PRODUCTIVE EFFICIENCY: A DYNAMIC CONSIDERATION*

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摘 要

效率乃經濟理論之精義。本文探討準固定要素下之生產效率。因準固定生產要素之本質，一動態生產效率之廠商於決定要素投入時，並非根據當前要素之價格，而是根據未來預期要素價格之折扣加權平均值，此預期值可能是理性之預期結果。本文假定動態成本函數爲一超越泛函數，生產效率服從隨機性截斷正常分配。理論模型應用於美國製造業，實證結果顯示：準固定性顯着地影響要素使用之決定，資本和勞動爲準固定要素，而能源爲一變動投入。實證結果並揭露：此產業之總生產效率逐時改善，但呈一循環現象；資源錯誤配置所導致之效率損失，主要來自技術面。

ABSTRACT

Efficiency is the essential theme of economic theory. This study investigates productive efficiency in a production process where quasi-fixed factors may appear. With the nature of quasi-fixed factors, a dynamically efficient firm will adjust factor proportions not to prevailing input prices but rather to some weighted average of discounted future input prices, probably rationally expected. To assess dynamic productive efficiency, a Translog dynamic cost function and a stochastic truncated normal distribution of productive efficiency are assumed. The constructed dynamic productive efficiency structure is applied to the U.S. manufacturing sector. The empirical results show that the quasi-fixity of factor inputs is significant in firm's input decisions, especially capital and labor, while energy is a variable or flexible input. The estimates of productive efficiency in the Farrell measurement reveal that the sector has improved its productive efficiency over time with a cyclical pattern, and that technical inefficiency accounts for the major source of the efficiency loss from resource misallocation.

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I. Introduction

This paper investigates productive efficiency in a dynamic setting. The dynamic framework incorporates the current variables and the relative levels of the expected arguments to the realized ones. An empirical estimation is carried out to disclose dynamic productive efficiency of the U.S. manufacturing sector 1954-1977, in the Farrell productive efficiency measurement.

Since the Farrell work (1957), the study of productive efficiency has attracted many researchers' and economists' attention. Especially, recently the Journal of Econometrics (1980), for example, issued a collection of such works. All empirical studies so far are devoted to the static framework in which an activity is technically efficient if it is on the efficient isoquant, and allocative efficient if the input marginal productivity ratios among the inputs are equal to the current input price ratios among them.

However, evidence derived from the static framework may be unsatisfactory or misleading. For example, the instantaneous adjustment and cost free adjustment assumptions of the static model are not appealing in reality. It fact, the adjustment process is staggered over time due to quasifixity of some input factors. Such quasifixity raises the so-called "internal adjustment cost" involving in production planning activity and learning activity [Lucas (1967)].[1] In the treatment of the impact of quasifixity, it is assumed that the investment rate enters a production function with a negative and decreasing marginal productivity, and that internal cost is positively related to the investment level.

In the second example, the nature of productive efficiency makes evidence from the static models controversial. It questions whether productive efficiency should be judged in a static criterion or a dynamic sense. As claimed by Førsund, Lovell and Schmidt (1980), in a putty-clay technology case, dynamically efficient firms will adjust factor proportions not to input prices prevailing at the time of installation, but rather to some weighted average of discounted expected future input prices. Thus productive efficiency derived from the static models will not correctly reflect the efficiency of a dynamic production activity.

Furthermore, the quasifixity of some productive factors affects the measurement of productive efficiency. If productive efficiency is used to evaluate the firm's controllable mistakes, then the negative effect of the investment of quasi-fixed factors on production should not be regarded as a source of productive inefficiency. This conclusion is plausible because the negative effect is the nature or the inherent property of such factors, which is out of the firm's control.

With the quasi-fixed factors in production, current demands for inputs have been proved to be the function of the levels and rates of changes of the factor prices, and the rate of demand shifts [e.g., Lucas (1967), and Kokkelenberg (19-81)]. To account for quasifixity and dynamic productive efficiency, this paper incorporates the prevailing input price levels, the realtive levels of current input prices to the expected input prices, and the output level into the firm's current cost function. To make the approach tractable, a Translog functional form is adopted. The model will be seen to degenerate to the static model as the expected input prices are fully realized or there is no quasi-fixed factors in poduction.

The plan of the paper is as follows: Section II develops a dynamic cost structure justifying the existence of quasi-fixed inputs. Section III models the distribution of productive efficiency for a four-input production technology. Empirical results of productive efficiency in the dynamic setting for the U.S. manufacturing sector are discussed in Section IV. It does so by modifying the Kopp-Diewert (1981) estimation technique in decomposing the Farrell overall productive efficiency into technical and allocative efficiency. Section V concludes the findings and proposes further studies.

