{\lambda_1 c}{\lambda p^*(t)}\right).$$

PROPOSITION 5. *Consider a given time interval $[0, T_1]$, where $T_1 \le t_0$.*

(i) *The optimal price is a constant given by* (18)*, and the optimal leadtime is given by*

$$(1/\lambda_1)\ln(\lambda_1 c/(\lambda p^*(t)) - t.$$

(ii) *The profit margin associated with the optimal price and leadtime is $1 - \lambda_1/\lambda$.*

(iii) *The total expected profit over the time interval is*

$$EPR_1(T_1) = \beta p^*(t)(1 - \lambda_1/\lambda)[1 - \Phi(z^*)]T_1.$$

The proposition indicates that the optimal strategy in the time interval $[0, T_1]$ is to simply have a constant posted price and reduce the leadtime linearly to maintain the same profit margin.

PROPOSITION 6. *In time interval (t_0, T_2), $T_2 > t_0$, the optimal leadtime is* 0*, and the optimal price is*

$$p^*(t) = e^{-\lambda t}(\mu + z^*\sigma),$$

where z^* is the solution to $g(z^*, 1, (\mu - ce^{-(\lambda_1-\lambda)t})/\sigma) = 0$. The total expected profit is

$$EPR_2(T_2) = \int_{t_0}^{T_2} \beta[p^*(t) - ce^{-\lambda_1 t}][1 - \Phi(z^*)]dt.$$

Note that Proposition 6 is a special case of the fixed leadtime considered in Subsection 3.1.

TABLE 1
Optimal Solutions

Model	z^*	z_2^* or z_3^*	$p^*(0)$	$p_1^*(0)$	d_0[§]	d_1[§§]	$r(p^*(0), 0)$
Basic	−1.5922	—	8.1756	—	0.9443	0.9443	7.7205
Common discounted price	−1.3612	−1.5997	8.4003	8.1684	0.9133	0.9452	7.9323
Customized discounted price	−1.3572	−1.5959	8.4042	8.1721[+]	0.9126	0.9447	7.9349

[§] Realized demand rate, in 10,000, under the posted price $p^*(0)$.
[§§] Total realized demand rate in 10,000.
[+] The lowest customized discounted price.

Using Propositions 5 and 6, the total expected profit in time horizon $[0, T]$ is given as:

$$EPR(T) = \begin{cases} EPR_1(T) & \text{for} \quad T \le t_0 \\ EPR_1(t_0) + EPR_2(T) & \quad T \ge t_0 \end{cases}.$$

Note that numerical integration is required to evaluate $EPR_2(T_2)$ because both z^* and $p^*(t)$ are functions of time.

4. Numerical Results

In this section, we present numerical results and analyses.[4] In Subsection 4.1, we compare the common and customized discounted price models and the basic model with respect to the effects of three model parameters, λ, τ, and σ, on the optimal solutions. In Subsection 4.2, we study the dynamic leadtime policy and compare it with the fixed leadtime policy.

4.1. *Common and Customized Discounted Prices*

Consider a BTO-DS manufacturer. The customer's utility of the product of interest is described by $U(t)$ in (1) with $\lambda = 0.002$/d, $\mu = 10$, and $\sigma = 1$. With these parameter values, the utility decreases on average 5.8% in 30 days and 45.1% in 300 days. Let τ be 14 days, T be 360 days, and the demand rate at time t be given by (2) with $\beta = 10{,}000$ customers.

The optimal solutions for the three models are given in Table 1. Note that we report only optimal prices and expected realized demand and revenue rates at time 0, because the optimal prices and expected revenue rates at time t, $0 < t \le T$, can be obtained by multiplying these values by $e^{-\lambda t}$, and the demand rates remain unchanged over time. For convenience, the expected demand and revenue rates are reported in units of 10,000 in the entire subsection.

For case A of the common discounted price model, z^* and z_1^* are −0.37585 and −2.11469, respectively. This solution is not feasible, however, because $p_1^*(t)$ is smaller than $p^*(t)e^{-\lambda\tau}$ [i.e., constraint (4) is violated]. Therefore, the case B solution is reported. The posted price of the basic model is significantly lower than that for either of the other two models. Furthermore, the optimal posted price ($p^*(t)$) and the lowest price ($p^*(t)e^{-\lambda\tau}$) of the customized discounted price model are larger than their corresponding prices for the common discounted price model. However, the differences in the prices between the two models and the resulting demand rates and revenue rates are very small.

