INCENTIVE OF LOSS RESERVES MANIPULATION AND RATE REGULATION IN PROPERTY-LIABILITY INSURANCE INDUSTRY

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摘　　要

　　損失準備金估計之正確與否，對於保險費率之公平性具有重要影響力。根據以往之研究發現，損失準備金可能受到保險人之人為操縱，以配合公司之財務目標。為防止造假與不適當費率，監理機關通常要求費率必須事先核准，然而冗長之審查程序往往影響費率反映損失經驗之時效性，進而部份抵銷事先核准規範之效果。
　　本文之主要目的在於提出一理論模型，以說產險費率決策過程中，由於保險人與監理機關雙方之資訊不對稱，保險人操縱準備金之道德危險與事先核准規範二者間之關係。本文之研究方法主要是以最佳控制理論 (optimal control theory) 為基礎，分析監督稽核策略對於保險人操縱造假程度之影響。本文之研究結論主要包括下列二點：(1)保險人造假程度程度愈嚴重，則愈容易受到監理單位之稽核；(2)監理審查時間長短將影響保險人之造假行為，審查拖延愈久，則造假程度愈高。因此根據研究結果，本文建議監理機關可藉由改善審查效率而抑制保險人之操縱造假行為。

Abstract

　　The accuracy of loss reserve estimation is important for determining the fair price of an insurance policy. According to the previous studies, the loss reserves are vulnerable to manipulation by the insurer in order to pursue certain financial purposes. To prevent the manipulation and inadequate rates, the insurer is required to obtain the approval of rates from the regulator. However, the regulatory review process may cause time lag between price and claim losses information and thus offset the effect of prior-approval.

　　The purpose of this paper is to provide a theoretical model to explain the relationship between the moral hazard of loss reserves manipulation and prior-approval rate regulation for property-liability insurance rating decision, with consideration of asymmetric information between the insurer and the regulator. Based on the optimal control theory, this paper develops a dynamic optimization model to analyze the manipulation behavior of the insurer. The findings of this paper imply the following two points. First, the higher level of manipulation, the higher probability for the insurer being audited. Second, the regulatory reviewing lag will induce the level of manipulation, i.e., the longer the lag, the

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higher the level of manipulation. The results of this study suggest that the regulator can reduce the level of loss reserves manipulation through the improvement of efficiency in the regulatory review process.

1. Introduction

The principle of insurance rate regulation is that the rate must be adequate, not excessive, and not unfairly discriminatory to the consumers. Although this principle looks simple and obvious, it is difficult for the consumers to judge whether the rate is fair or not due to the complexity of insurance losses information and calculation techniques. Therefore, in most of the countries the insurance premium rate is not determined based on market completition, but is set subject to certain regulatory interventions such as rating methods and prior-approval requirement.

It has been a long debate whether the regulatory intervention would increase the price level due to the delayed response of claim costs, or reduce the rates because of political pressure from the consumers and regulators. Some studies (e.g. Witt and Miller, 1981) based on the empirical analysis of the U.S. private auto insurance market suggest that regulation will lead to higher prices during the periods of downward pricing and to lower prices during the periods of rising prices. Cummins and Outrevile (1987) suggest that regulatory lag may be a cause to profit cycles in property-liability industry. On the other hand, Grabowski et al (1989) conclude that regulation decreases the unit price of automobile insurance, especially for liability coverage. Harrington (1984) finds that there is no significant difference in prices between the regulated and unregulated markets.

According to the fair rate principle, insurance premium rate is equal to the present value of expected losses. The estimation of expected losses usually must refer to the incurred losses on the financial reports. In fact, one of the most popular rating methods for property-liability insurance is the loss ratio method (see Brown, 1993), in which the information of incurred losses plays a crucial role. The incurred losses reported on the financial statement are equal to the sum of paid claims and loss reserves of this year minus the loss reserves of last year (see Troxel and Brouchie, 1990). Since a large proportion of incurred losses are loss reserves, the accuracy of loss reserves estimation has a significant impact on insurance price level. It has been an interesting topic in insurance research to evaluate the accuracy of loss reserves because the forecast of future losses opens wide space for the insurer to manipulate the losses data. Weiss (1985) and Grace (1990) conduct empirical

analyses and both conclude that loss reserve errors are not random and may be manipulated by the insurer. They suggest that the loss reserve errors are related to taxable income and income smoothing, as well as the unanticipated inflation.

The accuracy of loss reserves estimation is important not only to the premium rate but also to the solvency of the insurer. According to the study by A. M. Best's Company (1991), deficiency of loss reserves (i.e. underpricing) is the primary factor for financial distress of the insurance companies, accounted for 28 percent of the insolvent cases examined. Therefore, how to control the manipulation of loss reserves becomes the top-concerned subject by the regulator.