II. Dynamic Cost Function

It has been soundly established that a theory of the firm which is underpinned with the dynamic considerations of input interrelatedness and quasi-fixed inputs [e.g., Lucas (1967), Treadway (1970), McLaren and Cooper (1980), and Kokkelenberg (1981)] is more theoretically and empirically correct than that based on static assumptions. In that, the production function with quasi-fixed factors as concerned is conventionally written as $F(X, \dot{X})$,[2] where X and $\dot{X}$ refer to vectors of inputs and the net changes of inputs, respectively. The following assumptions are usually made: $F_1 > 0$, $F_2 < 0$, $F_{11} < 0$, $F_{12} < 0$, and $F_{22} < 0$.

The quasi-fixed factor production decision is in general investigated through the intertemporal optimization problem. In any such models, current demands for inputs, especially for quasi-fixed inputs, appear to depend on future exogenous variables. Thus, the current cost function could be hypothesized as

(2.1) $C = C(Q, W, RW, t)$

where C = current production cost,

Q = current output,

W = vector of current input prices,

RW = relative levels of the expected input prices to the realized ones, and

t = time trend increasing one unit per year and reflecting technical progress.

The specification (2.1) is indeed an ad hoc treatment, and called quasi-dynamic, but it merits several plausible considerations. The following comments are intended to substantiate the specification desirability:

(1) The integral in the conventional intertemporal optimization can be considered as the composite of W and RW in expression (2.1).

(2) The issue of quasi-fixed inputs is captured by the inclusion of RW which involves the expected future input prices.

(3) The structure (2.1) will degenerate to the static framework if expectations are fully realized. This is true for the case in which the cost structure is presented as the Translog functional form (Christensen et al., 1971) presented below. This, in turn, verifies that the static model is a special case of the dynamic framework.

Finally but most relevant to this study, provided that a particular distribution of productive efficiency is assumed, the issue of dynamic productive efficiency as addressed by Førsund, Lovell and Schmidt (1980) appears to be captured. A dynamically effcient firm facing the putty-clay technology will adjust factor proportions not to input prices prevailing at the time of installation, but rather to some weighted average of discounted expected future input prices.

To make the framework (2.1) analytical, a certain functional form could be assumed. In this study, a Translog function is assumed to describe production technology and cost structure. Let

$P_{i,t} = \ln W_{i,t}$, the natural logarithm of ith input price at time t,

$\dot{P}_{i,t} = \ln RW_{i,t}$, the natural logarithm of ith input relative price trend at time t,

and $Y_t = \ln Q_t$, the natural logarithm of output at time t.

The Translog cost function without restriction is written as

$$
\begin{aligned}
(2.2)\quad \ln C = {} & \alpha_0 + \alpha_y Y + \alpha_t t + \Sigma\alpha_i P_i + \Sigma\beta_i \dot{P}_i + \Sigma\alpha_{yi} YP_i + \Sigma\beta_{yi} Y\dot{P}_i \\
& + \alpha_{yt} Yt + \Sigma\alpha_{it} P_i t + \Sigma\beta_{it} \dot{P}_i t + \tfrac{1}{2}\Sigma\Sigma\alpha_{ij} P_i P_j + \tfrac{1}{2}\Sigma\Sigma\beta_{ij} P_i \dot{P}_j \\
& + \tfrac{1}{2}\Sigma\Sigma\gamma_{ij} \dot{P}_i \dot{P}_j + \tfrac{1}{2}\alpha_{yy} Y^2 + \tfrac{1}{2}\alpha_{tt} t^2
\end{aligned}
$$

For simplicity and the consideration of degrees of freedom, homotheticity and Hicksian neutral technical progress are assumed. Several parameters are also constrained to zero; but the simplified version is still capable of capturing the effect of dynamic input price variables on input shares. Thus one has

$$(2.3)\quad \ln C = \alpha_0 + \alpha_y Y + \alpha_t t + \Sigma\alpha_i P_i + ½\Sigma\Sigma\alpha_{ij} P_i P_j + ½\Sigma\Sigma\beta_{ij} P_i \dot{P}_j$$

For the function (2.3) to be appropriately defined, the assumptions of positive linear homogeneity in current input prices and symmetric cross input price effects are required. The two assumptions imply that

$$(2.4)\quad \Sigma\alpha_i = 1,\ \sum_i \alpha_{ij} = 0,\ \sum_j \alpha_{ij} = 0,\ \sum_i \beta_{ij} = 0, \text{ and } \sum_j \beta_{ij} = 0$$

and

$$(2.5)\quad \alpha_{ij} = \alpha_{ji}, \text{ and } \beta_{ij} = \beta_{ji}$$

Differentiating the cost function logarithmically and applying Shepard's Lemma (1953) yields input cost share equations as linear functions of the arguments

$$(2.6)\quad S_i = \alpha_i + \sum_j \alpha_{ij} P_j + \sum_j \beta_{ij} \dot{P}_j$$

It is obvious from equations above that individual factor shares are determined not only by current forces, but also by past and future (or expected) forces.