To study the effect of λ on the optimal solutions, we obtained the optimal solutions under selected values of λ ranging from 0.002 to 0.02. The optimal prices, realized demand rates, and expected revenue rates are reported in Figures 1–3, respectively.

In Figure 1, the middle curve is the optimal price of the basic model, and the two upper curves are the posted prices of the common discounted price and customized discounted price

[4] A FORTRAN program with IMSL subroutines (Microsoft Corporation 1995) was developed to perform numerical integration and to implement the solution procedures. For the examples considered, the time required for finding the optimal solutions was about 2–10 minutes on a Pentium II PC.

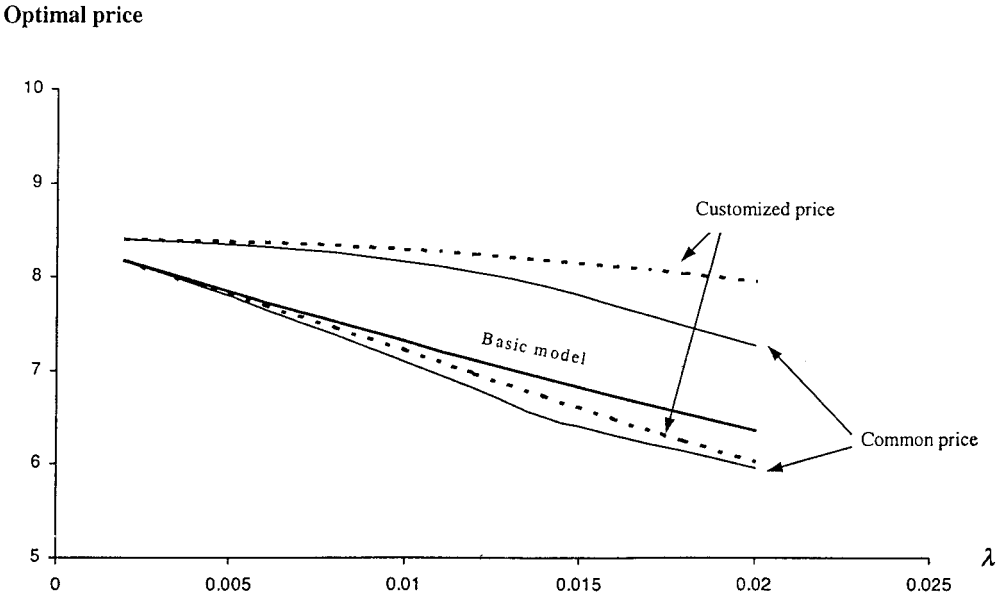

FIGURE 1. Optimal Prices as Functions of λ.

models. The lower two curves are the common discounted price and the lowest discounted prices of the common discounted price and customized discounted price models, respectively. The figure shows that the basic model has the lowest posted price (cf. Propositions 2 and 3). It also shows that the customized discounted price model has a larger posted price than the common discounted price model, and the difference is larger as λ increases. However, the discounted price of the common discounted price model and the lowest purchasing price of the customized discounted price model remain quite close. Consequently, the customized discounted price model has the largest price range (the difference between the posted price and the lowest discounted price) among the three models. The price range of the

