MR. KEYNES AND THE CLASSICS: A SUGGESTED TURNING POINT

DEPT OF ECONOMICS
WASHINGTON UNIVERSITY
ST. LOUIS, MISSOURI 63130

(Hsih-Chia Hsieh)*

謝 世 佳*

摘 要

這個總體理論是以個體經濟理論作根據的。國民產量的成長率不同，因國民所得的均衡點不僅僅是一個點。許多總體經濟理論發生相互矛盾，因這裏存在一個轉捩點。這轉捩點是最適當的利率或工資率，亦即最適當的資本或勞工的邊際生產力。在計算時，原始資料有時不能微分，本文使用未滑順的最適控制方法，發現貨幣和財政政策應調整，以減少景氣循環的波動。

Abstract

A turning point of macro theories is verified in which the parameters change in an expected direction. When the prices are incorporated into Hicks' (1937) LM-IS curves, they become non-linear or nonmonotonic. First, the policy is found not neutral, but the world real interest rate may not be the optimal interest rate whereby its second order derivative is negative instead of zero. Second, so far as the domestic inflation rate rises above the optimal level rather than inflation abroad, it tends to reduce real, financial, and innovation activities while the domestic currency is overvalued. The turning point is the optimal rate rather than the inflation abroad or the world real interest rate, whereby the output growth is maximized, and whereby contradictory macro theories ensued. Such turning points may shift due to institutional or technologial innovations. As far as prices remain sticky, Hicks' interpretation holds; otherwise it is substanstially refined.

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（作者曾任本校經濟研究所兼任副教授現為華盛頓大學教授）

1. Introduction

As a result of the Central Banks' stabilization process (West, 1988), many nonstationary series, such as income, interest rates, stock prices, and exchange rates are found (Hsieh, 1989; Scheinkman, 1989). Money is found non-neutral if the inflation rate abroad is assumed to be nonzero (Chen, et al., 1989). In contrast, our finding is that so fas as any real interest rate differential exists intertemporally, across countries, or between the private and the public sectors, the factor movement could arise and alter the resource constraint and the optimal growth path, regardless of whether the inflation abroad is zero or not. The nonneutrality may result in parameter instability after government's policy intervened. Thus in simulation, the macro models are found to be nonlinear (Ashley and Patterson, 1989).

The level of real activities can be measured through impacts of shocks on interest rates or inflation. According to the Keynesian economics, when the prices are sticky and money wages are very high, an excess supply and unemployment may arise, while expansionary policy has mutiplier effects upon income, promoting output or inflation and reducing unemployment. Conversely, as the classic economists suggested, a rise in price inflation reduces excess demand. Nevertheless, a very high inflation, such as in Argentina and Uruguay, could cause high unemployment (Buffie, 1985). Therefore, when all shocks occur simultaneously, Hicks' IS-LM curves could be parallel and there may be no solutions (Wang, 1989). Under Hicks' analysis, an expansionary fiscal policy cum a contractionary monetary policy will raise both interest rate and output but we find that it may reduce the domestic investment if it raises the interest rate beyond the optimal rate. Our intent is to analyze the optimal policy mix and the its transmission mechanism, in which as a result of nonneutrality, previous conflictory studies become special cases (Calvo, 1977; Frenkel el al., 1988; Kawai, 1985). Here, as a distinction from Hicks' monotonic IS-LM curve, three dimensions, policy, prices, and real and financial variables, interact simultaneously.

A question to be explored here is when and how macro policy sometimes has a positive and sometimes a negative impact on the real and financial variables. A nonlinear model is used here because there exist the turning points, around which sequences of changes in institutional or technological innovation, and then asset prices have nonstationary and time-lag impacts, and are serially correlated. Consequently, the Central Banks became independent; interest rates became the policy target. The parameters change expectedly because output is determined by both demand and supply forces, and these forces vary over business cycles and have non-monotonic impacts. Wages and employment have a non-monotonic systematic relation. In the command economy, low repressed wages lead to an involuntary employment, a low quality and a shortage of output supply. Nevertheless, an excessive rise in income or wages tends to raise unemployment, or reduce the effort or the hours worked (Winston, 1966). During a recovery, a rise in wages may be accompanied by a rise in employment and a procyclical inflation whereas during booms, a rise in wages may cause a decline in employment, while inflation falls counter-cyclically (Sumner and Silver, 1989).

In this study, in Section 2, a true model reconciles conflictory theories and policies and provides a nonlinear macro theory. In Section 3, this model is reduced into a controllable and identifiable form. A nonsmooth policy optimization method is introduced, because it is applicable even when the planner has an objective function, which may be not exactly known. In Section 4, this model is verified through empirical studies; and Section 5 concludes with remarks.

In the present study, structural changes are verified under F and t tests on regression coefficients, on residual heteroskedasticity, and on Granger- causality tests. Such a relationship is plotted and detected under a nonlinear curve. The data on both developing and developed countries are found largely consistent with such non-monotonic macro theories and our policy diagnoses. The figures plotted show common features for countries, such as the United States, the United Kingdom, Mexico, Brazil, Taiwan, and Saudi Arabia. But there are many variations. For example, in Brazil and Mexico, output growth is more sensitive to variations in real interest rates than in inflation rates, perhaps owing to using the price-indexed regime. In Brazil, high real interest rates are associated with high real exchange rates and capital inflow whereas in the United States, beyond the optimal real interest rate, a contractionary monetary policy cum an expansionary fiscal policy leads to a very high real interest rate and tends to appreciate the domestic currency. Further, it leads to a decline in domestic investment and reduces output growth. An inverse policy mix beyond the optimal level, however, will cause high inflation, reduce foreign direct investment and causes capital outflow.