The cost function (2.3) and cost share equations (2.6) form the base for empirical estimation. Since the sum of cost shares is unity at all the times, one of the four cost shares should be deleted so that disturbance terms in the cost shares assure a nonsingular covariance matrix.

III. Parametric Frontier Modelling

To facilitate the empirical estimation in next section, we assume there are four inputs in production process: capital (K), labor (L), energy (E), and net materials (M). To make the previous section's models amenable to analyzing productive efficiency statistically, distribution assumptions of productive inefficiency and random errors in the cost function and cost share equations are imposed.

First, the minimality of a cost function requires a non-negative disturbance term associated with it to acocunt for productive inefficiency. This study assumes that productive inefficiency obeys a non-zero mode truncated normal distribution.[3]

Secondly, the stochastic approach is adopted by imposing a symmetric normal distribution of random errors in the cost function. With these two assumptions, the estimated cost function is written as

$$(3.1)\quad \ln C = \alpha_0 + \alpha_y Y + \alpha_t t + \Sigma\alpha_i P_i + ½\Sigma\Sigma\alpha_{ij} P_i P_j + ½\Sigma\Sigma\beta_{ij} P_i \dot{P}_j + u + e,$$

$$i, j = K, L, E, M,$$

where u reflects productive inefficiency, and has a p.d.f.

(3.2) $f(u) = \frac{1}{(1-F(-\mu/\sigma_u))(2\pi)^{1/2}} \exp(-\tfrac{1}{2}(\frac{u-\mu}{\sigma_u})^2)$ for $u \geqq 0$

$= 0$ otherwise

where $F(\cdot)$ is the standard normal distribution function,

σ_u is the standard deviation of u, and

μ is the mode of u;

e reflects pure statistical errors (random errors), and has a p.d.f.

(3.3) $f(e) = \frac{1}{(2\pi)^{1/2}\sigma_e} \exp(-\tfrac{1}{2}(\frac{e}{\sigma_e})^2)$,

where σ_e is the standard deviation of e.

The joint density function of a truncated normal and a symmetric normal distributions is derived by M.A. Weinstein (1964). The p.d.f. of $\epsilon = u + e$ is given by

(3.4) $f(\epsilon) = \sigma^{-1} f(\frac{\epsilon-\mu}{\sigma}) [1-F(\frac{\mu}{\sigma\lambda} - \frac{\epsilon\lambda}{\sigma})] [1-F(-\frac{\mu}{\sigma_u})]^{-1}$

where $\sigma = (\sigma_u^2+\sigma_e^2)^{1/2}$, $\lambda = \frac{\sigma_u}{\sigma_e}$, and $f(\cdot)$ and $F(\cdot)$ are the standard p.d.f. and distributed function evaluated at the relevant locations, respectively.

Third, no systematic bias in input utilizations is assumed on the ground that one has no reason a priori to assert which factor(s) is(are) going to be overutilized. It is generally understood that under the rate of return regulation overcapitalization would occur,[4] but it is still testable. Nonetheless, one can conclude that at least one input has been overutilized as compared with the cost-minimizaton input mix. The input cost share equations are then estimated in the form of

(3.5) $S_i = \alpha_i + \sum_j \alpha_{ij} P_j + \sum_j \beta_{ij} \dot{P}_j + v_i$,

i, j = K, L, E, M.

where the v_i's are assumed to be symmetric and normally distributed with a p.d.f.

(3.6) $f(v_i) = [(2\pi)^{1/2}\sigma_{v_i}]^{-1} \exp[-\tfrac{1}{2}(\frac{v_i}{\sigma_{v_i}})^2]$,

i, j = K, L, E, M.

Furthermore, the three independent cost shares are assumed to have the joint p.d.f.

(3.7) $F(V) = (2\pi)^{-N/2}\ |\Omega|^{-½} \exp(V\Omega^{-1}\ V'/2)$

where N = 3 is the number of the independent cost share equations,[5]

$V = [v_K, v_L, v_E]$ is a (1 x 3) vector at time t, and Ω is a (3 x 3) covariance matrix.