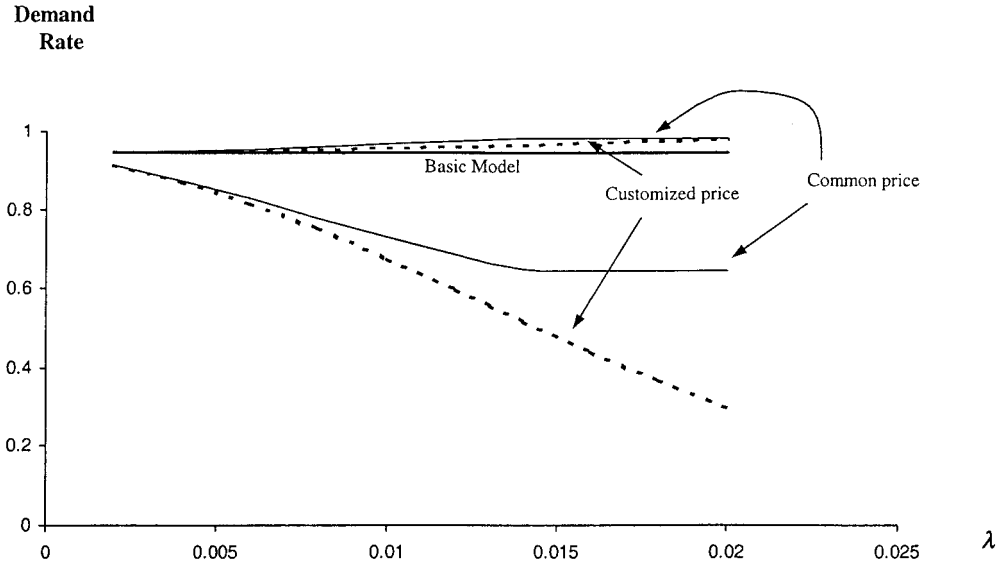

FIGURE 2. Realized Demand Rates as Functions of λ.

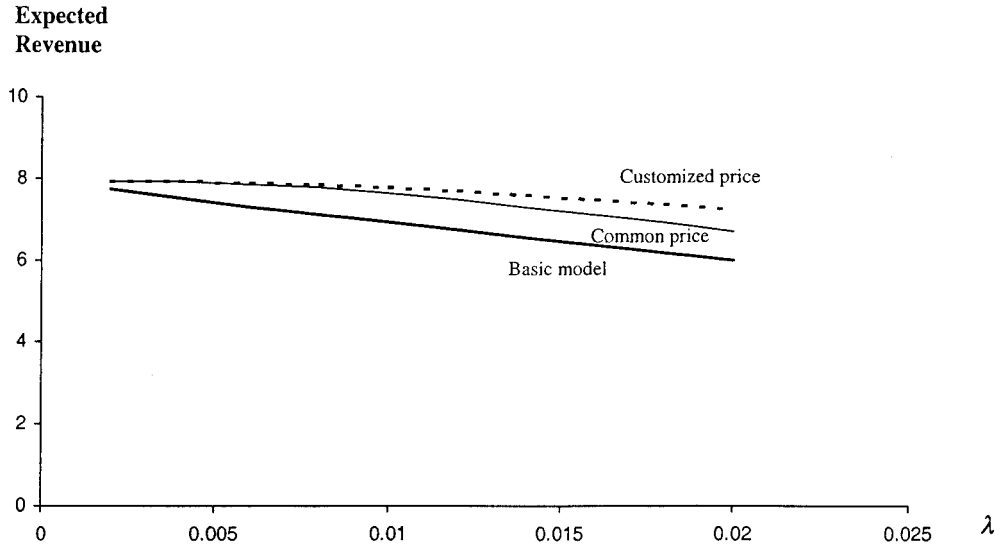

FIGURE 3. Expected Revenues as Functions of λ.

customized discounted price model becomes larger as λ increases, whereas the price range of the common discounted price model is relatively stable. In Figure 2, the upper two curves are the total realized demand rates, and the two lower curves are the realized demand rates under the posted prices for the common discounted price and customized discounted price models. Furthermore, for each price discount model, the vertical distance between the upper and lower curves represents the expected demand rate generated from discounted prices. It is clear that using customized pricing requires more efforts in discounted price determination, especially when λ is large. Figure 3 indicates that, when λ is small, the performances of the common discounted price and customized discounted price models are very close. However, the benefit of the customized price model becomes more significant as the utility of the product decreases faster.

In Figure 4, the expected revenues are shown as a function of σ ranging from 0.25 to 3, under two λ values, 0.002 and 0.02. In general, as σ increases, the expected revenue decreases for all the three models. The effects of σ on the differences among the three models depend heavily on λ. When λ is small (e.g., 0.002), the difference between two discounted price models is negligible. However, when λ is large, the difference becomes significant. Similar results are seen in Figure 5, where we study the effects of leadtime on the three models. As expected, when the leadtime increases, the expected revenue decreases for all the three models. When λ is small, the expected revenue remains quite stable as the leadtime increases because the product's utility decreases very slowly. However, the adverse effects are clearly seen on the basic model even when λ is small. When λ is large, the customized discounted price model becomes attractive when the leadtime is large.

As indicated in Propositions 2 and 3, we found that the two price discount models outperform the basic model and that significant differences occur when λ is large. The two price discount models perform similarly when λ is small; however, the customized discounted price model significantly outperforms the common discounted price model when λ is large. The benefit of the customized discounted price model is attributed mainly to the flexibility in discounted prices, which allows it to take full advantage of customer heterogeneity. As a result, the posted price and expected revenue remain quite stable as λ changes. The common discounted price model, on the other hand, has only one discounted price. Furthermore, the effects of σ and τ are more substantial when λ is large. Therefore, we conclude that λ is the most critical model parameter to a BTO-DS manufacturer.

