CHILDREN'S LEARNING GEOMETRIC CONCEPTS: LOGO AS A LEARNING ENVIRONMENT

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摘　要

孩童自幼即生活在幾何的世界中，幾何概念實爲他們生活中的一部份，例如許多小孩的玩具，如積木、球、盒子等均與幾何圖形有關。因此，如何利用小孩現有的生活環境，來幫助他們學習幾何概念實具有相當重要性。主張兒童學習抽象概念，特別是數學概念之前，一定要經過具體物感官經驗時期的皮雅傑（Jean Piaget）教學理論也因此日益受到重視。

LOGO 是一種根據皮雅傑的學習理論而發展出來的電腦程式語言，其目的在提供學童一種主動學習的環境，使他們能掌握他們自己的學習，並協助教師以一種新的方式來觀察學習過程。

本研究的目地在於，利用皮雅傑的教學理論以探討何以LOGO是學童幾何概念認知的一種理想環境，所欲探討的問題爲：1）從皮雅傑的觀點來說，小孩的幾何概念是如何形成？2）LOGO 究竟對學童幾何認知，提供了何種學習環境？3）LOGO能否作爲有效的學習工具？4）LOGO 學習環境對教師角色的影響如何？

Abstract

Children view their world through geometry. Geometry is an integral part of their everyday life. Many of their toys, i.e. blocks, balls and boxes teach them about geometric concepts. Mathematical educators have always sought ways of making the best use of a child's living environment to teach him/her geometric concepts. Piaget's theory of teaching children mathematical abstractions using concrete materials from the world around them is currently receiving much attention.

Logo is designed on the basis of Piaget's theory of learning to provide children with a learning environment that would enable them to control their own learning and would help teachers look at learning in new ways.

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The purpose of this study is to examine the educational rationale of Logo learning in geometry using Piaget's educational theory. The general questions which will be explored in this study are: 1) From Piaget's point of view what are children's conceptions of geometry? 2) What kind of learning environment does Logo provide for children's learning of geometry? 3) Can Logo be used as an efficient learning tool? 4) How does a Logo learning environment effect a teacher's role?

I. INTRODUCTION

Geometry is denoted as the mathematical "study of space and shapes in space" (Heddens, 1984, p. 461). Geometric concepts are an integral part of everyday life and currently geometry has taken on increased importance in the education of young children. Children live in a geometric world. Even in their everyday life, they have many experiences with geometry. As an example of this, we note that many of their toys are geometric, i.e. blocks, balls, boxes, and other items which are a combination of shapes. Due to this geometric environment, geometry should be emphasized at every grade level, not only taught as a discipline in the high school. Geometry has great potential for the elementary and secondary mathematics curriculum. However, the issue of great concern is that the majority of students are either overly anxious about mathematics, alienated from it, or simply bored by it. For the most part, the reason that children in schools are rarely encouraged to become actively involved in mathematical activity is because it is not expected to demand their thought or creativity. The children view it as only a necessary evil, it is not presented in an interesting or in depth manner.

The primary goal of geometry instruction should be have student develop useful intuition and knowledge about shape and their relationships. Secondly, students need to abstract or take notice of shapes from their environment; they need to analyze these shapes to learn what makes the shapes useful and then they need to apply what they have learned about shape to solving real-world problems (Kerr, 1979, p. 14). A simple example children must determine how best to fit the square couch through the rectangular door.

The teaching of geometry should be presented in a way that allows children to experience concrete materials from the world around them and allows them to actively explore these things in their own control. Academician A. D. Alexandrov, an outstanding Soviet geometrician, has this to say about geometry and its goals for teaching: "Geometry, in its essence, is such a combination of vivid imagination and strict logic at which they mutually organize and guide each other . . . The task of geometry teaching is to develop pupils' three corresponding qualities, that is spatial imagination, practical comprehension and logical thinking'' (Chernysheva et al., 1986, p. 102).