Our assumption is that wages are subject to union bargainings, fixed in labor contracts, and not competitive in the short run. But even if capital is perfectly mobile, we find that although a rise in the domestic interest rate beyond the foreign interest rates will lead to the expected appreciation in exchange rates, beyond the optimal rate, it may cause capital outflow and a depreciation. Hence, although it is often assumed that when the market is efficient, the future exchange rate converages towards the spot exchange rate in the future we find that such a covered interest rate parity does not hold exactly due to overshooting, segmentations of markets and risk differentials among assets. Its short-run effect is for the capital inflow or outflow to adjust the productivity and output. Our theory is largely verified using the data of several countries, as reported in the *International financial Statistics* for the 1954-1982 period.

2. A Nonlinear Model and A General Macro Theory

Consider the following dynammic true model: The money demand and supply (LM) function is derived from

(1) $M^d_t - P_t = a_1 Y^d_t - a_2 i_t + a_3 i^2_t + v_{it}$

$$M^d_t = M^s_t$$

The saving and investment (IS) function is derived from

(2) $Y^s_t = b_1 (P_t - W_t) - b_2 (P_t - W_t)^2 + b_3 (r^*_t + ex_t) - b_4 (r^*_t + ex_t)^2 + v_{2t}$

(3a) $Y^d_t = - c_1 r_t + c_2 r^2_t + c_3 G_t - C_4 G_t^2 + v_{3t}$

(3b) $P_t = f_1 (Y^d_t - Y^s_t)$

The money supply is derived stochastically from

(4) $M^s_t = d_1 (g_t - T_t) - d_2 (g_t - T_t)^2 + V_{1t} + v_{2t} + v_{3t}$

The identity of budget deficit and trade balance is

(5a) $(p_t S_t - p_t I_t) + (p_t T_t - p_t g_t) - (p_t X_{xt} - Ex_t p^*_t X_{mt}) = 0$

The long run real interest rate parity holds, but in the short-run, it leads to overshootings or undershootings around the purchasing power parity:

(5b) $(dz_t/dt) / z_t = r_t - r^*_t$

and $z_t = Ex_t p^*_t / p_t$

The expected real interest rate is nominal interest rate minus the expected inflation rate, (minus tax rates plus subsidies):

(5c) $r_t = i_t - E(\log p_t - (\log p_{t-1}))$

In the long run equilibrium, the control policy steers the state variables from y_0 toward the targets of full employment, $y_t = y^*$. An identity is defined as:

(5d) $y_t = y_{t-1}(1 + Y_t) = Y_0 \ \exp(Y_t)\, t$

The government savings identity is defined as

(6) $g_t - T_t = m_t / p_t + b_t / p_t$

where all parameters a_i, b_i, c_i, and d_i are positive. E denotes an expectation operator.

The Notation

A = the autonomous component of output $py = pc + pI + pg + (p\,X_x) - (Ex\ p^*)\,X_m$

b_t = the new issue of public bonds

$\log C_t - \log C_{t-1}$ or dC_t / C_{t-1} the growth rate of real private consumption

Ex_t and z_t = the nominal and the real exchange rates

ex_t = the rate of changes in the nominal exchange rate;

G_t = the growth rate of real government spendings, including the interest payment on public bonds;

g_t = the real government spendings;

i_t = nominal interest rate;

$\log I_t - \log I_{t-1}$ or dI_t/I_{t-1} the growth rate of real private investment;

M_t = the growth rate of money supply;

m_t = the nominal increase in money supply;

P_t = the price inflation of domestic goods; p the price level;

P^*_t = the price inflation of foreign goods. (For the sake of convenience, the foreign price is set equal to unity, and its logarithm $\ln p^*_t = 1$; its growth rate is zero); p^* is the foreign price level;

r_t = real domestic interest rate, $r = i - P$;

r^*_t = foreign interest rate (for convenience set equal to a unity; and the logarithm of $r^*=1$ is zero);

S_t = real savings.

T_t = real tax revenues;

$\log X_{xt} - \log X_{xt-1}$ = the growth rate of real exports;

$\log X_{mt} - \log X_{mt-1}$ = the growth rate of real imports;

u = the unemployment rate;

U = the social welfare or utility

W_t = the growth rate of nominal wages

w_t, w^*_t = the real wage at the domestic and world levels.

v_{it} = a random variable, including institutional and technological innovation;

Y, Y^d, Y^s, and Y* are the real growth rates of the actual output, output demand and supply, and maximum output.

y and y* = the levels of real output and the optimal output.

Note: In the above nonlinear model:
Time-t exogenous variables are
G, T, i, m
$(a_1, a_2, b_1, b_3, c_1, c_3, d_1, f_1)$ = set of positive constants;
$(v_{1t}, v_{2t}, v_{3t}, v_{4t})$ = set of random variables;
Time-t endogenous variables are
$(M^d, r, Y^s, Y^d, P, W, Ex)$

When the model is linearized, or when the quadratic terms are assumed constant, the variables are classified as follows

Time-t exogenous variables are G, T, and i or M;
Time-t endogenous variables are Y, P, and W.

In Equation 1, an increase in income tends to raise the demand for money, and an increase in the nominal interest rate tends to lower it. During a depression and a recovery, a rise in output growth could raise both money demand and the interest rates. Then, the relationship between interest rates and money demand turns from a negative into a positive one (Poole, 1986 and Gibson, 1970). In contrast, according to Hicks (1937), LM curve, income and interest rates are negatively correlated. Figure 1 shows our non-monotonic LM curve. In developing countries, high inflation tends to be characterized by high nominal interest rates, while its real interest rate remains relatively low or negative. Conversely, for the developed countries, the real interest rate is often relatively high, though its low inflation is often accompanied by a low nominal interest rate. The term structure of interest rates adjusts slowly. Under expansionary policies, the short-term nominal rate turns upward sloping during good times, but it is downward sloping during deflation and recessions (Mishkin, 1981a; 1981b).

Proposition 1:

Nominal interest rates are positively correlated with inflation rates while the real interest rates are negatively correlated with inflation rates. Excess demand is associated with low real interest rates, and vice versa.