The remaining problem is the relationship between (u + e) and V. The exact statistical relationship between u and V appears complicated even for a Cobb-Douglas production structure. Thus, for the sake of simplicity and tractability, u and V are assumed independently distributed.[6] Finally, it is reasonable to assume e and V, are independent. With such assumptions, the joint density function of (u + e) and V is given as

(3.8) $$f(\epsilon, V) = \sigma^{-1}(2\pi)^{-½(N+1)}\exp[-½((\frac{\epsilon-\mu}{\sigma})^2 + V\Omega^{-1}V')]$$

$$[1-F(-\frac{\mu}{\sigma\lambda}-\frac{\epsilon\lambda}{\sigma})][1-F(\frac{-\mu}{\sigma_u})]^{-1}\ |\Omega|^{-½}$$

Assuming there are T observations for the sample, then the joint loglikelihood of the cost function and the three independent input cost shares is

(3.9) $$L(\epsilon, V) = L(u + e, v_1, v_2, v_3)$$

$$= -T\ln\sigma - ½T(N+1)\ln 2\pi - ½\sum^{T}((\frac{\epsilon_t-\mu}{\sigma})^2 + V_t\Omega^{-1}V_t')$$

$$-T\ln(1 - F(-\frac{\mu}{\sigma}(\lambda^{-2}+1)^{½})) + \sum^{T}\ln(1 - F(-\frac{\mu}{\sigma\lambda}-\frac{\epsilon_t\lambda}{\sigma})) - \frac{T}{2}\ln|\Omega|$$

Before transforming the unobservable arguments, ϵ and V, to the observable variables, C and S_i's, some mathematical manipulations are made on the cost function and the three independent cost shares by directly imposing the restrictions (2.4) and (2.5). Rearranged appropriately, a matrix representing the reduced form is obtained as shown in Table 3.1. To express this clearly, let the matrix correspond to the following notations:

(3.10) $\bar{C}_t = Z_t B + \epsilon_t$ for the normalized cost function,

and

(3.11) $S_t = R_t B + V_t$ for the three independent cost share equations.

The loglikelihood in terms of the observable variables, $\bar{C}$ and S, apart from the irrelevant constants, is

Table 3.1
Translog Quasi-Dynamic Cost Structure

$$
\begin{bmatrix} \ln(\frac{C}{W_M}) \\ S_K \\ S_L \\ S_E \end{bmatrix}
=
\begin{bmatrix}
1 & Y & t & (P_K-P_M) & (P_L-P_M) & (P_E-P_M) & .5(P_K-P_M)^2 & .5(P_L-P_M)^2 & .5(P_E-P_M)^2 & (P_K-P_M)(P_L-P_M) & (P_K-P_M)(P_E-P_M) & (P_L-P_M)(P_E-P_M) \\
0 & 0 & 0 & 1 & 0 & 0 & (P_K-P_M) & 0 & 0 & (P_L-P_M) & (P_E-P_M) & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & (P_L-P_M) & 0 & (P_K-P_M) & 0 & (P_E-P_M) \\
0 & 0 & 0 & 0\;\;0 & 0 & 1 & 0 & 0 & (P_E-P_M) & 0 & (P_K-P_M) & (P_L-P_M)
\end{bmatrix}
$$

Table 3.1 (continued)
Translog Quasi-Dynamic Cost Structure

$.5(P_K\dot{P}_K+P_M\dot{P}_M-\dot{P}_KP_M-P_K\dot{P}_M)$	$.5(P_L\dot{P}_L+P_M\dot{P}_M-\dot{P}_LP_M-P_L\dot{P}_M)$	$.5(P_E\dot{P}_E+P_M\dot{P}_M-\dot{P}_EP_M-P_E\dot{P}_M)$	$P_M\dot{P}_M+.5(\dot{P}_KP_L+P_K\dot{P}_L-\dot{P}_KP_M-P_K\dot{P}_M-\dot{P}_LP_M-P_L\dot{P}_M)$
$.5(\dot{P}_K-\dot{P}_M)$	0	0	$.5(\dot{P}_L-\dot{P}_M)$
0	$.5(\dot{P}_L-\dot{P}_M)$	0	$.5(\dot{P}_K-\dot{P}_M)$
0	0	$.5(\dot{P}_E-\dot{P}_M)$	0