The introduction of geometry into school mathematics education in conformance with Piaget's learning theory is currently receiving much attention because it provides mathematics educators with new insights into how children learn. It is suggested that some rational way of structuring and guiding experiences which facilitate children in constructing, in a qualitative manner, an understanding of the logical meaning of spatial operations is necessary and important to revive school geometry (Rahim & Sawada, 1986, p. 236).

Logo is designed on the basis of Piaget's theory of learning to provide children with a learning environment that would enable them to control their own learning and would help teachers look at learning in new ways. The develoment of Logo, to a certain extent, has paralleled the recent development of Piagetian theory. The Logo environment makes it possible for children to experiment and obtain intimate contact with concepts from subject areas like science and mathematics and allows children to play with their imagination and creativity. Therefore, some researchers suggest introduce children to Logo as early as their early years when they are beginning to formulate an understanding about the world and develop ideas (Vaidya & McKeeby, 1985, pp. 81-82). For example, Johnson and Swoope believe: ''Children who have access to computers from an early age are likely to develop skills and attitudes that will give them a distinct advantage over youngsters who lack this experience'' (Johnson & Swoope, 1987, p. 14).

The purpose of this paper is to examine the educational rationale of Logo learning environment in geometry learning in light of Piaget's educational theory. The general questions which will be explored in this paper are: (1) What are children's conceptions of geometry from Piaget's view? (2) What kind of learning environment does Logo provide for children's learning geometry? (3) Can Logo be used as an efficient learning tool? (4) What are the implications of Logo environment on the teacher's role?

The premise of this paper is that meaningful learning in mathematics occurs when children link their understanding with the symbols and procedures they are taught. In this context, Logo has the potential for being a powerful tool to facilitate children's learning of geometric concepts.

II. THE DEVELOPMENT OF CHILDREN'S GEOMETRIC CONCEPTS FROM PIAGET'S VIEW

Geometry is a mathematical science, dealing with position or location in space. Namely, the study of the spatial properties of a variety of figures excerpted from the concrete world of physical objects. And the study of the concept of space, or of the ideas involved in the concept of space, is an indispensable part of child psychology. "If the development of various aspects of child thought can tell us anything about the mechanism of intelligence and the nature of human thought in general, then the problem of space must surely rank as of the highest importance," said Piaget (Piaget & Inhelder, 1967, p. vii). Among many different kinds of geometries, in general, topology, projective geometry, and Euclidean geometry are closely related to children's experiences (Copeland, 1974, p. 209). In topological space, figures are not thought to be rigid or fixed in shape. To the contrast, topology is a sort of "rubber sheet" geometry, in which openness-closedness and boundary are kept constant and distance, angular or straightness are meaningless concepts. Therefore, simple closed figures, such as triangles, squares, and circles, are equivalent topologically because they can be squeezed or transformed to form each other. In topological spatial conceptualizations, there is also no global coordination of space. Projective conceptualizations take account, not only of topological properties, but also of the shapes of figures and their relative positions. Projective space begins psychologically at the point when the object or idea such as a straight line is no longer viewed in isolation, but in relation to how it looks from a special 'point of view'. Euclidean conceptualizations have the same properties as do projective concepts, plus a property of distance/size. To the contrast of topological space, in Euclidean space, figures are considered to be rigid in shape. Curved figures are first distinguished from straight-edged figures (Piaget & Inhelder, 1967, pp. xi, 153, 467; Cohen, 1987, p. 72; Copeland, 1974, pp. 210-215, 345).

Although a variety of approaches have been conducted in relation to the development of children's geometric concepts, such as those by Weir (1987), Gagne (see Forman & Sigel, 1979, pp. 39-46), Van Hiele (see Wirszup, 1976, pp. 77-84), Kuchemann (1980, pp. 12-13), etc., the study in this section will adopt a Piagetian point of view to examine two basic issues. They are: (1) What is the developmental sequence of the child's geometric concepts? (2) How the child develops geometric concepts from his environments? The main reason to study from Piaget's view is that his theory, more than any other, is to date increasingly being used to support the discovery approach of geometric learning. According to Piaget, children develop their knowledge and concepts of space because they "(1) learn ways to represent spatial relations and consequently (2) interrelate more aspects of space" (Forman & Sigel, 1979, pp. 133-134).