Equation 2 is an output supply equation. Output supply varies nonlinearly with the real wages, W − P, and real interest rates, $r = r^* + ex$. When inflation is high and the trade deficit accumulates, an increase in real interest rates will reduce excess demand and inflation rates, and induce capital inflow. It raises real wages, and stimulates labor supply and output growth. Thus if the low wage leads to trade surplus, a rise in real wages encourages labor effort and induces investors to use capital stocks as a substitute for labor. Nevertheless, when interest rates or wages rise above their marginal productivity of capital or labor, it will increase production costs, raise unemployment and reduce capital utilization and output supply.

In Equation 3a, the demand for real output growth is determined nonlinearly by the real interest rates and real government spending if the tax revenue remains unaltered. During a depression, a fall in interest rates or a rise in government spending tends to stimulate effective demand and reduce trade surplus, whereas when inflation is high, it tends to crowd out consumption or investment, and increase trade deficits. Equation 3b shows that in disequilibrium, inflation adjusts the actual output growth to minimize the difference between output demand or supply. When aggregate demand is less than aggregate supply, $Y^d < Y^s$, a deflation or low inflation leads to the Keynesian unemployment, and causes a high real interest rate, resulting in recessions, and reducing output supply. Conversely, insofar as there is an excess demand, and the domestic inflation stays higher than the optimal level, a rise in interest rates will encourage savings, capital inflow, investment, output growth and effective demand.

In Hicks (1937), the IS curve is negatively slopped; a rise in interest rates tends to reduce output demand. In contrast, Figures 2.1 shows our IS curve. When inflation is high, a rise in real interest rates tends to raise output growth whereas beyond an optimal rate, it tends to reduce output growth. This relationship holds regardless whether the output growth is decomposed into consumption growth and investment growth or it is decomposed into wage growth and profit growth.

During depressions, a rise in interest rates may raise savings but reduce investment and output growth whereas when inflation is high, it raises both saving and investment. Output growth is more correlated with investment rather than with savings. Although a rise in income could increase imports and deteriorate the terms of trade, it also may increase exports and cause an appreciation insofar as inflation is relatively low. In Figure 4, whether or not there is undervaluation and overshooting depends on whether exchange rates deviate away from the differential of the domestic over and above the purchasing parity and causes the capital outflow or the direct foreign investment. If there exists trade surplus, the optimal interest rate would be lower than the world interest rate.

Equation 4 is an identity in which the money supply is determined by government budgets, and all exogenous demand and supply shocks. During a recovery or wars, a tax cut or a budget deficit initially may be accompanied by a contractionary money supply. However, an increasing budget deficit finally need be either monetarized or paid for through a tax increase. Equation (5a) is another identity which depicts the relationship between budget deficits and trade deficits relative to the national output. Equation (5b) is a behavior equation and depicts a convergence for exchange rates toward the purchasing power parity in the long-run but as in Fig. 4, the capital outflow leads to a depreciation and deviates away from the interest rate parity in the short-run. But production takes time to adjust and the parity does not hold exactly in the short run because the nominal interest rate, i, is exogenous.

Proposition 2.1

When inflation is lower than the optimal inflation rather than inflation abroad, a decline in interest rates tends to raise stock prices (or returns), long-term bond prices, and appreciate the exchange rate. When inflation is high, the relationships turn negative (as shown in Fig. 3).

During price inflation, as Lach et al (1989) pointed out, the demand pulls technological innovation and investment. When inflation is high, it may induce speculative activities since the prices of real estates rise more rapidly than equity prices (Benari, 1989). In Fig. 2.2, the optimal real interest rate maximizes output and consumption growth while the trade-off between the growth rates of investment and wages are optimized, while stock prices (or rate of returns) rise moderately, and the domestic currency appreciates towards the purchasing parity in Figs. 3 and 4.

Proposition 2.2:

With the sticky contracts on wages, loans, and exchange rates, inflation or an overvaluation of domestic currency tends to reduce the real wage and raise domestic demand and trade deficits. A rise in interest rates not beyond the optimal rate tends to reduce domestic excess demand and profit growth, but promotes capital inflow and output growth (as shown in Fig. 2.1, 2.2 and 3).

In Equation (5c), the expected real interest rate is defined as the nominal interest rate minus the expected inflation. An excesses supply and unemployment are often associated with a relatively low inflation or a high real interest rate (Hamilton, 1985). Hence, at the long-run optimal interest rate and inflation, the maximal output growth is located at the optimal equilibrium among commodity, money, and bond markets. Eq. (5d) is an identity between the growth rate and the level of variables. In the short-run disequilibrium, the actual output growth determines the deviation of actual output from the equilibrium output level. Equation (6) is an identity of the budget deficits. In this model, the money supply and demand (LM) curve and the investment and saving (IS) curve are nonlinear in interest rates.

3. Controllability and Identifiability

In the above multivariate polynomial model, the four exogenous control variables, Z = (G, T, i, M), include government spending, G, and taxes, T, the interest rate, i, and money supply, M. The interest rate and money supply are not independent of each other. Since both demand and supply-sided shocks can affect this nonlinear system, we minimize the fluctuations or variances of interest rates while using money supply as the policy target. In equilibrium, supply is equal to demand, $M^d = M^s = M$; and $Y^d = Y^s = Y$. Of the first four equations, only three are independent. Using three control variables, Z = (G, T, M), we can solve for the three unknown endogenous variables, X = (Y, P, W) and let the exchange rate balance the external trade.

The simultaneous system used here is nonlinear. Our purpose is to minimize the disturbance errors and reduce parameter instability. The nonlinear LM and IS curves have multiple local intersected points or equilibria. First, for example, equalizing money supply and demand, $M^s = M^d$, yields the nonlinear LM curve.