To prevent the manipulation of reported losses and the inadequate premium rate, the insurer is usually required to submit the losses information and schedule of premium rates to the commissioner for the approval. However, because of the information asymmetry between the regulator and the insurer, the insurer has more information and better techniques in estimating the loss reserves. The truthfulness of reported losses is not easy to judge. The regulator may monitor the insurer by way of auditing and charging a penalty in case the insurer is found manipulating the reported losses. The insurer's expected profit will of course reduce if being audited and charged a penalty, which can deter the manipulation of loss reserves. However, the auditing cost is high in practice and the regulator will not conduct the auditing for every case but only do with probability. Random sampling for auditing may induce the incentive of manipulation because the insurer may be opportunistic. Therefore, it is interesting to study the relationship between the incentive level of manipulation and the auditing probability.

In practice the prior-approval approach is popular and widely adopted in most of the regulated industries. The regulatory lag resulted from prior-approval procedure is important to pricng decision of the firm. The length of time over which the prices are subject to regulatory review and thus are fixed is a crucial factor in determining price schedules (see Laffont and Tirole, 1993). The objective of prior-approval regulation in insurance industry is to provide some barrier for misreporting of losses and inadequate premium rate, but the prior approval procedure may cause regulatory lag and delay the adjustment of premium rate to the claim costs (Cummins, 1990). That is, there is a contradictory incentive scheme in the prior-approval rate regulation.

To prevent the potential profit loss from regulatory lag, it is highly possible that the firm will take into account of the length of time lag when setting the price. For example, if the regulatory review requires six months, the firm may submit a price schedule which is a forecast for the price level of six months later instead

of next day. Under such situation, the insurer must consider more factors for forecasting and it becomes more difficult for the regulator to inspect the insurer. Therefore, the regulatory lag offsets the effect of prior-approval regulation. Due to the complicated loss settlement process of property-liability insurance, the claim costs cannot be realized at the end of accounting year (see section II for details). Even if the financial statement are audited by the certified public accountants (CPA), it is impossible to decide the accurate level of claim costs for the current year.

Gort and Wall (1988) have shown that the impact of regulatory lag on the investor's foresight may reduce the power of the regulator for the public utility industry. Several empirical studies have contributed to this subject and suggested that the regulatory lag dose have an impact on insurance price (e.g., Tennyson, 1993), but it lacks of theoretical analysis to show whether the regulatory lag will induce the incentive of manipulating loss reserves and thus affect the price level.

The primary purpose of this paper is to provide a theoretical model to explain the relationship between the incentive of loss reserves manipulation and rate regulation for property-liability insurance pricing decision, with consideration of regulatory lag and information asymmetry between the insurer and the regulator. Numerous studies of agency theory and regulation economics have pointed out that asymmetric information will induce the incentive of moral hazard (Laffont, 1994). Regulatory lag has been identified as a medium through which the commissioners may be able to affect the behavior of the firm in other industries (Atkinson and Nowell, 1994). While several studies have proved that the regulatory lag has an impact on insurance price as indicated in the above, the direct analysis of loss reserves manipulation behavior in the insurance industry has not been investigated yet.

Sine the insurer's rating decision and forecast of loss reserves usually involve the timing problem of regulatory lag as well as the probability of auditing, the model must take into account of these dynamic elements. The methodology used in this study is based on the optimal control theory which is considered as a useful technique for dynamic optimization problems and has been applied in economic research extensively (Chiang, 1992).

The structure of this paper is organized as follows. Section II describes the underlying relationship between insurance cost and premium rate. Section III introduces the basic model of insurer's pricing decision without the regulatory lag to provide a preliminary understanding of the analysis procedures. Section IV extends the pricing model with consideration of the regulatory lag. The interpretation

of the findings and concluding remarks are provided in section V.

II. Insurance Cost and Price

The rate regulation requires that the insurance price should be adequate, not excessive, and not unfairly discriminated. In principle, the price of insurance is composed of pure premium as well as expense and profit loading. Pure premium which accounts for the major proportion of insurance price is primarily based on the present value of expected claim costs. The expense and profit loading is equal to a percentage of pure premium. Since the loading is a constant percentage of pure premium, it is usually omitted in most traditional insurance literature. Therefore the insurance premium rate in theory is formulated as the discounted value of total expected claim cost.[1] Then the forecast of the expected losses and the selection of discount factor become the major concerns in insurance literature (Cummins and Harrington 1987). For example, Fairley (1979) applies CAPM theory and proposes an underwriting beta for insurance price. The forecast of expected losses involves complicated actuarial techniques as indicated in the study by Taylor (1986).

The premium rate in theory is equal to the present value of expected losses. However, since the insurer's objective is to maximize the total profit, it is highly possible the price will be deviated from the fair rate principle if without monitoring by the commissioner. As indicated in the study by Wilson (1981), the investment return is also an important factor for insurance price. The insurer may adjust the premium rates and underwriting standards to pursue the investment income or other financial purpose such as taxation. Consequently, the insurer's ratemaking decision in fact is not just based on the expected losses.