(A) The Developmental Sequence of Children's Geometric Concepts

The development of geometry has traditionally followed the order of the historical sequence, that is, from Euclidean, then projective, to topological geometry. Historically the earliest geometric operations were developed to deal with practical problems of land measurement with the emphasis on the concepts of similarity, congruence, and proportion; hence had a Euclidean character, after a long process of developing these concepts. the child arrives at the more general, abstract, and non-metrical concept found in projective geometry and finally in topology (Dodwell, 1971, p. 179). The present practice reflects that the introduction of geometry to children in school is to begin with Euclidean geometry, a study of figures, such as line segments, triangles, squares, and circles, that might be referred to as 'rigid' in shape. This is how geometry developed, historically speaking.

In sharp contrast to the historical approach, Piaget and Inhelder contend that actually children's spatial concepts develop in reverse order to the traditional approach. Piaget claims that mathematically and psychologically, in the development of children's spatial concepts, topological concepts develop prior to projective and Euclidean geometry concepts. From the point of view of mathematical construction, "a projective correspondence or homology may be considered as a topological homeomorphism having the added property of conserving straight lines and certain quantitative relationships. As for general metric relationships, these derive directly from topological relationships with euclidean metrics considered as a particular case," said Piaget (Piaget & Inhelder, 1967, p. 301). From the psychological view, the ideas of the straight line and basic projective relations derive from topological concepts incorporated in a system of viewpoints. Similarily, the child's euclidean concepts of distance and measurement come from these same topological ideas (Piaget & Inhelder, 1967, p. 301).

Piaget asserts that the evolution of children's spatial concepts proceeds at two distinct levels: the perceptual level (as learned through the sense of touch and seeing) and the conceptual level (Piaget & Inhelder, 1967, p. 3). There is an interval of several years between perceptual and conceptual construction of space. The later does not develop logically from the former as most people might assume, but each develops along its own path. Children's perceptual space has reached projective or quasi-metric levels during the early years of life, when their conceptual space has barely begun. The ignorance of this fact, in Piaget's view, results in the misconception that space concepts begin with simple Euclidean rather than topological characteristics. This misconception is based on the assumption that like adults, children's knowledge and concept of space have developed through their sensory experiences of touching and seeing objects into a meaningful idea on the basis of their sensory perceptions. Piaget found that this assumption was incorrect. He argued that in fact 'shape' is not constant or fixed as perceived by the young child and that there are still great differences which exist between the young child and the adult as regards size constant (Copeland, pp. 238-239).

In their attempt to illustrate the development of perceptual space as seen in children's perception of shape, Piaget and Inhelder divided the development of the child's perception of shapes into three stages. During Stage I (0 — 4 years), known as the sensory-motor level, only the shapes of simple topological relations, such as openness or closure, proximity and separation, surrounding, etc., aıe recognized and draw by young children. At Stage II (4 — 6 years), Euclidean and projective ideas, such as line, angle, circle, square, and other simple figures, begin to emerge, but they are still overshadowed by topological concepts. At this level, the distinction between straight and curved lines, angles of different sizes, parallels, and relations between equal or unequal sides of figures comes into existence. Finally, at Stage III (6 — 7 years) the child is able to return systematically to a fixed point of reference while exploring an object (Smock, 1976, p. 47). It is clear that children's spatial conceptions take shape through a developmental stage sequence. At first it is based on topological relationships, and later on projective and Euclidean relations.

As to the development of conceptual spaces, it was clearly illustrated by asking the child to draw geometrical figures. During Stage I, the child has difficulty to drawing the rigid shapes. Hence, the circle is drawn as an irregular closed curve, squares and triangles are not distinguished from circles, and the drawing is of two more or less intersecting lines. These are the topological properties. The child has preserved the topological properties of closure, order, and proximity. Later at Stage II, the child can distinguish a square from a closed curve. He has preserved the projective properties of straightness. This marks the beginning of Euclidean shapes. Finally at Stage III, the child's idea of shapes is at the operational level, he can draw figures quickly and correctly, and thus he has preserved the Euclidean properties of distance and angularity (Forman & Sigel, 1979, p. 147). The study of the children's drawings demonstrated that for the child, the earliest and easiest spatial concepts to grasp (in a very intuitive way) are those related to general features such as proximity, separation, order, enclosure, and so on. These ideas are all topological in nature and quite different from the Euclidean concepts of rigid shape, distance, straight lines, and angles.