Fig. 1 The LM Curve in 1955-81 in the United States

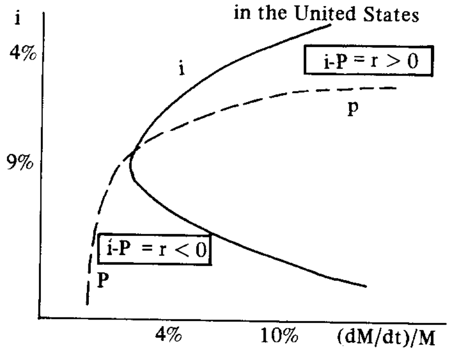


Fig. 2.2 The Saving and Investment in Brazil and Taiwan 1965-81

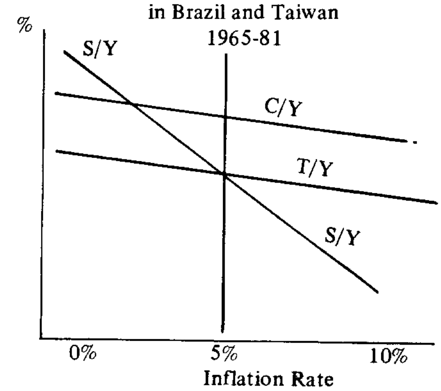


Fig. 21 The IS Curve in 1955-81 in the United States

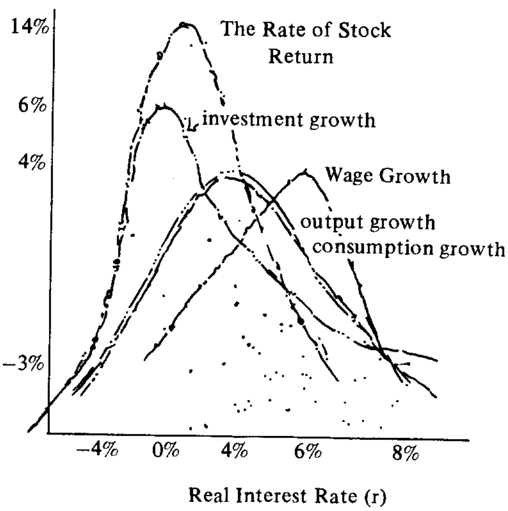
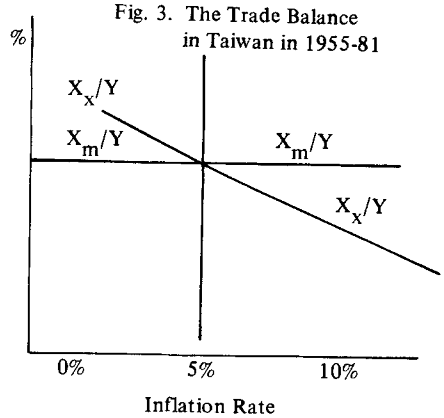


Fig. 3. The Trade Balance in Taiwan in 1955-81



(7) $$M_t = P_t + a_1 Y_t d - a_2 i_t + a_3 i^2{}_t + v_{1t}$$
$$= P_t + d_1 (g_t - T_t) - d_2 (g_t - T_t)^2 + v_1 t + v_2 t$$

By altering the budget deficit, g − T, and minimizing the variation in the interest rate, i_t, while controlling money supply, M_t, we stabilize the real interest rate and the inflation rate, P_t. In the steady state all expected values equal the actual values, such as $E(P_t) = P_t$. In the reduced forms, the random disturbance terms are assumed to be zero. According to Hicks, when the real interest rate is expressed on the vertical axis and the real output on the horizontal axis, the iterative solution of fiscal expansion behaves as if the LM curve becomes horizontal, while we can increase government spending and shift the IS curve rightwards. It raises output, inflation or interest rates. Nevertheless, we find that the IS curve bends backwards. High interest rates may lead to recessions while low interest rates may result in hyperinflation, both reducing output growth.

Now, equalizing the output demand and supply, $Y^d = Y^s$, yields the IS curve where investment equals savings.

(8) $$b_1 (P_t - W_t) - b_2 (P_t - W_t)^2 + b_3 (r^*_t + ex_t) - b_4 (r^*_t + ex_t)^2 + v_{2t}$$
$$= - c_1 r_t + c_2 r^2{}_t + \ldots + v_{3t}$$

where the profit, $P_t - W_t$, varies nonlinearly with the level of the real interest differential, $r_t = r^*_t + ex_t$, in the long run. By using Eqs. 5c, 7 and 8, wages, prices, and output are simultaneously determined by interest rates, i_t, government spendings, g_t, and taxes, T_t. As shown in Figs. 2 through 4, at the maximal output growth, the exchange rate adjusts itself and balance exports and imports. In Eqs. 1, 4, 5b, and 5c, the level of real interest rate, r, depends on the size of supply and demand shocks, G, T, M, and v_i, i=1, 2, 3. It, in turn, determines the price level and output growth at an intersected point of the LM and the IS curves (Fukuda, 1989; Penati, 1985; Dixit, 1983). Sometimes the solution, positive or negative, could be one of multiple local solutions, and may not yield the maximal output growth, however.

The wages, nominal or real, are rigid in the short run. Through a rise in productivity, Y, a decrease in price inflation, a currency appreciation, or a shortening of the labor-contract period, the real wages can rise competitively towards the marginal productivity of labor. Conversely, insofar as the domestic inflation is lower than the optimal inflation, or the real interest rate is relatively high, it tends to undervaluate the domestic currency, $Ex < p/p^*$, because low inflation stimulates productivity, and enhances import prices and balances the trade balances. Like interest rates, the short-run deviation of wages can cause the disequilibrium of output, employment and trade balances. Finally, when the money and the commodity markets are in equilibrium, the bond market will clear itself according to the Walrasian equilibrium.