As discussed in the above, the insurance premium rate in practice is based on loss ratio method. The regulator gives approval of rate modification after reviewing the loss ratio of the insurer. Thus the insurer may manipulate the loss ratio if

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[1] Usually the (pure) premium rate P_t in theory is equal to the following equation:

$$P_t = \Sigma_{i=1}^{n} \{ \alpha_i \ E(L_t) \ / \ (1+r)^i \}$$

where, P_t is premium rate of an insurance policy at time t
  α_i is the proportion of claim cost paid at time $t+i$
  $E(L_t)$ is the expected claim cost of an insurance policy of time t
  r is the discount rate.

he intends to adjust the premium rate to meet his financial purposes. Since *premiums earned* are just premium incomes adjusted with accounting period, there is little space for manipulation. On the other hand, the *incurred losses* with large amount of loss reserves highly depend on the insurer's forecasting techniques, which are difficult to be detected by the regulator and thus may be subject to manipulation by the insurer. Therefore, this paper focuses the discussion of manipulation on the loss reserves.[2].

In order to develop a realistic pricing model which is consistent with the ratemaking decision behavior in insurance market, the action of manipulation (i.e., moral hazard) must be taken into consideration. The total cost and claim losses of an insurance policy in this paper are expressed by the following equations.

$$C = L + k \tag{1}$$

$$L = \hat{L} + \varepsilon \tag{2}$$

$$\hat{C} = \hat{L} + k \tag{3}$$

Equation (1) presents that actual total cost of an insurance policy C is equal to the claim losses L plus a fixed cost k such as the administration expense an commission. Since the actual losses are a random variable and not realized at the beginning of the year, the insurer must make a forecast of the claim losses. Let $\hat{L}$ is the rational (unbiased) estimate of L based on all the information available and the ε is white noise for forecasting error. The relationship between the actual losses and its unbiased forecast is shown by equation (2). Therefore, the expected total cost of an insurance policy at the beginning of the year $\hat{C}$ is equal to $\hat{L}+k$ as shown by equation (3) since the expense is assumed a fixed cost and not involved with forecasting.

Equation (4) shows the action of manipulation. The reported incurred losses $\bar{L}$ is equal to the unbiased forecast of claim losses plus an adjustment $X(\theta)$ due

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[2] The "loss reserves" in this paper are referred to total loss reserves which include bulk reserves and case reserves, The "incurred losses" on the financial statement for year t is equal to total loss payments during year t plus the loss reserves for year t minus loss reserves for year t-1. The definitions of terminologies and the details of property-liability insurance accounting procedures are provided in the appendix 1.

to the manipulation action θ[3]. Equation (5) means that insurer's price is a function of reported incurred losses, expense and profit loading e, and investment return r in the market, while the reported incurred losses can be decomposed of unbiased forecast of losses and the manipulation parameter.

$$\overline{L} = \hat{L} + X(\theta) \tag{4}$$

$$P = (\overline{L}, \ e, \ r)$$

$$= P(\hat{L}, \ e, \ r, \ \theta) \tag{5}$$

We consider that $\partial P/\partial \hat{L} > 0$, $\partial P/\partial e > 0$, $\partial P/\partial r < 0$, and $\partial P/\partial \theta > $ or < 0. Price usually is an increasing function of claim losses and expenses, while it is a decreasing function of investment return because the insurer may reduce price and underwriting standard to attract premiums written for the investment funds. The effect of moral hazard parameter θ on the price level may be positive or negative depending on the purpose of ratemaking. The insurer may intentionally raise the price to have excessive premium income, or reduce the premium rate to pursue the investment income.

According to the previous studies (e.g. Grace 1990), the insurer may have intention to manipulate the loss reserves information on accounting reports due to some financial purposes. Since the loss reserves are one of the most important factor in estimating the claim cost, their manipulation will have an impact on price and profit. The insurer may lower down the loss reserves to satisfy the regulatory requirements,[4] or reduce price level to attract the investment funds, but sometimes the loss reserves are overstated to have more conservative under-

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[3] The exact definition of θ is the parameter of moral hazard or manipulation action as used in the agency theory literature, e.g., Barron and Besanko (1984). The action of moral hazard itself usually cannot be observed directly, but the observation of moral hazard may be done indirectly through auditing the financial data of the firm. That is, the amount of manipulated losses $X(\theta)$ is observable which is a function of θ. However, such distinction is omitted in the following discussion of this paper because it is redundant and makes no additional contribution. In the following discussion, we will simply use the parameter θ for either the action or the amount of manipulation $X(\theta)$, depending on the context.

[4] For example, the NAIC early warning system requests that the one-year reserve development to prior year's surplus and the two-year reserve development to second prior year's surplus must be less than 25% (see Troxel and Bouchie, 1990).

writing performance.[5] No matter understatement or overstatement of the loss reserves, we consider the purpose of manipulation is to increase the expected profit.