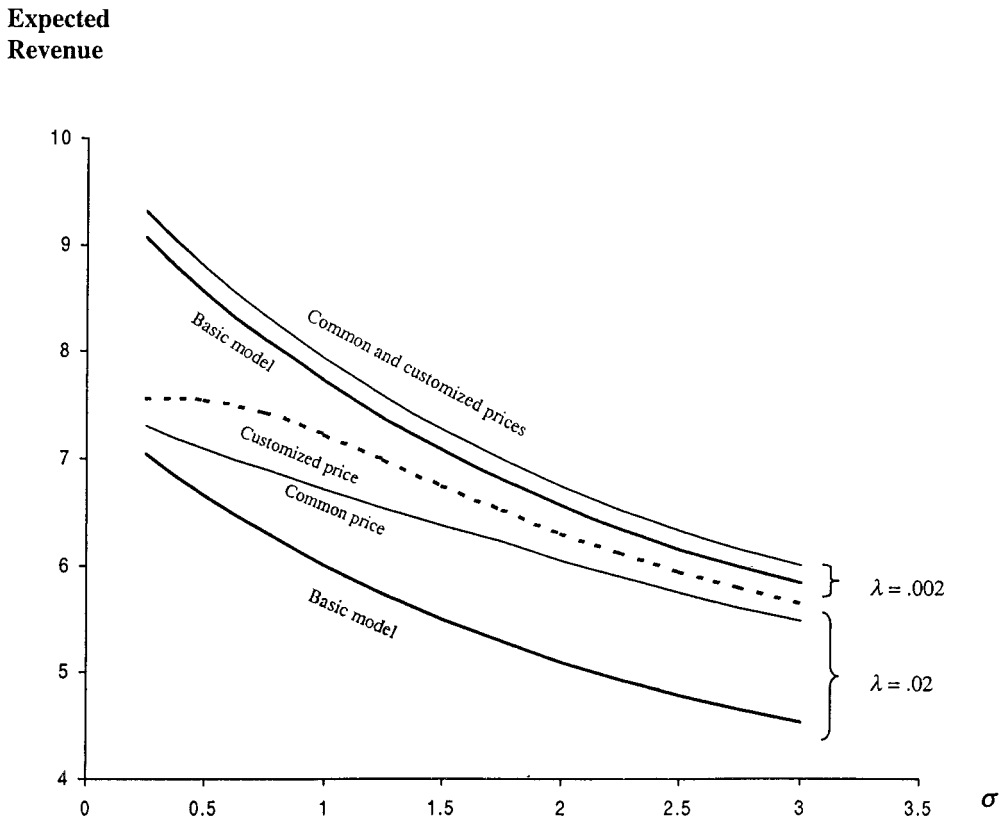

FIGURE 4. Effects of σ on Expected Revenue.

4.2. *Fixed and Dynamic Leadtimes*

Consider the example in the last subsection. Assume that the production cost is given by (12) with $c = 5$ and $\lambda_1 = 0.01$. Because the performances of the fixed and dynamic leadtime policies are affected by the length of the planning horizon, we report the expected unit time profit ($EUPR = EPR/T$) instead of the EPR as the criterion. We obtain the optimal fixed leadtime and the corresponding $EUPR$ for selected values of T, ranging from 30 to 360. The result shows that, as T becomes larger, the leadtime becomes smaller and the unit time profit decreases. To explain this result, we first point out that the fixed leadtime policy can be viewed as a special case of the dynamic leadtime policy. Consequently, we can compare the optimal fixed leadtime and dynamic leadtime policies in terms of the effects of the length of the planning horizon. We find the optimal solution for the dynamic leadtime policy to be: $p^*(t) = 6.5223$ and $t_0 = 134.36$. Using the solution, the optimal strategy is to keep the price fixed at 6.5223 per item and to linearly decrease the leadtime from 134.36 to 0 in time interval (0, 134.36), resulting in a constant realized demand rate of 0.92879β in the same interval. After $t = 134.36$, the leadtime is 0, and the price decreases over time. In contrast, the fixed leadtime policy uses the same leadtime in the entire planning horizon. Our computational experience also shows that the optimal fixed leadtime policy actually deviates more from the optimal dynamic leadtime policy as the length of the planning horizon increases. Consequently, as the length of the planning horizon increases, the expected profit decreases and the policy becomes less attractive.