Since the identifiability is a weak form of controllability in nonlinear systems, this system can be reduced into its implicit controllable equations (Brown, 1983). Specifically, the Hamiltonian for this optimization problem is

$$\text{Maximize } H = U(Z) + h\,F(X, Z)$$

where h is the Lagrangian multiplier. With the different initial conditions under the simulation method, the first-order derivatives of H may not converge towards a unique optimal solution. Alternatively, we may follow the null controllability approach (Underwood, 1988). The nonlinear model, $F(X, Z) = 0$, is transformed into a linearized form by neglecting the second and higher order terms or by assuming the quadratic terms as a constant variance. Since the state variables could have lag adjustment to policy impacts, our disequilibrium model of $F(X,Z) = 0$ can be written as differential equations:

$$(9) \qquad dX/dt = A\,X + BZ$$

As time goes to infinity, it becomes stable, $dX/dt = 0$. The subcript, t, is omitted in the optimal stationary equilibrium. In the discrete form, Eq. (9) is written as:

$$X_{t+1} = K\,X_t + H\,Z_t$$

Let $K = (K_1, K_2) > 0$ and $H = (h_1, h_2) > 0$ be parameter matrices. In Section 4, we empirically estimate this nonlinear model as a decomposed piecewise regression:

$$(10) \qquad X_{t+1} = K_1\,X_t + h_1\,Z_t \qquad \text{for } P^* < P \quad \text{or } r^* > r$$

and

$$X_{t+1} = K_2\,X_t - h_2\,Z_t \qquad \text{for } P^* > P \quad \text{or } r^* < r$$

where k_i and h_i, $i = 1,2$, are constants; P^* and r^* denotes the optimal inflation and interest rates. Random errors in this piecewise model are minimized. When the control function of z is nonlinear, it may oscillate and respond instantaneously to the change of the coefficients (Artstein, 1989). The necessary and sufficient condition for its identifiability and controllability is that the full rank of the matrix is three. The coefficients K and A are kept constant. Three control variables, Z = (G, T, M), steers the three state variables X = (Y, P, W). Eq. 9 can be transformed as

(11) $X = \exp(At)\, BZ = (1 + A + A^2 + \ldots)\, BZ$

The controllable condition is

$$\text{rank}\,(B, AB, A^2B) = 3$$

where A and B are parameter matrices. The rank condition can be verified by the Gaussion elimination method. The identification problem of the time-varying or nonlinear system is converted into a problem of estimating the linear time-invariant system with unknown parameters of Eq. (10). For a nonlinear or uncertainty system, the controllability for an infinite horizone is the minimization of the norm or the variance of X

$$E\,(X\text{-}X^*)'\,(X - X^*) < C$$

or $E\,\|\, X'X - X'LZ \,\| < C$

where C is a small positive constant; X′ is a transpose of X; and the control law is $Z = -LX$, here A and L are constant.

Suppose that the Central Banks do not exactly know the objective function nor the functional forms and parameters with any degree of confidence. The following linearized reduced form (Hsieh, 1987) will be used as a function of a control variable, say, interest rates. By letting M,G, and T be constant, and neglecting their high-order terms, we can take a total differentiation of the utility, U, subject to the model or the data constraint, with respect to each policy variable. Partial differentials are solved for $dU/dr = 0$ or $\partial U/\partial M = 0$, $\partial^2 U/\partial M\partial T = 0$, $\partial^3 U/\partial M\partial T\partial G = 0$ or $\partial^2 U/\partial M\partial\,(T\text{-}G) = 0$. The solution between the unconstrained and constrained optimization should be the same as far as it is feasible. In this study, the data constraint of previous experiences insures such a feasible solution.

By controlling the nominal interest rate, the optimal real interest rate, r*, is a solution of maximizing utility U = U(r) or the real activities Y = f(r). The linearized form (10) becomes:

(12a) $Y^s = a_1 r$ for $Y < Y^*$ and $r < r^*$

$Y = Y^*$ for $r = r^*$

and $Y^d = -a_2 r$ for $Y > Y^*$ and $r > r^*$

where $a_i > 0$ for $i = 1,2$. To minimize the difference between output supply and demand, the two convex functions, in Eq. 12a, can be combined into a concave function in addition to the turning point (Tuy, 1987):

$$Y = f(r, Y^*)$$

Its second order derivative is negative, $d^2Y/drdr = -a_2 < 0$. As Figure 2.1 shows, since a further rise beyond the optimal rate, r*, tends to reduce the output growth, the interest rate policy should respond to any unsystematic shocks, thereby controlling money supply (Snower, 1984). Next, we take the optimal interest rate as given, the optimal taxation or budget deficit are obtained through further differentiation.

For example, suppose that utility increases with intertemporal consumption and income but decreases with inflation. The wages are assumed to be given. Our problem is to find an optimal monetary growth, which maximizes the output growth, Y, and minimizes inflation, P, and their variations. These variances are assumed to be constant. Thus

(12b) $$\text{Maximize } U = Y - Y^2 - P - P^2$$

subject to the constraint of the reduced model. The dynamic stability is attained around the long-run equilibrium values. The Taylor series is expanded to the first order as:

(13) $$\begin{vmatrix} dM/dt \\ d(g\text{-}T)/dt \end{vmatrix} = \begin{vmatrix} \partial U/\partial M & \partial U/\partial (g\text{-}T) \\ \partial U \; \partial M & \partial U/\partial (g\text{-}T) \end{vmatrix} \begin{vmatrix} M - M^* \\ (g\text{-}T) - (g\text{-}T)^* \end{vmatrix}$$

where the stability of the solutions should satisfy the Routh-Hurwitz's condition. The asterisk denotes the optimal values. Let $X^* = (Y^* \; P^*)$ be targets for which the actual values should strive to achieve. As one of three first derivatives of U, $dU/dM = 0$ is derived approximately from:

$$dU/dZ = Z - h(X) = 0 \qquad \text{for } X > X^*$$

To achieve at least our target, $X > X^*$, our target is

$$\min_{X^*} \max_{X} U(X) \qquad \text{for } X = (Y, P)$$

subject to $dX/dt = Ah(X) + BZ$
and the optimal control law

$$Z = h(X) \qquad \text{for } X > X^*$$

Proof: The objective, U(X), is a semi-differentiable function with uncertain coefficients. According to the Lipschitz continuity, the necessary and sufficient condition of the optimality principle is approximated as $Z \to Z^*$