With their successful experiments of the child's spatial concepts, Piaget and Inhelder have demonstrated that the first spatial concepts a child develops are topological concepts and that once these concepts are constructed, projective and Euclidean concepts are derived from them (Smock, 1976, p. 47). This theory of sequential development of spatial conceptual abilities is supported by several other researchers such as Dodwell (1963, pp. 141-161), Forman & Sigel (1979, pp. 132-158), Cohen (1987, pp. 71-78), etc. Some other studies, on the other hand, fail to support Piaget's this aspect of Piaget's argument. For example, the results of Martin's research didn't show that topological concepts dominate the 4-year-old children's representational space (as contrast with perceptual space). However, Martin admitted that his study should not be interpreted as a deniel of Piaget's theory of the representational space because only two topological propeties (connectedness and openness and closedness of curves) were tested in the study (Martin, 1976, pp. 26-38).

(B) How Children Learn Geometric Concepts from Their Environment

Piaget has conducted a great number of experiments dealing with the child's spatial and geometric concepts. His research is within the broader context of his theory of cognitive development and the nature of knowledge. In Piaget's view, the understanding of geometric and other spatial concepts is closely connected with both perceptual and intellectual development. As is usual in the Piagetian scheme of things, these developments occur through the child's own active exploration of, and interaction with, his environment (Dodwell, 1971, pp. 179-180).

According to Piaget, the child's geometric and other spatial concepts are constructed through coming into contact with the perceived environment and are only aspect of general cognitive development. The construction of geometric concepts proceeds through a continuous transformation of thought processes (Piaget & Inhelder, 1967, pp. 3-5). At this juncture, we need to know that the primary concern of Piaget's research on space is on conceptual space rather than perceptual space. Thus acquisition of spatial concepts is a product of the child's general intellectual development. Spatial representations are structured through "the process of organization of actions and/or logico-mathematical experience. Initially, these are sensory-motor actions . . . which later are internalized actions that culminate in operational systems" (Smock, 1976, p. 33). Therefore, Piaget assumes that through their own dealing with, and active exploration of their perceived environment, children can develope their own personal geometric concepts (Dodwell, 1971, pp. 179-180).

Children become aware of the environment through interaction with it, through the interpretations their brains make of the impulses that are signaled from their six senses. Within their brains, geometrical activity is interpreted from a refining of the raw data the children's senses emit to their nerve center of the brain. So, information about space, shape, and movement that helps children to think usefully about what they see and feel, helps them to clarify their world. What is seen is space and shape, colour and movement; what is felt is texture. The feeling and seeing together join forces with the kinaesthetic sense which then allows the child to become aware of shape, of size, of space, and of movement (Glenn, 1977, p. 19).

A drawing is a kind of representation. By drawing a picture to represent an object that the child has seen previously, the child uses his ability to construct a mental representation. "It marks the transition from visual perception to ideo-motor representation" (Piaget, 1967, p. 5). In this respect, the evolution of children's conceptual spaces can be clearly illustrated in their drawings of geometrical figures. Similarily, the teaching of geometry can be of value in providing children with an environment to help them use their eyes and develop skills with their own fingers to explore activities relevant to space, shape and movement.

The child at the stage of "concrete operation" (age 7 to 11 or 12), by assimilating information derived from his actions, no longer reasons solely on the basis of his perceptual judgements. Although at this stage, thought processes are more logical than those of the preceding stage, they are still largely dependent upon the use of things he can see and touch, for he has not yet fully developed the capability for abstract thought. In brief, he can reason logically but only about concrete phenomena (Reys, 1973, pp. 33-34).