In Figure 6, we compare the fixed leadtime and dynamic policies under selected values of T and λ_1. As expected, the unit time profits increase as λ_1 increases for both policies, because

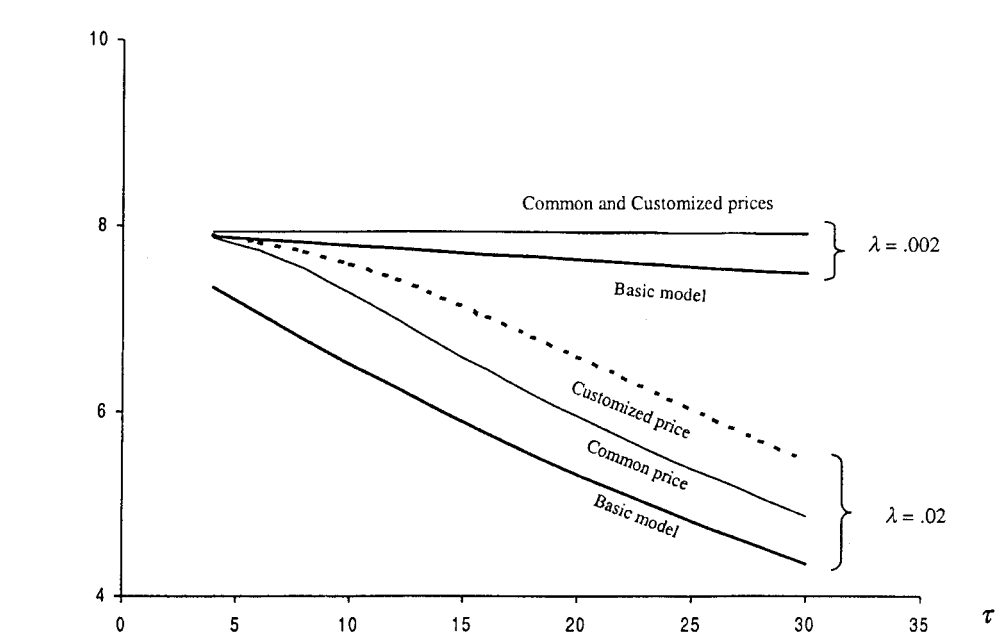

FIGURE 5. Effects of Leadtime on Expected Revenue.

a larger λ_1 implies a faster decrease in production cost over time. The dynamic leadtime policy has consistently outperformed the fixed leadtime policy, and the differences in *EUPR* are very significant. Furthermore, as λ_1 increases, the influence of T becomes more significant.

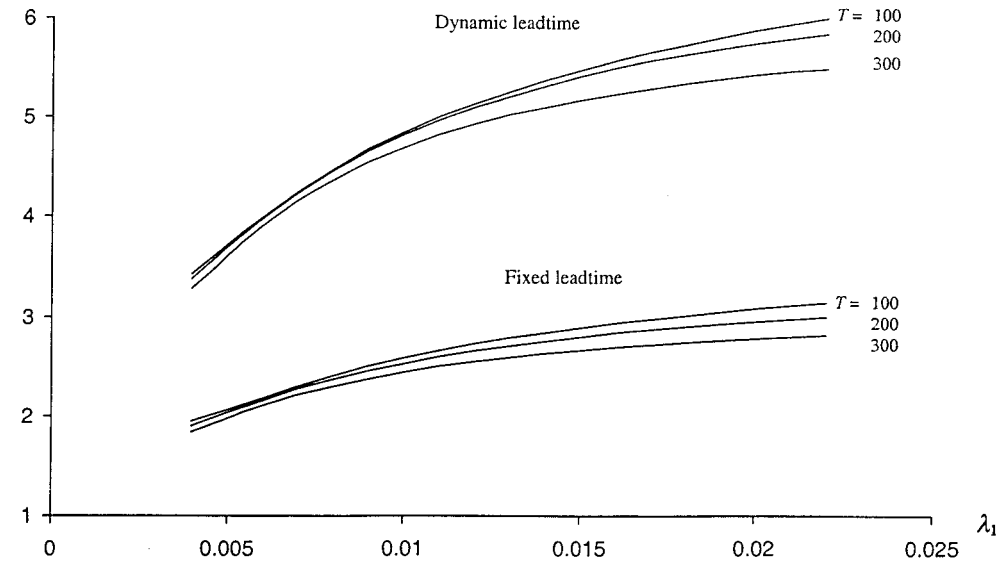

FIGURE 6. A Comparison of the Fixed Leadtime and Dynamic Leadtime Policies.