Table 3.1 (continued)
Translog Quasi-Dynamic Cost Structure

$$
\left.\begin{array}{cc}
P_M P_M + .5(\dot{P}_K P_E + P_K \dot{P}_E - \dot{P}_E P_M - P_E \dot{P}_M - \dot{P}_K P_M - P_K \dot{P}_M) & P_M \dot{P}_M + .5(\dot{P}_L P_E + P_L \dot{P}_E - \dot{P}_L P_M - P_L \dot{P}_M - P_E P_M - P_E \dot{P}_M) \\
 & \\
 & \\
.5(\dot{P}_E - \dot{P}_M) & 0 \\
 & \\
 & \\
 & \\
0 & .5(\dot{P}_E - \dot{P}_M) \\
 & \\
 & \\
.5(\dot{P}_E - \dot{P}_M) & .5(\dot{P}_L - \dot{P}_M) \\
 & \\
 & \\
 & \\
 & \\
 & \\
 & \\
 &
\end{array}\right]
\begin{bmatrix}
\alpha_0 \\ \alpha_y \\ \alpha_t \\ \alpha_K \\ \alpha_L \\ \alpha_E \\ \alpha_{KK} \\ \alpha_{LL} \\ \alpha_{EE} \\ \alpha_{KL} \\ \alpha_{KE} \\ \alpha_{LE} \\ \beta_{KK} \\ \beta_{LL} \\ \beta_{EE} \\ \beta_{KL} \\ \beta_{KE} \\ \beta_{LE}
\end{bmatrix}
+
\begin{bmatrix}
\epsilon \\ \\ \\ \\ \\ v_K \\ \\ \\ \\ v_L \\ \\ \\ \\ \\ v_E \\ \\ \\ \\
\end{bmatrix}
$$

$$(3.12)\quad \bar{L} = \bar{L}(\bar{C}, S|B, \lambda, \sigma^2, \mu, \Omega)$$

$$= -\frac{T}{2}\ln\sigma^2 - \frac{1}{2\sigma^2}\sum^{T}(\bar{C}_t - Z_tB-\mu)^2 - \frac{T}{2}\mathrm{tr}\,(\Omega^{-1}\omega)$$

$$+\sum^{T}\ln(1-F(\frac{1}{\sigma}(-\frac{\mu}{\lambda}-(C_t-Z_tB)\lambda))) - T\ln(1-F(-\frac{\mu}{\sigma}(\lambda^{-2}+1)^{1/2})) - \frac{T}{2}\ln|\Omega|$$

where $\omega = \frac{1}{T}(S_t-R_tB)(S_t-R_tB)'$. The loglikelihood (3.12) is found estimable through the concentrated likelihood function

$$(3.13)\quad L^* = L^*(\bar{C}, S|B, \lambda, \sigma^2, \mu)$$

$$= -\frac{T}{2}\ln\sigma^2 - \frac{1}{2\sigma^2}\sum^{T}(\bar{C}_t - Z_tB-\mu)^2 + \sum^{T}\ln(1-F(\frac{1}{\sigma}(-\frac{\mu}{\lambda}-(\bar{C}_t-Z_tB)\lambda)))$$

$$-T\ln(1-F(-\frac{\mu}{\sigma}(\lambda^{-2}+1)^{1/2})) - \frac{T}{2}\ln|\omega|$$

which is to be maximized with respect to B, λ, σ^2, and μ, see Appendix A. The first order conditions for the maximization are presented in Appendix B.

IV. The Empirical Result

The models developed above are applied to U.S. manufacturing 1954-1977. The manufacturing data comes from J.R. Norsworthy, The Bureau of Labor Statistics, U.S.A. The section delves to the issues of quasi-fixed factor(s) and dynamic efficiency.

The efficiency measure is confined to the Farrell measurement types: overall productive efficiency, technical efficiency, and allocative efficiency. The Kopp-Diewert (1981) technique in decomposing overall efficiency is applied. However, there are some modifications needed for the present model's considerations of dynamic and stochastic efficiency measures. The implementation of estimation of the objective function (3.13) is not an easy task.[7] Here some important comments on obtaining three types of productive efficiency are in order.

First of all, relative trends of input prices are measured as the current price levels to the expected price levels. Expected ones are measured by the exponential smoothing method which fits a trend model across time such that the most recent data is weighted more heavily than data in the early past of the series, see Brown (1962).

Secondly, overall efficiency is calculated as the exponential of the negative of the estimate of u in the cost function. This is seen as follows. Let

(4.1) $C = C^* \exp(u + e)$

where all notations are the same as in the previous sections. Taking the natural logarithm of the expression (4.1) yields

(4.2) $\ln C = \ln C^* + u + e$

It is clear that $\ln C^*$ in expression (4.2) can be considered as the Translog cost function (2.3). Then, following Aigner, Lovell and Schmidt (1977), overall productive efficiency (OE) in the stochastic framework should be calculated as

(4.3) $OE = C^* \exp(e)/C = \exp(-u)$,

which indicates that OE ranges from unity to zero as u goes from zero to positive infinity. Recall that u is specified to obey a truncated normal distribution which ranges between zero and positive infinity.

Third, in order to capture the meaning of the Farrell production frontier under the stochastic specification, the stochastic errors should be removed. For this purpose, it is assumed that the random errors can be obtained from the average cost function estimation. Therefore, decomposition of productive efficiency is conducted with respect to predicted inputs and costs, rather than with respect to the observed ones.