In sum, children learn most effectively when they are actively involved in his perceived environment. Through their own active exploration of, and interaction with, his environments, the development of children's cognition occurs. In this connection, Logo provides an ideal experience for the child needs, because "the Logo learning environment is based on the developmental psychology of Piaget as well as an approach to the design of computer languages as problem solving tools developed in the study of Artificial Intelligence" (Weir & Watt, 1980, p. 4).

III. THE FEATURES OF LOGO AS A LEARNING ENVIRONMENT FOR GEOMETRY STUDY

(A) Logo Programming and Problem-Solving Skills

Currently, many claims have been made about the positive benefits of the use of microcomputers in education. Papert strongly recommands teaching children how to program. He believes that in learning to program Logo, children are allowed to create their own learning environment whereby they are not only developing their logical (mathematical) thinking and problem-solving skills, but also gaining knowledge about geometric concepts (Weir & Watt, 1980, p. 6). Specifically, Logo programming has the effects on children's problem-solving ability and their ability of learning geometric concepts.

In considering the programming, in Papert's view, it is "a human problem-solving activities involving the application of specific, identifiable 'procedures' and strategies" (Bass, 1985, p. 108). In this context, Logo programming is an excellent medium for children to develop their problem-solving skills, because it is built up through the use of procedures which are lists of instructions. Papert believes that once children begin to experience Logo programming, they begin also to start thinking about the precise strategies to teach the turtle to do what they desire. These strategies are of problem-solving in nature (Nolan & Ryba, 1986, p. 8). Through the manipulation of movement of the turtle, children learn the problem-solving skills in a concrete way. This is the most important part of Papert's claim. Papert argues that learning Logo enhance children's problem-solving skills by "providing concrete experience that promote thinking at a formal operational level" (Horner & Maddux, 1985, p. 45). Formal operational thinking, in Piaget's view, is "the ability to construct relationships, make inferences, and hypothesize" (Horner & Maddux, 1985, p. 45).

According to Polya, classic models of problem-solving consists of four basic steps: they are understanding the problem, devising a soultion, working out the solution plan, and evaluating the solution. Papert believes that thee problem-solving skills can be acquired through the Logo experience in several ways (Krasnor & Mitterer, 1984, p. 134):

Firstly, the strategy of breaking problems into managable subprocedures. Logo encourages learners to build their own procedures as chunks of larger projects. Using a small set of primitives, children can give the turtle instructions to draw their designs by defining procedures. After that, they can develop a set of procedures into higher order procedures to create more complex designs.

Secondly, means-end analysis or the systematic planning of actions to achieve goals. Systematic analysis and planning with Logo require that learners use both duductive ('top down') and inductive ('bottom-up') analytical thinking methods in combination. In this way, learner "can learn more, and more quickly by taking conscious control of the learning process" (Papert, 1980, p. 113).

The third one is debugging, the principle that problem solution can be successively refined. In Logo, debugging indicates improving the procedures of programs that do not work as desire. To be specific, debugging means "the process whereby learners judge the worth of their self-made concepts, products and projects and asses the adequacy of the methods used to create them" (Nolan & Ryba, 1986, p. 10). This is the process of learning from mistakes. It makes learners think about their own thinking.

Finally, the development of a more positive attitude toward mistakes is a powerful idea. Dubugging is a kind of strategy which consists of both a set of skills and an attitude. In using the dubugging strategy, learners must take the attitude that programs that do not work as desired are merely unfinished, rather than bad finished products. Programming bugs are considered as being unavoidable and constructive in the process of problem solving (Krasnor & Mitterer, 1984, p. 134).

Papert argues that these skills learned through dealing with Logo programming can be transferred to other problem-solving domains. According to Woodworth, the transfer of skills may occur in two conditions. One is that transfer between tasks requires that "some of the process or knowledge used in the tasks be essentially identical." The second one is that "the learner must recognize that a new problem situation is similar to one encountered previously" (Krasnor & Mitterer, 1984, p. 134). Thus, the possible condition for the transfer to be met is that Logo problems are similar to problems in other domains.