The expected total profit of the insurer with manipulation on reported losses, $\pi_I(\theta)$, is shown as equation (6).

$$\pi_I(\theta) = \{P(\hat{L}, e, r, \theta) - \hat{C}\} + I_I(r, \theta)$$

$$= \{P_I(\theta) - \hat{C}\} + I_I(\theta) \tag{6}$$

The bracket in equation (6) represents the expected underwriting income which is equal to premiums minus the expected total claim cost $\hat{C}$. The second term $I_I(r, \theta)$ is the expected investment income. The investment return r is of course a factor for expected investment income. Since the sources of investment come from the premiums written at manipulated price, it is also related to θ because the insurer earns extra profit from manipulating the loss reserves. Assumed all the other variables $\hat{L}$, e, and r are given and omitted in the following discussion, then the profit equation is revised as function of one variable θ. Since the purpose of misreporting is to increase the total profit, the more effort of manipulation made by the insurer is expected to generate the higher expected total profit, but with a decreasing rate. Provided all the other variables $\hat{L}$, e, and r are given and omitted in the equation, then the profit function is assumed to be a strictly concave function of moral hazard parameter. That is, $\pi_I'(\theta) > 0$, $\pi_I''(\theta) < 0$.[6]

The moral hazard parameter is only known to the insurer. It is not observable directly by the regulator. However, the regulator may observe the misreporting of losses by way of auditing. The profit is reduced if the regulator makes auditing on the financial statements and charges a penalty for misreporting. Under such

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[5] Due to the characteristics of underwriting cycles in property-liability insurance industry, the insurer may become more coonservative for their underwriting standards during the years of increasing underwriting losses (see Cummins and Outreville, 1987).

[6] The model developed in this paper is emphasized on the total expected profit which is the sum of the underwriting profit and the investment profit. Although the reduced premium rate may result in lower underwriting profit, it is assumed that the reduction in underwriting profit will be compensated by the increase of extra investment profit. That is, we assumed the insurer is rational and he will not reduce the premium rate if investment profit is not good enough to cover the lost underwriting profit. Therefore, the manipulation will always make the expected total profit higher if without auditing. The decreasing rate for the increment of expected total profit is simply a common mathematic assumption for utility function as applied in most of the economic literature.

situation the insurer must set price at the level of fair rate. The expected profit based on fair rate is not a function of the moral hazard parameter θ, indicated by the following equation.[7]

$$\pi_2 = \{P(\hat{L}, e, r,) - \hat{C}\} + I_2(r)$$

$$= \{P_2 - \hat{C}\} + I_2 \tag{7}$$

The difference between π_1 *and* π_2 reflects the potential extra profit from manipulation. The extra profit $(\pi_1 - \pi_2)$ will equal to $(P_1 - P_2)$ plus $(I_1 - I_2)$. Besides, the insurer will be charged a penalty if being audited. According to the study by Baron and Besanko (1984), the optimal solution of penalty to induce true reporting is a flat amount of maximum value $\overline{N}$, $0 < \overline{N} < \infty$.

Finally, it is assumed that the insurer is risk neutral and sets the price with objective to maximize the expected total profits.

III. The Basic Model

According to the description of previous section, the objective of ratemaking decision of the insurer is to maximize the expected total profit. The insurer believes that there are two possible levels of expected total profit: (1) profit level if not being audited (π_1), and (2) profit level if being audited (π_2), and a penalty $\overline{N}$. The profit function π_1 is assumed to be a strictly concave function of moral hazard parameter θ. That is, $\pi_1'(\theta) > 0$, $\pi_1''(\theta) < 0$, as indicated in the previous section. The expected maximum profit with auditing, π_2, is to be independent of θ since it is equal to underwriting income and investment income but without extra profit from manipulation. It is obvious that $\pi_2 < \pi_1$.

Since insurance price involves complicated calculation and it is impossible for the consumers to monitor the insurer by themselves, the regulatory surveillance is always required. However, the regulator will not audit every insurer because of the auditing cost. Whether (or when) the regulator will conduct the auditing is not known to the insurer at the moment of ratemaking. Thus, the insurer may take chance of not being audited and charge a biased rate in order to earn the extra profits.

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[7] The investment income is replaced with I_2 because the sales volume based on fair-rate price may be different from the previous case and the sources for investment (premiums written) will be rearranged. Consequently the investment income may be different.

Let the probability for the insurer being audited by time t be $F(t)$, with $F(0) = 0$, $F(t) \leq 1$. The conditional probability density of auditing at time t, given that there is no occurrence of auditing prior to time t, is $F'(t)/[1-F(t)]$. It is assumed that the conditional probability density of auditing is an increasing, convex function of θ. This assumption is to reflect the situation in practice that the probability of auditing will increase as the level of manipulation rises.[8] Let $\theta(t)$ denote the level of moral hazard (manipulation) at time t which is assumed a piecewise continuous function of time, and define $g(\theta(t)) = F'(t)/[1-F(t)]$, then $g(0) = 0$, $g'(\theta) \geq 0$, $g''(\theta) \geq 0$.