Fourthly, in finding the Kopp-Diewert optimal hypothetical inputs and input prices, input prices are normalized with capital input price, and the dynamic variables, $\dot{P}$, are treated as given.[8] Technical efficiency is measured as the inner product of the hypothetical least cost inputs vector and observed input prices vector; allocative efficiency is then derived as the ratio of overall efficiency to technical efficiency.

The estimation results of the maximum likelihood estimates (MLE) of the frontier cost function and the iterative three stage least square estimates (IT3SLSE) of the average cost function are reported in Table 4.1. The IT3SLSE provides the insights into the effect of quasi-fixed factor(s). As indicated in Table 4.1, the R^2 statistic and the t-ratios show the desirability of the dynamic modelling. Five out of the six dynamic terms are significant at the 1% level. In general, the significance of the inclusion of dynamic variables can be assessed by testing the null hypothesis $H_0: \beta_{KK} = \beta_{LL} = \beta_{EE} = \beta_{KL} = \beta_{KE} = \beta_{LE} = 0$. In fact, the F test has rejected the H_0 in the present case.

Specifically, among the dynamic terms, the first three parameters' estimates, i.e., estimates of β_{KK}, β_{LL}, and β_{EE}, show that dynamic capital and labor input

Table 4.1

ME and IT3SLS Estimates of Dynamic Cost Function
(asymptotic t ratios in parentheses)

(1) parameters	(2) MLE	(3) IT3SLSE
α_0	6.1247	6.3135 (417.56)
α_y	0.8109	.7544 (23.70)
α_t	0.0007	.0008 (.66)
α_K	0.1078	.1065 (102.29)
α_L	0.2571	.2501 (141.11)
α_E	0.0156	.0156 (30.81)
α_{KK}	0.0772	.0766 (15.74)
α_{LL}	0.0306	.0306 (2.93)
α_{EE}	0.0234	.0234 (6.15)
α_{KL}	−0.0325	−.0324 (−8.32)
α_{KE}	0.0136	.0137 (3.73)
α_{LE}	0.0010	.0011 (.42)
β_{KK}	−0.0538	−.0535 (−5.88)
β_{LL}	0.5055	.4803 (12.82)
β_{EE}	−0.0069	−.0070 (−.82)
β_{KL}	0.0649	.0645 (3.88)
β_{KE}	−0.0331	−.0330 (−4.28)
β_{LE}	−0.0423	−.0422 (−4.87)
λ	1.1932	
μ	0.1842	R^2 = .9972
σ^2	0.0003	

Maximum Likelihood = 5991

prices are significantly correlated with the current costs. This indicates that capital and labor factors are quasi-fixed inputs so that current costs appear to be influenced not only by the current input price levels, but also be the expected input price levels which involve the past and future input prices. On the other hand, the dynamic energy price variable appears insignificantly related to the current input configuration; this implies that the energy factor is a "flexible or variable" input which can be adjusted freely; it is also compatible with the nonreservable nature of energy, especially electricity. As for the last three dynamic terms, they are capturing the interactions among them on current cost, and are seen as significantly related to current cost.

Table 4.2

Indexes of Dynamic Productive Efficiency

Year	Overall Efficiency	Technical Efficiency	Allocative Efficiency
1954	.81699		
55			
56	.81699	.81713	.99982
57	.81705	.81722	.99979
58	.81327	.81343	.99980
59	.81827	.81844	.99979
60	.81927	.81945	.99978
61	.81924	.81943	.99976
62	.82308	.82327	.99976
63	.82562	.82958	.99522
64	.82857	.82878	.99974
65	.83226	.83245	.99977
66	.83531	.83550	.99977
67	.83627	.83647	.99976
68	.83909	.83931	.99973
69	.84039	.84061	.99973
70	.83796	.83818	.99973
71	.83936	.83959	.99972
72	.84358	.84380	.99973
73	.84678	.84699	.99975
74	.84516	.84534	.99978
75	.84098	.84117	.99977
76	.84563	.84583	.99976
77	.84852	.84873	.99975

Discussion turns to the estimates of three types of the Farrell productive efficiency. As indicated in Table 4.2, overall efficiency has on average, increased over the study period; also, it has exhibited a cyclical pattern with declines in 1958, 1961 (insignificantly), 1970, 1974, and 1975. As for trends of individual efficiency, technical efficiency resembles the pattern of overall efficiency; allocative efficiency is almost perfect with an average estimate of .999.

Through such evidence, one could observe a general picture of the performance of the manufacturing sector during the period 1954-1977 that this sector increases efficiency during expansion periods, and loses efficiency during contraction periods. One would be interested in why it shapes up so. The main reason we claimed for such productive efficiency patterns is the existence of quasi-fixed factor inputs in the production process and the consideration of dynamic economic decisions. Let us scrutinize the economic reasons behind it.