(B) Logo Programming and Geometric Concepts Learning

Being a language and being a way of thinking, mathematics requires not only a concrete environment to enhance the learning process but also an active learner. Therefore, in order to facilitate the development of children's geometrical cognition, it is essential to provide children with an environment where they are active participants. According to Hartfield,

> Mathematics learning is primarily a person-centered, constructive process: students build and modify their knowledge from experiences with task-oriented situations characteristic of mathematics. Students must experience opportunities and develop feelings of responsibility for revising, refining, and extending their ideas as the ideas are being constructed . . . Computers can be effective instructional contexts for such constructive approaches to learning (Hartfield, 1979, p. 53).

The computer indeed has the potential to provide students with an interactive learning environment in which they can be actively engaged in constructing and exploring mathematical concepts and problem-solving skills. The computer should be integrated into the curriculum as an interactive link between the concrete and abstract levels of thought (Berlin & White, 1986, pp. 468-469).

As children move from preoperational thinking toward concrete operations, they still need to experience directly with concrete materials. They are, however, ready to begin to manipulate symbolic references to real objects. Therefore, a good preliminary program for mathematics should include knowledge of symbol systems, understanding of the concepts represented by symbols, and practice in exploring them. To meet the need, Logo has potential for providing the necessary practice in symbolic manipulation by its ability of turtle graphics. For example, as children use Logo they must use symbols to achieve their objectives. However, within the confines of the Logo language, they are free to design their own procedures. Subsequently, they are not only in command of the "turtle" but they are also in control of their own learning. "They are free to be as playful as they like" (Barnes & Hill, 1983, p. 13).

Based on Piaget's philosophy that children learn best through actively interacting with and exploring objects within their environment, the creation of Logo primarily aims to provide children with a highly interesting learning environment in which they can actively participate in constructing and exploring the concepts of mathematics, particularly geometry, by actual manipulation of the turtle (Vaidya & McKeeby, 1985, p. 82).

The central part of the work on turtle graphics is the idea of developing a new kind of geometry — "turtle geometry" — which offers children a powerful and yet easily accessible means to manipulate shapes and motions. Logo is an excellent tool for children to express certain ideas in visual form, such as to draw a square and a triangle to form a house, which will encourage children to develop and practice problem-solving techniques and promote learning. This may very well be attractive to any "visually literate" child that has grown up with television and other visual media (Streibel, 1983, p. 23).

With its turtle graphics ability, Logo is especially useful in geometry learning because geometric figures and concepts can be created by directing the movement of the "turtle" on the screen. For example, using the initial commands **FORWARD, BACK, LEFT, RIGHT,** young children can explore the concepts of distance and angles. A child may also write a procedure to instruct the turtle to draw a circle, then write a main procedure to teach the turtle to combine the circle procedures into the figure of a snowman. By playing with the turtle, the child learns the concept of a circle. Just as Feurzeig and Lukas said that Logo "provides an operational universe within which students can define a mathematical process and then see its effects unfold" (Feurzeig & Lukas, 1972, p. 39).

Examining Turtle geometry and Euclidean geometry, we will find their initial concepts to be fundamentally similar, but stylistically they seperate. Euclidean geometry builds its geometry from a set of fundamental concepts, one of which is the point. Similarly, turtle geometry has also the fundamental entity of point. Yet this entity (turtle) is dynamic instead of being static. The dynamic nature of turtle will capture students' interest. Above all, Turtle geometry has not only the property of 'position' but also that of 'heading'. In this sense, the turtle is like a man, having not only its position, but also its direction. From these properties come the Turtle's special ability to serve as a first representative of formal geometry for a child (Thompson, 1985, p. 466). Although the child and the turtle are in different physical world, they actually share many important traits. Its two essential traits, position and heading, and ability to obey 'turtle talk' commands make the turtle body syntonic with the child and do geometry (Papert, 1980, pp. 55-56).