5. Conclusion and Future Studies

In this paper, we discuss two critical time-based policies on pricing and leadtime for a BTO-DS manufacturer. We study dynamic pricing policies for two price discount strategies, namely, common price discount and customized price discount, assuming a fixed leadtime. Then, we studied two leadtime policies, fixed leadtime and dynamic leadtime, jointly with a dynamic pricing policy under a decreasing production cost. We analyze optimal solutions and their properties and discuss their managerial insights.

The results suggest that price discounts and dynamic leadtime are attractive for products with values decreasing rapidly over time. We also find that the customized discounted price policy significantly outperforms the common discounted price policy when the product value decreases rapidly. In implementation, however, the customized discounted price policy requires more effort in determining individual price discounts and has a larger variation in prices. Furthermore, we find that the dynamic leadtime policy significantly outperforms the fixed leadtime policy. The optimal dynamic leadtime policy, however, often requires that a large volume of product be shipped at the same time, which may be difficult to implement in practice.

The results are derived when an unlimited process capacity is assumed. When process capacity is limited, the optimal policy may be different and inventory costs may have to be incorporated into the model formulation, which will make the model difficult to analyze. In addition to process capacity, the models developed in this paper can be extended by considering additional issues, such as diffusion effects on demand, quantity discounts in production cost, and stepwise leadtime. Furthermore, in this paper, the customized pricing policy is developed under the assumption that the manufacturer has an efficient process to command the highest possible prices. Negotiation and bidding methods in e-commerce and their effects on pricing are rich areas for future study (Moodie 1999). Finally, another possible extension is a customized leadtime policy, which allows negotiating with different customers for different leadtimes when variation exists in customer impatience in receiving the order (Abad 1996). Because product demand may be a function of leadtime as well as price (Palaka, Erlebacher, and Kropp 1998), using the approach in this paper, a multi-attribute utility function of price and leadtime can be used to relate demand with price and leadtime. These possible extensions have been included in the authors' future research plans.[5]

[5] We thank Prof. Amiya K. Chakravarty and two anonymous referees for many helpful suggestions that improved this paper.

Appendix

PROOF OF LEMMA 1. For simplicity, write $g(z)$ for $g(z, a, b)$. Note that $g(z)$ is continuously differentiable in z. Denote the two finite solutions of $g'(z) = 0$ as $z_L(<0)$ and $z_H(>0)$, respectively. Then it can be shown that $g''(z_L) < 0$ and $g''(z_H) > 0$, and that $g(z)$ is strictly increasing in $(-\infty, z_L)$, strictly decreasing in (z_L, z_H) and then strictly increasing in (z_H, ∞). Because $\lim_{z\to-\infty} g(z) = 1$ and $\lim_{z\to\infty} g(z) = 0$, there is a unique solution to $g(z) = 0$, denoted by z^*, between z_L and z_H. Moreover, from (3) and the fact $z_L < -b$, z^* must be in the interval $[-b, z_H]$. For any fixed $a(0 < a \le 1)$, since $\partial g(z)/\partial b = -a\phi(z) < 0$, a larger b will give a smaller $g(z)$ at any z. Similarly, for any fixed b, $\partial g(z)/\partial a = -(z + b)\phi(z) \le 0$, and hence a larger a will give a smaller $g(z)$ value at any $z \in [-b, z_H]$. As a result, z^* is smaller for a larger a or b.

PROOF OF PROPOSITION 1. The optimal $p(\cdot)$ is the solution to the Euler's condition (Fox 1987), $\partial ER(p(t))/\partial p(t) = 0$, which is basically the solution of (3) with $a = 1$, $b = \mu/\sigma$, and $z = (p(t) - \alpha(t)\mu)/\alpha(t)\sigma$. Therefore, part (i) is proved. Parts (ii) and (iii) are obtained by evaluating $D(p(t), t)$ and $r(p(t), t)$ at $p(t) = p^*(t)$.