Because the property-liability insurance claim settlement is very lengthy, especially for the long-tail lines, it may take several years to finish a claim. Therefore, the insurer's ratemaking decision usually must take into account of time value, that is, the decision is made based on the present value of expected profit. Supposed the instantaneous force of interest rate is δ, then the discount factor for the case of continuous time is $e^{-\delta t}$ (see Kellison, 1991). Therefore, the objective of ratemaking decision of the insurer is to maximize the present value of expected profit subject to the probability constraint of being audited. We express this problem by an optimal control model as indicated by the following equations (see Kamien and Schwartz, 1981).

$$\max_{F, \theta} \quad \int_0^T e^{-\delta t} \{ \pi_1(\theta(t))[1-F(t)] + \pi_2 F(t) - \overline{N} F(t) \} dt \qquad (8)$$

$$\text{s.t.} \quad F'(t) = g(\theta(t))[1-F(t)], \ 0 \leq t \leq T \qquad (9)$$

$$F(0) = 0, \ F(T) \text{ is free within } [0,1] \qquad (10)$$

T in the model is the maximum time point to finish claim settlement of the insurance policy. The value of T in practice varies with insurance lines. It can be as short as one year, or can be very lengthy for more than fifteen years. The equation (8) shows the present value of expected profit at time t, which composes of π_1 if no auditing conducted by t, as well as π_2 and $\overline{N}$ if with the regulatory

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[8] Although the moral hazard parameter itself is not observable, the regulator can observe the price level $(p(\theta))$ and losses information. Based on the fluctuations of price level, the regulator will decide to audit the insurer or not. Because of the underwriting cycles in property-liability insurance industry, manipulation may raise or reduce the premium rates, i.e., $\delta P/\delta \theta$ may > 0 or < 0 from time to time, but its purpose is always to maximize the profits. It is impossible to express an increasing convex function based on cyclic price p. Thus we express the density function on θ.

intervention. The state variable in this problem is the auditing probability $F(t)$, and the control variable is $\theta(t)$. Thus the insurer's target is to choose an optimal $\theta^*(t)$ to maximize his expected total profit. Based on the above equations, the current-value Hamiltonian H_c is shown as follows (see Chiang, 1992).

$$H_c = \pi_1(\theta(t))[1-F(t)] + \pi_2 F(t) - \overline{N}F(t) + \lambda(t)g(\theta(t))[1-F(t)] \qquad (11)$$

where, $\lambda(t)$ is the current-value Hamiltonian multiplier.

The typical procedures to find the optimal solution for an optimal control model are taking the first order conditions of H_c with respect to the control variable, the state variable, and the multiplier. That is,

$$\partial H_c/\partial\theta = 0 \qquad \longrightarrow \qquad \pi_1{}'(\theta)[1-F(t)] + \lambda g'(\theta)[1-F(t)] = 0 \qquad (12)$$

$$\partial H_c/\partial F = -\lambda' + \delta\lambda \qquad \longrightarrow \qquad \lambda' = \pi_1 - \pi_2 + \overline{N} + \lambda[g(\theta) + \delta] \qquad (13)$$

$$\partial H_c/\partial\lambda = F' \qquad \longrightarrow \qquad F'(t) = g(\theta(t))[1-F(t)] \qquad (14)$$

According to Arrow's Theorem (Kamien and Schwartz, 1981), if F^* and θ^* are optimal solutions, then F^*, θ^* and λ^* must satisfy the constraints (9) and (10) and also the conditions (12)-(14). Therefore a solution to the necessary conditions (12)-(14) is also a solution to the optimization model of (8) and (9). It can be shown that the necessary conditions can be satisfied by constant values of θ^* and λ^* based on the study by Kamien and Schwartz (1971). They suggest that there is a unique solution θ^* which is some appropriate constant that satisfies $\pi_1(\theta^*) > \pi_2$. Since the differentiation of a constant is equal to zero, the constant $\lambda^*(t)$ satisfying equation (13) is as follows.

$$\lambda^*(t) = -(\pi_1(\theta^*) - \pi_2 + \overline{N}) / [g(\theta^*) + \delta] \qquad (15)$$

Because the multiplier equation $\lambda(t)$ is the marginal valuation of the associated state variable at time t, it measures the influence of the state variable F on the optimal value of the objective function. As indicated in the above, π_1 is greater than π_2 and $\overline{N}$ is a positive amount, consequently λ^* is negative in equation (15). The interpretation of the negativity of λ^* is interesting because it reflects the fact that an increment in the state variable F (auditing probability) will reduce the optimal expected value of profit. The higher the probability of being audited, the lower the expectation of profit for the insurer since he has less chance to earn the extra

profit through manipulation.

Since the value of the control variable $\theta*$ that maximizes the current Hamiltonian H_c satisfies equation (12), it will also satisfy the following equation (see appendix 2 for proof).

$$\pi_1'(\theta*) \; g(\theta*) \; + \; \pi'_1(\theta*) \; \delta \; = \; [\pi_1(\theta*) \; - \; \pi_2 \; + \; \overline{N}] \; g'(\theta*) \tag{16}$$

The equation (16) shows that under the optimal situation, the changes in the expected profit from insurer's manipulation in loss reserves are related to the penalty charged by the regulator $\overline{N}$, profit difference $\pi_1 - \pi_2$, and the conditional probability of auditing. The left-hand side of equation (16) is the expected marginal profit with interest incomes, while the right-hand side is the expected profit difference and regulatory penalty times the marginal probability density of being audited. Therefore, the optimal solution may be concluded with the situation that the expected marginal benefits through manipulation is equal to the marginal value of expected profit difference and regulatory penalty.