$$\lim_{M} (U(Y^*, p^*, Z^*) - U(Y_t, P_t, Z_t) / \| Z_t - Z^* \| \leqslant 0,$$

The utility is maximized as:

$$U (Y_t, P_t, Z_t) > U (Y^*, P^*, Z^*) \qquad \text{for all } Y_t > Y^* \text{ and } P_t < P^*$$

where the intent of constraints, $Y > Y^*$ and $P < P^*$, is to satisfy the Bellman's optimality principle or the sufficient condition of the second order derivatives. The optimization problem becomes a problem of estimating the parameter of the Euler equation,

$$\left(\frac{dU}{dZ} - \frac{d}{dt}\frac{dU}{dZ'}\right) = 0, \text{ or approximately } dU/dZ = h(X) - Z = 0.$$

The suboptimal solution is located at intersected point of two curves $Z = h(X)$ with and without constraints $Y > Y^*$ and $P < P^*$. It can be iteratively improved as new data are updated. The step of direction finding is $X^* > X_k > X_{k-1} > \ldots > X_0$, where k is the number of iterations. The coefficients vary with the state of X. At the optimum, $X^* = X_{k+1}$, and $U^* = U_{k+1}$ · Q.E.D.

4. Empirical Studies

4.1 The Optimal solution of the model

Using the Taiwan data over the period 1954-1981, as published in Taiwan's Income Data Book, by Executive Yuan, Rep. of China, we use a finite horizon to approximate the optimization for the infinite horizon. In this period, the maximal output growth, around 12%, was associated with an inflation rate of around 9%. By selecting those annual observations in the sample period in which the annual output growth is high, say, $Y > 8\%$ in Taiwan, and the annual inflation rate is low, say $P < 7\%$, we estimate a simultaneous system in which the piecewise regression, $M = h(Y)$, is one of the approximate first derivatives, $dU/dM = 0$, of the objective function. This is the government's optimal response rule:

$$M = a_1 Y + a_2 Y_2 + a_3 Y_3 \qquad \text{for } Y > 8\% \text{ and } P < 7\%$$

It can be rewritten as a lag adjustment of output towards monetary policy in disequilibrium:

$$M_t = b_1 Y_t + b_2 Y_{t-1} + b_3 Y_{t-2} \qquad \text{for } Y > 8\% \text{ and } P < 7\%$$

The above control rule is based on the prior constraint while a reaction function, Y = f(M), without constraints denotes the average response of the private sector. The intersected point of these two response rules is an approximately optimal or Nash solution. As Fig. 5 shows, the optimal constrained reaction curve, M = h(Y), of money growth to the output growth is negatively sloped, as the output growth rises beyond the target growth rate, the monetary growth should slow down. It is countercyclical since there are resource constraints. In contrast, the average response is procyclical and positively sloped. For example, the average monetary growth is accomodating to output growth. Similarly, the optimal response rules on government spending and tax revenues largely behave like the monetary response rule.

Finally, we estimate the optimal budget deficit for Mexico, the United States and Taiwan. As Figure 6 shows, the optimal budget response rule is positively sloped. The budget deficit should decline or turn into a surplus when the output growth rate exceeds the potential growth rate of output. It is also counter-cyclical. Thus the optimal policy is a constant time-consistent rule but not a constant since the government is bound by the intertemporal budget constraint and there are unexpected external shocks.

To verify that the merit function is roughly maximized, we simulate with the U.S. data for the period 1954-1981. The result is shown below:

Table 1. Payoffs of the U.S. Optimal Money Growth

	Actual (1954-1984)	Optimal (1954-1984)	Actual (1981-1982)	Optimal (1981-1982)
Inflation	4.58%	3.19%	6%	5%
Monetary Growth	4.6%	6.8%	5.5%	11%
Unemployment Rate	5.56%	5.22%	9.5%	5.2%
Output Growth	2.93%	5.49%	−1.08%	5.49%

Fig. 4 The Real Exchange Rate in Taiwan, Saudi Arabia and the United States in 1965-81

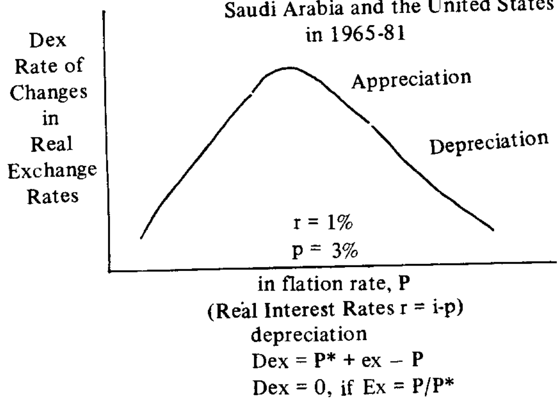


Fig. 5 The Optimal Macro Policy in Brazil, Saudi Arabia and Taiwan in 1965-81

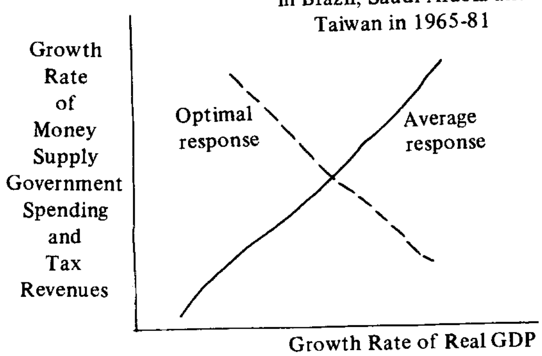
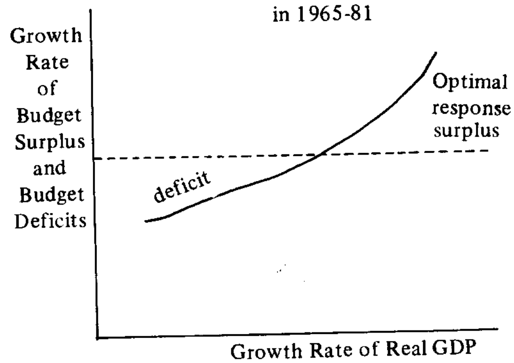