PROOF OF PROPOSITION 2. To prove this result, we need to show that z^* of the basic model is between z_1^* and z^* of case A, and between z_2^* and z^* of case B. Consider case A first. Because

$$-z_1 = \frac{\mu}{\sigma} - \frac{p_1(t)}{\alpha\sigma} \le \frac{\mu}{\sigma},$$

and from the proof of Lemma 1, z^* that satisfies (6) is greater than or equal to z^*, the solution to $g(z^*, 1, \mu/\sigma) = 0$ for the basic model. Furthermore, because $\Phi(z) < 1$, z_1^* that satisfies (7) is smaller than z^* that satisfies $g(z, 1, \mu/\sigma) = 0$ for the basic model. This proves the result for case A.

For case B, notice that the functional forms in the two pairs of brackets on the left side of (9) are identical to that of g in (3). Because $0 \le e^{-\lambda\tau} \le 1$ and z^* for the basic model satisfies $g(z^*, 1, \mu/\sigma) = 0$, the solution in case B must satisfy $g(z^*, 1, \mu/\sigma) > 0$ and $g(z_2^*, 1, \mu/\sigma) < 0$, and therefore, following the proof of Lemma 1, z^* of the basic model is between z^* and z_2^* of the common discounted price model. Parts (i) and (ii) are direct consequences of the above result. Part (iii) is true because the basic model is a special case of the common discounted price model with $p_1(t) = p(t)$.

PROOF OF PROPOSITION 3. Let z_B^* denote the z^* value for the basic model in Subsection 2.1, satisfying Proposition 1 (i)

$$g\left(z_B^*, 1, \frac{\mu}{\sigma}\right) = 1 - \Phi(z_B^*) - \left(z_B^* + \frac{\mu}{\sigma}\right)\phi(z_B^*) = 0 \tag{A1}$$

To prove part (i) of the proposition, we need to prove $z_3^* \le z_B^* \le z^*$, where z_3^* and z^* are associated with the customized discounted price model, which satisfy (10) and (11).

Note that, because we assumed in Subsection 2.1 that $Pr\{U(t) < 0\}$ is negligible, it must be true that $\mu/\sigma \ge \sqrt{\pi/2}$. As a result, $g(0, 1, \mu/\sigma) = 1 - \Phi(0) - (\mu/\sigma)/\sqrt{2\pi} \le 0$, and, from (A1) and the increasing and decreasing patterns of the graph of g in Lemma 1, we have $-\mu/\sigma \le z_B^* \le 0$.

To prove $z_B^* \le z^*$, we consider the following function:

$$g_1\left(z, e^{-\lambda\tau}, \frac{\mu}{\sigma}\right) = 1 - \Phi(z) - e^{-2\lambda\tau}\left(z + \frac{\mu}{\sigma}\right)\phi\left(e^{-\lambda\tau}\left(z + \frac{\mu}{\sigma}\right) - \frac{\mu}{\sigma}\right).$$

Note that g_1 is similar to g in (3) that the graph of g_1 is not significantly different from that of g. Hence, to prove $z_B^* \le z^*$, it is sufficient to prove $g_1(z_B^*, e^{-\lambda\tau}, \mu/\sigma) \ge 0$, because $g_1(z^*, e^{-\lambda\tau}, \mu/\sigma) = 0$. Now, because $e^{-\lambda\tau}(z_B^* + \mu/\sigma) - \mu/\sigma \le z_B^* \le 0$, $z_B^* + \mu/\sigma \ge 0$, and from (A1) and the property of $\phi(\cdot)$, we can prove that $g_1(z_B^*, e^{-\lambda\tau}, \mu/\sigma) \ge 0$, and hence $z_B^* \le z^*$.

Similarly, to prove $z_3^* \le z_B^*$, we consider

$$g_2\left(z_3, e^{-\lambda\tau}, \frac{\mu}{\sigma}\right) = 1 - \Phi\left(e^{\lambda\tau}\left(z_3 + \frac{\mu}{\sigma}\right) - \frac{\mu}{\sigma}\right) - e^{-\lambda\tau}\left(z_3 + \frac{\mu}{\sigma}\right)\phi(z_3).$$

Note that z_3^* satisfies $g_2(z_3^*, e^{-\lambda\tau}, \mu/\sigma) = 0$, and the graph of g_2 is similar to that of g and g_1. Hence, to prove $z_3^* \le z_B^*$, it is sufficient to prove $g_2(z_B^*, e^{-\lambda\tau}, \mu/\sigma) \le 0$. In fact, we will show

$$g_2\left(z_B^*, e^{-\lambda\tau}, \frac{\mu}{\sigma}\right) \le g_2\left(z_B^*, e^0, \frac{\mu}{\sigma}\right) = 0, \qquad \text{for all } \lambda\tau \ge 0 \tag{A2}$$