Furthermore, the optimal function of auditing probability $F*(t)$ may be found through integrating equation (9) by setting $\theta(t)=\theta*$, which is shown as follows (see appendix 3 for proof).

$$F*(t) \; = \; 1 \; - \; e^{-g(\theta*)t} \tag{17}$$

Equation (17) explains that the probability of being audited is positively related to the level of manipulation. The higher the level of moral hazard, the larger the chance of being audited. Since $g(\theta)$ is an increasing convex function of θ, it will become extreme large as $\theta*$ increases. Consequently, the $e^{-g(\theta*)t}$ will approach to zero and F will be close to one. The interpretation of this result for the insurance industry is that the highly manipulated financial reports are more possible to be audited and cause regulatory intervention than the slightly misreported cases. This result is reasonable in practice. Due to the consideration of auditing costs and impact on insurance price, the regulator usually concerns more on the severe manipulation and starts auditing from these insurers.

IV. Ratemaking Decision with Regulatory Lag

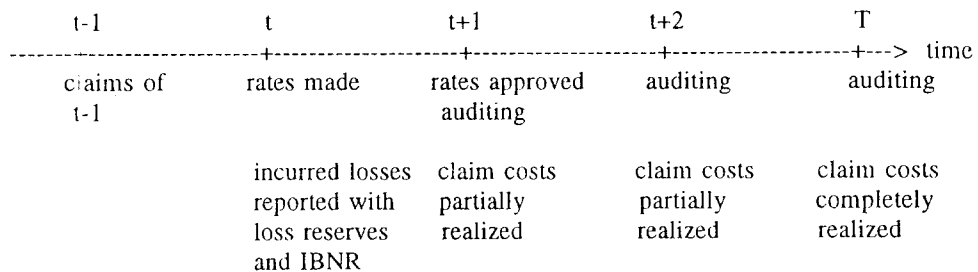The model developed in the previous section shows that the probability to

conduct auditing will be related to the current financial reports, which implies that the commissioner can investigate premium rates immediately. However, it may be more realistic that the regulators's intervention usually will be somewhat delayed. The regulator may be not able to conduct auditing immediately because the review of premium rates takes a long time. Therefore, there is delayed response between the regulatory intervention and the insurer's ratemaking decision.

Figure 1 illustrates that concept of time lag for insurance rate making process. Due to the lengthy claim settlement process of the property-liability insurance, the insurer must make the rates based on the estimated claim costs, i.e. incurred losses, which include loss reserves and loss payments (see appendix 1 for the details). Therefore, the insurer may manipulate the claim cost information. At the moment of ratemaking, the commissioner may not be able to audit the insurer immediately because he lacks of the actual claim cost information. The regulator may take auditing at some future time point since the claim cost information related to ratemaking will be revealed from time to time. When the insurer sets up price for the insurance policy of current period t, he may take into account the probability of being audited in the future periods. Thus the model developed in the previous section must be revised with consideration of regulatory lag.

Figure 1.
A Simplified Time Line for Property-Liability Ratemaking Process

t-1	t	t+1	t+2	T
----------+----------------+-------------------+-----------------+---> time
claims of	rates made	rates approved	auditing	auditing
t-1		auditing		
	incurred losses	claim costs	claim costs	claim costs
	reported with	partially	partially	completely
	loss reserves	realized	realized	realized
	and IBNR			

Provided the basic assumptions and notations are the same as before, the revised model is presented as follows (see Kamien and Schwartz, 1981).

$$\max_{F,\ \theta} \quad \int_0^T e^{-\delta t}\{\pi_1(\theta(t))[1-F(t)] + \pi_2 F(t) - \overline{N}F(t)\}dt \qquad (18)$$

$$\text{s.t.} \quad F'(t) = g(\theta(t-\tau))[1-F(t)], \qquad \tau \leq t \leq T \qquad (19)$$

$$\theta(t) = 0, \qquad\qquad\qquad -\tau \leq t < 0 \qquad (20)$$

$$F(t) = 0, \ F(T) \text{ is free within } [0,1], \ -\tau \leq t < 0 \qquad (21)$$

The major revision made in the model is the timing of auditing as presented by the equation (19). Equation (19) indicates that the manipulation of financial reports made at time t will have an impact on the probability of auditing at time $t+\tau$. τ denotes the regulatory lag which is a difference in time period between ratemaking and auditing.