Fig. 6 The Optimal Budget Definiti in Brazil, Taiwan and the United States in 1965-81



4.2 Testing the Turning Points of Piecewise Macro Theories

We use the annual data on the United States, Saudi Arabia and Taiwan over the period 1954-1981, on Brazil and Mexico over 1965-1981, as reported in the *International Finance Statistics.* By using Eqs. 9 and 10, the macro variables are regressed nonlinearly upon interest rates or inflation rates. The data for the United States include Benderly et al's (1985) data on the real rate of return, the inflation rate and output growth, and Eisener's (1986) data on budget deficit. The data on wages and unemployment rates are available in the U.S. President's Economic Report. We also consider Barro's (1987) data on 83 countries.

For each country, respectively, several simultaneous systems are estimated using three stage least squares in the IBM Shazam package. By using the t tests, Chow F tests, and Goldfeld and Quandt F tests, we find that consumption, investment and exchange rate equations are subject to significantly structural changes, depending on the level of the real interest rate. Our approach is expressed in the domain of the real interest rate instead of the time period or the spectrum. Regardless of whether the disturbances come from the supply shocks or the demand shocks, v_{it}, our criteria of impacts on real activities are measured by the deviation of the optimal real interest rate or inflation rates towards which the world levels converge. Therefore, if an insufficient size of policy responses arise, ineffective demand may still exist owing to multiple equilibria.

The nonlinearity enters through altering the variances, which takes the form of conditional heteroskedasticity. In Table 2 through 4, as shown in F tests in all equations, when the real interest rates change, the explanatory powers, R^2, are significantly changed, and the hypotheses of homoskedasticity of residuals are significantly rejected at the 95 percent levels.

Next, Tables 5.1 and 5.2 show the Granger-causality tests. By using t statistics, we find that when the real interest is high, $r > 1.1\%$, a rise in the real interest rate insignificantly reduces the growth rate of consumption and investment and depreciate the real exchange rate (i.e. increase the deviation of the exchange rate from the purchasing power parity). In contrast, when the real interest rate is low, $r < 0.9\%$ or inflation is high, a rise in real interest rates will significantly promote consumption and investment and fairly significantly appreciate the real exchange rate. In Table 5.1, the high real interest rate has no significantly negative impact, implying that low inflation has no significantly positive impact on real activities. However, the impact of real interest rates on real exchange rates is significant in structural changes while it is less significant in Granger-

Table 2. Structural Changes (When r > 1.2%)

	dC/C (14.1)	dI/I (15.1)	Dex (16.1)
	0.02 (2.80)	0.02 (0.09)	−0.07 (−1.74)
	−0.51 r (−3.36)	− 0.05 r (−1.15)	4.22 r (2.26)
R^2	0.038	0.0001	0.28

Note:
1. dC/C and dI/I denote the growth rates of consumption and investment. The subscript, t, is omitted whenever possible. These equations are estimated first as the ordinary least squares and then as a system of simultaneous equations under the three stage least squares. With a simultaneous system, the coefficient estimates are not biased even when the endogenous variables serve as the optimality condition, $r > r^*$. Thus the result holds robustly even if the period when the low inflation happens is the period when the supply shocks occur; and vice versa.
2. Through this study, the values in the parentheses are t statistics.
3. Structural changes can be detected through changes in the cumulated sum of residuals (CUSUM); but it can not indicate why and how the structure changes in the sense of economic variables instead of time period.

Table 3. Structural Changes (When r < 1.2%)

	dC/C (14.2)	dI/I (15.2)	Dex (16.2)
	0.03 (4.80)	0.05 (2.51)	−0.4 (−2.1)
	1.08 r (1.63)	2.8 r (1.19)	1.21 r (5.2)
R^2	0.21	0.12	0.03

Note: The values in parentheses are t statistics. The deviation of exchange rates from the purchasing power parity is denoted as Dex = ex + P* −P. ex is the rate of changes in German marks per U.S. dollar. When Ex=p/p*, Dex = 0.

causality tests. Thus our findings are that when the real interest rate is high, or inflation is low, it has no significantly negative impacts on output, whereas when inflation is high, a rise in interest rates has a significant positive impact on output.

Table 4. The F Tests of Turning Points

The Deviation of Real Exchange Rates	Consumption Function	Investment Function
$F_1 = \frac{0.09/(12\text{-}2)}{0.08/(14\text{-}2)} = 3.5$;	$F_2 = 5.0$,	$F_2 = 1200$

Note: Two F tests used are

$$F_1 = \frac{(1 - R^2_1)/(T_1 - K_1)}{(1 - R^2_2)/(T_2 - K_2)} \quad \text{and} \quad F_2 = \frac{R^2_1/K_1}{R^2_2/K_2}$$

Where T is the number of observations; K the number of independent variables. The first F value is to test the hypothesis of homoskedasticity of residuals. The second F test is to test the hypothesis of the identical explanatory power of equations. The critical value of F tests, for example, is P $(F > 3.07) = 0.05$ with 12 degrees of freedom in the numerator and 10 in the denominator. P $(F>19) = 0.05$ with 2 degrees of freedom in both the numerator and the denominator.

Table 5.1 Granger-Causality Tests

When $r > 1.1\%$			$r < 0.9\%$		
dC/C	dI/I (17.1)	Dex (18.1)	dC/C	dI/I (17.2)	Dex (18.2)
−0.3 r (−1.5)	−2.39 r (−1.06)	1.24 r (0.36)	1.61 r (2.14)	1.03 r (2.16)	−4.90 r (−1.39)
$-0.13 dC/C_{-1}$ (−0.92)	$-0.31 dI/I_{-1}$ (−0.12)	1.03 Dex-1 (1.53)	$0.29\ dC/C_{-1}$ (2.52)	dI/I_{-1} 0.75 (4.11)	0.75 Dex-1 (2.14)

Note: t statistics are in parentheses.