Because z_B^* and μ/σ are fixed and independent of $\lambda\tau$, $g_2(z_B^*, e^{-\lambda\tau}, \mu/\sigma)$ is considered a function of $\lambda\tau$, but only through $e^{-\lambda\tau}$. The equality in (A2) holds, because from (A1) we have

$$g_2\left(z_B^*, e^0, \frac{\mu}{\sigma}\right) = g\left(z_B^*, 1, \frac{\mu}{\sigma}\right) = 0$$

To prove the inequality in (A2), it is easy to see from $\partial g_2/\partial(\lambda\tau)$ that, without the constraint $\lambda\tau \ge 0$, g_2 reaches its global maximum with $g_2(z_B^*, e^{\lambda\tau}, \mu/\sigma) = 0$ at $\lambda\tau = 0$, which will remain as the global maximum under the constraint $\lambda\tau \ge 0$. This proves the inequality in (A2) and completes the proof of part (i). Parts (ii) and (iii) are direct consequences of part (i).

To prove part (iv), it is sufficient to prove that any given common discounted price policy (including the optimal one) with a posted price $p(t)$ is dominated in the expected revenue by a customized discounted price policy with the same posted price. Under the common discounted price model, the utility at time $t + \tau$ of a customer who placed his/her order at time t can fall into one of the following three ranges (cases): $(p(t), \infty)$, $(p_1(t), p(t))$, and $(p(t)e^{-\lambda\tau}, p_1(t))$. If a customer's utility at time $t + \tau$ is in the first or second intervals, he/she pays the posted price and the common discounted price for his/her order, respectively, and if in the third interval, he/she cancels the order. If the same posted price is used for the customized discounted price model, the revenue rates for case 1 are identical for the common and customized discounted price models. However, the revenue rate generated from case 2 for the common discounted price model is smaller than that for the customized discounted price model because $U(t + \tau) \ge p_1(t)$ for the customer with utility in this interval. Furthermore, the expected revenue from case 3 is zero under the common discounted price model and non-negative under the customized discounted price model. Therefore, the customized discounted price model has a larger expected revenue from the three cases than the common discounted price model.

PROOF OF PROPOSITION 4. The following equality gives the necessary first-order condition for the optimal solution for $\tau(t)$:

$$1 - \Phi\left(\frac{p(t) - \alpha(t)\mu}{\alpha(t)\sigma}\right) - \frac{(p(t) - ce^{-\lambda_1(t+\tau(t))})\lambda p(t)e^{\lambda(t+\tau(t))}}{c\sigma\lambda_1 e^{-\lambda_1(t+\tau(t))}}\phi\left(\frac{p(t) - \alpha(t)\mu}{\alpha(t)\sigma}\right) = 0 \tag{A3}$$

Using (A3) and (14), we obtain the optimal $\tau(t)$ for a given $p(t)$ as:

$$\tau(t) = \frac{1}{\lambda_1}\ln\left(\frac{\lambda_1 c}{\lambda p(t)}\right) - t$$

and, thus, $EPR(\tau(\cdot), p(\cdot))$ in (13) can be expressed as a function of $p(t)$ as follows:

$$EPR(p(\cdot), \tau(\cdot)) = \int_0^T \beta p(t)\left(1 - \frac{\lambda}{\lambda_1}\right)\left[1 - \Phi\left(\frac{p(t)^{(1-\lambda/\lambda_1)}h - \mu}{\sigma}\right)\right]dt$$

where $h = (c\lambda_1/\lambda)^{(\lambda/\lambda_1)}$. Then, the Euler condition for finding the optimal $p(t)$ is

$$1 - \Phi(z) - \left(1 - \frac{\lambda}{\lambda_1}\right)\left(z + \frac{\mu}{\sigma}\right)\phi(z) = 0, \tag{A4}$$

where $z = (p(t)^{1-\lambda/\lambda_1}h - \mu)/\sigma$. Let z^* be the solution to (A4) and then the optimal $p(t)$ is a constant and is given in (18).

PROOF OF PROPOSITION 5. Part (i) of Proposition 5 is directly from Proposition 4. Evaluating the production cost at time $t + \tau^*(t)$, we find $C(t + \tau^*(t)) = p^*(t)\lambda/\lambda_1$. Accordingly, parts (ii) and (iii) are proved.

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