Based on the above model, the current-value Hamiltonian H_c is revised as the following equation.

$$H_c = \pi_1(\theta(t))[1-F(t)] + \pi_2 F(t) - \overline{N}F(t) + \lambda(t)g(\theta(t-\tau))[1-F(t)] \qquad (22)$$

The optimal solution which incorporates the effect of the delayed response must satisfy the following necessary conditions.

$$\partial H_c/\partial\theta_t + \partial H_c/\partial\theta_{t-\tau}\Big|_{t+\tau} = 0, \qquad 0 \leq t \leq T\text{-}\tau \qquad (23)$$

$$\lambda' = -\partial H_c/\partial F_t - \partial H_c/\partial F_{t-\tau}\Big|_{t+\tau} + \delta\lambda, \qquad 0 \leq t \leq T\text{-}\tau \qquad (24)$$

Equation (23) describes that the impact of manipulation by the insurer is partially realized at current time t, and partially at later time $t+\tau$ by way of the lagged effect. By the same reasoning for equation (24), the marginal impact on the multiplier is attributed partially to the auditing probability of the current period as well as to the probability of being audited in the future period.

According to the formulas of (23) and (24), it can be shown that the necessary conditions for the optimal solution of our model are:

$$\pi_1'(\theta(t))[1-F(t)] + \lambda(t+\tau)g'(\theta(t))[1-F(t+\tau)] = 0 \qquad (25)$$

$$\lambda' = \pi_1(\theta(t)) - \pi_2 + \overline{N} + \lambda(t)[g(\theta(t-\tau)) + \delta] \qquad (26)$$

By the same procedures as those in the previous section, we know that

there exist some appropriate constant values of $\theta*$ and $\lambda*$ for the optimal solution. Therefore, the following results are obtained.

$$\lambda* = -(\pi_1(\theta*) - \pi_2 + \overline{N}) / [g(\theta*) + \delta] \tag{27}$$

$$\pi_1{}'(\theta*)g(\theta*) + \pi_1{}'(\theta*)\delta = [\pi_1(\theta*) - \pi_2 + \overline{N}]g'(\theta*)\, e^{-g(\theta*)\tau} \tag{28}$$

$$F*(t) = 1 - e^{-g(\theta*)(t-\tau)}, \qquad t \geq \tau \tag{29}$$

The multiplier equation (27) which represents the marginal contribution of state variable to the optimal control program is the same as before. That is, the impact of auditing probability function F itself on the profit expectation is not changed with the lagged intervention.

On the other hand, although the marginal expected profit with the insurer's moral hazard is still proportional to the profit difference and penalty charged by the regulator, the effect is smaller than the case without regulatory lag. By comparing equation (28) with equation (16), we find that there is an additional term, $e^{-g(\theta*)\tau}$, included in equation (28). Since $e^{-g(\theta*)\tau}$ is less than one, the value $\pi_1{}'(\theta)$ will be smaller under the case of lagged regulatory intervention. The result is interesting because it shows the effect of penalty charged will be diluted when the regulatory intervention cannot be performed immediately.

The auditing probability $F*(t)$ is also reduced by the lagged response because the value of the exponential is increased by $e^{g(\theta*)\tau}$. That is, the probability of regulatory intervention or being penalized will be reduced since it is difficult to detect the manipulation by the insurer when the correspondence between price and claim cost information is not straightforward.

Furthermore, the regulatory lag may induce an incentive to increase the level of moral hazard. Based on equation (29), the conditional density of auditing can be derived as follows.

$$g(\theta*) = -ln(1-F*(t)) / (t-\tau), \tag{30}$$

Equation (30) shows that $g(\theta*)$ will increase when τ increases because the denominator becomes smaller and numerator is a positive number. As indicated in the previous section, the density function $g(\theta)$ is an increasing convex function of θ, therefore, the higher $g(\theta*)$ implies that there exists a higher level of $\theta*$. The result is reasonable in practice because the insurer will increase the level of manipulation once he knows that the financial data do not immediately respond

the cost of claims and the probability of being audited by the regulator is lower than before.

IV. Conclusion

The accuracy of loss reserves has been a controversial topic in property-liability industry. Due to the lengthy claim settlement process, estimation of incurred losses is always vulnerable to forecast errors and open the space for manipulation by the insurer. The incurred losses are the basis for calculating insurance premium rates and thus the accuracy of loss reserves is crucial to the consumers. To maintain the solvency of the insurer and protect the policyholders, the insurance commissioner must monitor the insurance industry. Rate regulation is one of the most important tools to maintain the fair transaction in the insurance market. On the other hand, the regulatory intervention itself may offset the effect because the lengthy regulatory process may delay the responses to the market conditions.

The purpose of this paper is to provide a theoretical analysis for the effect of the regulatory intervention on the insurer's behavior of manipulation on the loss reserves. Due to the constraint of auditing cost, the regulator cannot examine each insurer and must conduct the auditing with probability. The result of this study shows that the probability for the insurer being audited will increase with the level of manipulation, but the time lag of regulatory intervention process may provide additional opportunity for manipulation and reduce the effect of auditing because the insurer may take advantages on the delayed response between price and claim costs.