It is well established that the demand for factors of production is related to the demands for the final product produced. However, the response of the derived demand to changes in product demand may not be direct, but delayed and partially adjusted.[9] Among the sources of such phenomena, quasi-fixed inputs seem to be a major one. In the short run, employment contracts are subject to rigidity and then the function of quasi-fixed inputs staggers over time. For example, at the beginning of an economic upturn, due to the fixity of nonproduction labor and the delay in obtaining additional workers, the firms will tend to increase the hours of the existing laborers. It is often cheaper to do so even if it requires some overtime pay, because the firms do not have to incur the transaction costs of screening and training new employees. Therefore, a cost advantage is brought in at the beginning of boom years, and that results in an obvious increase in productive efficiency. Conversely, it is easy to reason that the quasi-fixity of factor inputs deteriorates productive efficiency in economic downturn years. Finally, the long term increase in productive efficiency could be attributed to cumulative managerial experience.

Furthermore, there is an apparent phenomenon that allocative efficiency is negatively correlated to the rate of output price change. The change in output price, in fact, could be considered as the proxy of an aggregate change of input prices.[10] It is theoretically plausible to infer that the more variation of price, the less allocative efficiency exists, because a higher variation of price, the less allocative efficiency exists, because a higher variation of input prices causes more difficulties in employing the right input proportions. In the present study, lower allocative efficiency occurs in 1963 and 1968-1974 which indeed have a higher inflation rate than other periods.

V. Conclusion

In this paper, a dynamic cost structure is developed to justify the price expectation and quasifixity of inputs. Through such an underpinning, dynamic efficiency is then assessed. With the appropriate assumptions of productive efficiency, the dynamic structure is applied to the U.S. manufacturing sector 1954-1977. The significant findings from the estimation are: (1) the existence of quasi-fixity of inputs is significant in production process, (2) productive efficiency has increased over time with a cyclical pattern, and (3) allocative efficiency is significantly correlated with the variation of prices.

Compared with previous studies of Burley (1980), Greene (1980), and Färe and Grosskopf (1982), the present study shows two distinctions: the sector has improved its productive efficiency, and the sector seems to have more difficulty in achieving technically efficient allocation than in hiring inputs at the right proportions.

Finally, we would like to have some points that are relevant for future research.

Although the paper presents a framework accounting for dynamic input allocation to measure productive efficiency, we think that the legitimate dynamic efficiency could be modeled on the base of optimal control underpinning. Such a method shall involve complicated mathematical manipulations; these contrast with and in turn, merit the simpler and tractable manipulation of the present model. Further, how the firm corrects its decisions in achieving efficient allocation seems to be an interesting issue to be dealt with. Othe intersting issues are input-specific efficiency and the stability of input-specific efficiency.

Footnotes

1. In Lucas' terminology, internal adjustment cost is the value of output foregone because of the investment in quasi-fixed factors.
2. Here generalizing all inputs to be quasi-fixed, and leaving the variable or "flexible" inputs case as testable.
3. The specification of the efficiency distribution should be based on information about the economic mechanism generating the productive efficiency. However, in empirical applications, econometricians generally do not have such information. Consequently, one needs to base the choice on statistical evidence. In his recent paper, Lee (1981) suggests the efficiency distribution could be based on the truncated Pearson family of distributions, leaving half-normal, exponential, truncated normal and gamma distributions as testable cases. His suggested Lagrangean multiplier test, however, is computationally complicated if the model is a system of equations.
4. See, for example, Baumol and Klevorick (1970).
5. Using N instead of 3 in the expression is intended to facilitate the general modelling.
6. Our approach is similiar to Greene (1980). Through correspondence with professor C.A. Knox Lovell, it became apparent that the independent assumption between u and V is not satisfactory since the "wrong" cost shares, in either direction, raise cost. However, the problem is ignored at the present time for two reasons: (1) as claimed by Greene (1980), given certain dynamic consideration, the independence assumption is not inappropriate, and (2) the p.d.f. of (u+e) and V are very complicated if u and V are correlated.
7. In the process of solving the nonlinear objective functions as expression (3.13) or solving the associated first order condition system, most researchers have observed and experienced problems and difficulties, such as finding good starting

points and scale problems. The methods designed by and presented in the International Mathematical Statistical Library (IMSL) in the IBM computer system are local methods. It has been observed that there is no such thing as a fool-proof nonlinear optimization technique. In this study, we use the IT3SLS estimates as the first starting point, then use ZSRCH routine in the IMSL to generate several starting points in a chosen rectangle. With such starting points, the nonlinear optimization routine ZXMIN are used to optimize the objective function. Finally, error function is used to achieve the boundedness conditions on the parameters.