In Table 5.2, the nonlinear Phillips curve also implies a significant trade-off. When the real interest rate in low, unemployment rates decline; and vice versa. Similarly, when the interest rate is low, a rise in real interest rate will significantly increase stock returns and vice versa. Our finding is largely consistent with both the Phillips curve and with the New Keynesian arguements because inflation has a positive trade-off while high inflation has a significant negative impact. But low

Table 5.2 Granger-Causality Tests (unemployment rate)

$u_t = 0.82u_{t-1}$ (11.7)	$-42.3\ r_t$ (−2.49)	for all r
$u_t = 0.75u_{t-1}$ (8.70)	$+57\ r_t$ (1.66)	for $r > 1.2\%$
	(Rates of Stock Returns)	
$e_t = 0.19e_{t-1}$ (1.02)	$-6.35\ r_t$ (−1.47)	for all r
$e_t = -0.2e_{t-1}$ (−1.19)	$+15\ r_t$ (2.73)	for $r < 0.9\%$

Note 1: The expected real interest rate used is $r = i - 0.5\ (P_t + P_{t-1})$

2: To incorporate the last two equations into the model, the Okun law is used, $(u - u^*) = -(y - y^*)/3$, and the national income is decomposed into profits (dividends), wage, interests, and rents. After taking the first derivative of the objective function, (12b), if we use three equations. 9, or 10 and 12b as a tent map model, the stability region of the Phillips curve could look roughly like a circle (For proof, see Chiang and Thorp, 1989, p. 1239). Expected inflation leads to high inflation and high unemployment rates; and finally they are reduced through a contractionary policies.

inflation has no very significant positive impact on real activities (Jung et al., 1986).

When a subsample is used, the simultaneous system yields $R^2 = 0.86$, $x^2 = 24.05$. When all sample observations are used, the system yields $R^2 = 0.98$, $x^2 = 56.4$. Both have 15 degrees of freedom. The results are statistically significant at the 95 percent level.

4.3 A Discussion on Interest Rates, Exchange Rates, and Stock Prices:

As Figs. 1, 3 and 4 show, low inflation is associated with high output growth because the demand pull leads to technological innovations and raises the marginal efficiency of capital or profit, thereby promoting investment and output, while it pulls down the nominal interest rate and raising the real interest rate. Low inflation promotes net exports and capital inflow, and appreciates the domestic currency.

Empirically, a long-run monetary expansion, on average, is accompanied by almost the same rate of inflation and the devaluation of exhange rates. However, the long-run output growth does not converge monotonically toward zero nor the steady state.

Proposition 3:

Money is not neutral since through currency substitution and output growth, high monetary growth is accompanied by an even higher rate of inflation. Whereas a smaller monetary growth is accompanied by less than a proportional rate of inflation, whereby domestic currency is held by foreign countries.

Insofar as the domestic inflation rate remains lower than the optimal inflation rather than inflation abroad, or the real interest rate is higher than the optimal rate, expansionary policies tend to raise the rate of returns, induce capital inflow and raise asset prices. Thus saving could exceed investment, thereby generating an excess supply and a trade surplus (as shown in Fig. 3). Then since the capacity remains underutilized, regardless of whether the world inflation rate is zero or not, an expansionary policy has a positive impact on output. When government spending is financed through debt issues instead of money supply or taxation, it has a less expansionary impact and tends to raise interest rates and overvaluate the domestic currency (Rankin, 1989). Thus, if the government increases its spending on non-tradable goods, it tends to appreciate the domestic currency.

In Tables 6.1 and 6.2, we imposed the conditions on Dornbusch and Fischer (1988 and 1990)'s proposition. The results are inverted when such conditions are inverted. p* and r* denote the optimal inflation or the optimal real interest rate. These conditions are applicable to a small and a large open economy as well as to the world economy.

Table 6.1 Impacts of Policies

	Current Account	Condition
Ep*/p depreciation	+	$r > r^*$ and $p < p^*$
Output growth	+	$r > r^*$ and $p < p^*$

Table 6.2 Impacts of Policies

Policy	Equilibrium interest	Equilibrium income	Conditions
Monetary expansion	–	+	$r > r^*$ and $p < p^*$
Fiscal expansion	+	+	$r > r^*$ and $p < p^*$

5. Remarks

Insofar as the domestic inflation stays lower than the optimal inflation or as the domestic real interest rate is higher than the optimal rate, when the expansion policies raise inflation, interest rates, and asset prices, it promotes trade surplus and output growth and appreciates the domestic currency. Conversely, when inflation is relatively high, a fall in real interest rates tends to exacerbate inflation, reduce income, and increase the imports and trade deficits. The macro relationships are found to be non-monotonic and have been verified through F tests and Granger-causality tests.

The LM curve and IS curve may take the C curve and inverted U curve over a cyclical process, respectively. The exchange rate should not be regarded as a target of a zero sum game across countries because the maximal output growth is attained at the balanced trade. Our new nonsmooth optimization shows that the time-consistent policy rule are counter-cyclical. It appears easily verifiable. In contrast, many dynamic programmings, such as the Quasi-Newton method, could yield infeasible solutions (Mahdavi-Amiri and Bartels, 1989), and its state trajectory may not make economic senses to policy makers.

Our finding rejects many previous hypotheses that a decline in real interest rates, a rise in inflation, or a devaluation of currencies will always encourage investment, raise the profits rate (or stock prices), reduce the real wages, and raise the output growth. There exist turning points, leading to institutional changes. At a very low real interest rate or a very high inflation rate, the expansionary monetary and fiscal policy will cause disequilibrium and yield a negative income multiplier on financial and real variables, and worsen the trade deficits and asset prices.

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