The finding of this study implies that the regulatory review process must be more efficient by shortening the time lag to make the insurance premium rate less manipulated. However, completely eliminating the prior-approval rate regulation may be not a good alternative for the commissioner. Due to the information asymmetry between the insurer and insured, it may result in unfair transaction without regulatory intervention because the consumers cannot judge whether the insurance price is adequate or not. Thus to some extent the regulatory review of premium rates is still a necessary procedure to keep the insurance market well performed. By way of improving the efficiency of the regulatory review, the regulator can reduce the level of manipulation. Maintenance of the surveillance authority itself to some extent may deter the insurer from manipulation of premium rates.

Incentive of Loss Reserves Manipulation and Rate Regulation in
Property-Liability Insurance Industry

Appendix 1.

Because the loss settlement process in property-liability industry is usually very lengthy, the claim costs for the policies of year t may not be paid completely during year t. That is, an insurance claim may involve several accounting years. On the other hand, the insurer must submit the financial statements to the commissioner every year. In order to report the cost of insurance operation. The insurer must estimate the total costs of incurred claim which are the sum of loss payments (i.e. paid claim costs) and total loss reserves (i.e. unpaid claim costs).

The total loss reserves for an insurer are composed of the case reserves and the bulk reserves. The case reserves are the loss reserves for the incurred and reported claims. The bulk reserves represents the estimated costs for the incurred but not reported (IBNR) claims, adjustment for the future case reserves, and the reserves for the closed claims but reopened. The bulk reserves involves even more uncertainty than the case reserves and are difficult to forecast accurately.

The incurred losses (*IL*) are the reported claim costs on the financial statement. Based on the accounting procedures, the incurred losses of year t is equal to loss payments (*LP*) and total loss reserves (*LR*) of current year t minus the total loss reserves of the previous year t-1 as shown by equation (A1). Since the total loss reserves contain case reserves and bulk reserves, they definitely involve the estimation problems.

$$IL_t = LP_t + LR_t - LR_{t-1} \qquad (A1)$$

The adjustment of premium rates is usually based on the loss ratio method. The loss ratio (ρ) is equal to the incurred losses divided by the premiums earned (PE) as shown by equation (A2).

$$p_t = IL_t \ / \ PE_t \qquad (A2)$$

Premium earned is simply the premium written adjusted for the account period, and does not involve estimation problems. For example, the accounting year is from Jan. 1 to Dec. 31, 1996, and the insured buys an insurance policy for one year coverage and pays the total premiums $1000 on May 1, 1996. Then at Dec. 31, 1996, the insurer has the premium earned for the period May 1 — Dec. 31, 1996. That is, $1000*(8/12) = \$667$.

Appendix 2.

Since the value of the control variable θ that maximizes the Hamiltonian H_c satisfies equation (12) in the text, that is,

$$\partial H_c / \partial \theta = \pi_1{'}(\theta)[1\text{-}F] + \lambda g{'}(\theta)[1\text{-}F]$$
$$= \{\pi_1{'}(\theta) + \lambda g{'}(\theta)\}[1\text{-}F]$$
$$= 0$$

Unless time $t = \infty$, otherwise the probability F will less than one and $[1\text{-}F] \neq 0$. Therefore,

$$\pi_1{'}(\theta) + \lambda g{'}(\theta) = 0 \tag{A3}$$

By substituting λ in equation (A3) with equation (15) in the text, we obtain

$$\pi_1{'}(\theta) + [-(\pi_1(\theta) - \pi_2 + \overline{N}) / (g(\theta) + \delta)]g{'}(\theta) = 0 \tag{A4}$$

Therefore,

$$\pi_1{'}(\theta) = [\pi_1(\theta) - \pi_2 + \overline{N}]g{'}(\theta) / [g(\theta) + \delta] \tag{A5}$$

$$\pi_1{'}(\theta)[g(\theta) + \delta] = [\pi_1(\theta) - \pi_2 + \overline{N}]g{'}(\theta) \tag{A6}$$

and

$$\pi_1{'}(\theta)g(\theta) + \pi_1{'}(\theta)\delta = [\pi_1(\theta) - \pi_2 + \overline{N}]g{'}(\theta) \tag{A7}$$

Appendix 3.

Based on the equation (9) in the text, we know that

$$F'(t) \ / \ [1-F(t)] \ = \ g(\theta(t)) \tag{A8}$$

Provided $\theta(t) = \theta^*$, we can obtain $F(t)$ by taking integration of the above equation as follows.

$$\int F'(t) \ / \ [1-F(t)] \, dt \ = \ \int g(\theta^*) \, dt \tag{A9}$$

$$\ln \ [1-F(t)] \ = \ -g(\theta^*)t \tag{A10}$$

Therefore, by taking exponential for both sides of the equation (A10), we have:

$$[1-F(t)] \ = \ \exp[-g(\theta^*)t] \tag{A11}$$

and

$$F(t) \ = \ 1 \ - \ \exp[-g(\theta^*)t] \tag{A12}$$

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