8. The Levenberg-Marquardt nonlinear algorithm is adopted to find the optimal hypothetical inputs and input prices. In fact, we have used ZXSSQ routine in the IMSL.
9. For the principal reasons see Myers (1969), and Oi (1981).
10. The price of output is in itself a function of input prices, known as the output price function, see Samuelson (1953).

Appendix A

The optimization of (3.12) over Ω implies that

$$\begin{aligned}\frac{\partial \bar{L}}{\partial \Omega} &= \frac{\partial}{\partial \Omega}\left(-\frac{T}{2}(\ln|\Omega| + \mathrm{tr}\,(\Omega^{-1}\omega))\right) \\ &= -\frac{T}{2}\Omega^{-1} - \frac{T}{2}(-\Omega^{-1}\omega\Omega^{-1}) \\ &= -\frac{T}{2}\Omega^{-1}(\Omega-\omega)\Omega^{-1} \\ &= [0]\end{aligned}$$

This implies that, given B, λ, σ^2, and μ, ω is the maximum likelihood estimate of Ω. Note that the derivation of the partial derivative of $\mathrm{tr}(\Omega^{-1}\omega)$ with respect to Ω involves

(1) $\dfrac{\partial \Omega^{-1}}{\partial \Omega} = -\Omega^{-1}\Omega^{-1}$, since $\dfrac{\partial(\Omega\Omega^{-1})}{\partial \Omega} = \Omega^{-1} + \Omega\dfrac{\partial \Omega^{-1}}{\partial \Omega} = 0$, and

(2) $\dfrac{\partial}{\partial \Omega_{ij}}(\mathrm{tr}\Omega^{-1}\omega) = \mathrm{tr}(\omega\dfrac{\partial \Omega^{-1}}{\partial \Omega_{ij}}) = \mathrm{tr}(-\omega\Omega^{-1}\triangle_{ij}\Omega^{-1}) = \mathrm{tr}(-\triangle_{ij}\Omega^{-1}\omega\Omega^{-1})$,

where Ω_{ij} is the (i, j) th element of Ω; $\triangle_{ij}$ is an NxN matrix with zeroelements except the (i, j) th and the (j, i) th elements both being unity, see Graybill (1969).

Appendix B

Taking partial derivatives of (3.13) with respect to B, λ, σ^2 and μ implies the following first-order conditions for optimization

$$\frac{\partial L^*}{\partial B} = \frac{1}{\sigma^2}\sum^{T}(\bar{C}_t - Z_t B - \mu)Z_t' - \frac{\lambda}{\sigma}\sum^{T}\frac{f_{2t}}{(1-F_{2t})}Z_t' + \sum^{T} R_t'\omega^{-1}(S_t - R_t B) = [0]$$

$$\frac{\partial L^*}{\partial \lambda} = -\sum^{T}\frac{f_{2t}}{(1-F_{2t})}\left(\frac{\mu}{\lambda^2} - (\bar{C}_t - Z_t B)\right)\frac{1}{\sigma} + \frac{T\mu}{\sigma\lambda^3}(\lambda^{-2}+1)^{-1/2}\frac{f_1}{(1-F_1)} = 0$$

$$\frac{\partial L^*}{\partial \mu} = \frac{1}{\sigma^2}\sum^{T}(\bar{C}_t - Z_t B - \mu) + \frac{1}{\lambda\sigma}\sum^{T}\frac{f_{2t}}{(1-F_{2t})} - \frac{T}{\sigma}(\lambda^{-2}+1)^{1/2}\frac{f_1}{(1-F_1)} = 0$$

$$\frac{\partial L^*}{\partial \sigma^2} = -\frac{T}{2\sigma^2} + \frac{1}{2\sigma^4}\sum^{T}(\bar{C}_t - Z_t B - \mu)^2 + \frac{1}{2\sigma^3}\sum^{T}\frac{f_{2t}}{(1-F_{2t})}\left(-\frac{\mu}{\lambda} - (\bar{C}_t - Z_t B)\lambda\right)$$

$$+\frac{T\mu(\lambda^{-2}+1)^{1/2}}{2\sigma^3}\frac{f_1}{(1-F_1)} = 0$$

where f_1 and F_1 are the standard normal density and distribution functions, respectively, evaluated at $(-\frac{\mu}{\sigma}(\lambda^{-2}+1)^{1/2})$; f_{2t} and F_{2t} are the standard normal density and distribution evaluated at $(\sigma^{-1}(-\frac{\mu}{\lambda} - (\bar{C}_t - Z_t B)\lambda))$.

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