By making the turtle put images on the screen and making them to move, children acquire a dimension completely lacking in traditional pencil-and-paper drawing. As Abelson and diSessa have noted, with Logo, students can "regard plan-geometric figures not as static entities but as tracings made on a display screen by a computer-controlled 'turtle'" (Papert, 1980, p. xiv). A circle, for example, is described from the perspective of a person who is not only part of, but also creating the actual circle. Within the Logo's system this translates into instructions such as "move forward one unit" and turn right one degree until you get back to where you started." Papert made it clear: "Children can identify with the turtle and are thus able to bring their knowledge about their bodies and how they move into the work of learning formal geometry" (Papert, 1980, p. 56). To be specific, Papert believes Logo can create a "mathland" where mathematics is the spoken language, and children can learn to speak mathematics as easily and as successfully as they learn to speak their native language without memorizing formulas.

Logo also makes it possible for children to engage in abstract thinking by relating new ideas to concrete experiences they are familiar with already, thus strengthening existing foundations while allowing new seeds to grow. It forms "a playground somewhere between concrete models and abstract formalisms for developing intuitions of abstract concepts" (Thompson, 1985, p. 466). For example, rather than teach geometric ideas through abstraction such as points, lines, and coordinates, Logo encourages students to relate the turtle's geometric movements to their own body movements. So what we create is an insightful analysis into the "freedom" of movement geometrically, and our students realize that anything can be mapped once the concept is understood. Also because young children can create spectacular pictures with the turtle, their motivation to learn and play with it is usually very high, and hence their learning becomes "personally meaningful and emotionally charged" (Vaidya & McKeeby, 1985, p. 82).

A child's initial efforts to create geometric shapes are likely to be faulty. He learns by using his experiences and observations to correct initially mistaken belief about concepts. In Papert's and Piaget's view, this is the natural process for children to understanding. Children evolve slowly and through a process of creating, testing, reformulating, and retesting (Willis, 1987, p. 349). Logo, which combines the capabilities of artificial intelligence with Piaget's theories, encourages this process. It provides children with an environment for testing definitions, for refining definitions, and for formulating definitions of geometric concepts. For example, a student intends to create a procedure to have the turtle draw a triangle. If his definition has irrelevant attributes or is incomplete, his instructions to the turtle will have irrelevant directions or will be incomplete. For instance, the child creates the following procedure to draw a triangle.

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TO TRIANGLE
RT 30
REPEAT 3[FD 40 RT]
END
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In this case, the student is orienting the triangle with RT 30. The motion is not part of the triangle. Also, not all triangles are equilateral, nor do all triangles have 40 turtle steps on each side. Logo provides an environment to explore these ideas, to refine the definition (Moore, 1984, pp. iv-v).

Logo is not just a computer language. It is a philosophy of education. In many ways, the development of Logo has in accordance with the recent development of Piagetian theory. Logo was developed to meet the need to design a learning environment in which children are enabled to master the first computer language while developing attitudes and intellectual structures that help them overcome anxieties and approach problem solving constructively in mathematics (Bass, 1985, p. 108). The proponents of making Logo an important aspect of children's education generally advocate the discovery learning approach supported by Piaget. They believe the use of Logo in classes allow children to pursue their own course, to discover things on their own with only a small amount of teacher guidance. Logo also allows the child to experience directly the consequences of his actions. The computer is given instructions, the child observes the consequences of those instructions, and may then modify them to better accomplish a goal. The Logo approach thus creates "microworlds" where students can discover the rules governing the way that world operates. They discover those rules through manipulation of that environment.

In sum, Logo is not only a mathematical but also a pedagogical language (Martin, K., 1985, p. 271). Being a mathematical language, Logo is a mathematics-rich environment where mathematical objects and ideas are joyfully shared, played with, discussed and encouraged. It was created because of a fundamental belief that mathematical intuitions and language can be learned without excessive formal instruction just as children learn verbal language without formal instruction. Being a pedagogical language, it can encourage children to think about their own thinking processes. "What lies behind all of this is a Piagetian view of children who learn because they naturally make and revise theories about things they are interested in" (Riordon, 1982, p. 49). Piaget believed that knowledge naturally grows as a child actively interacts with the environment. The more enriched the environment, the more opportunity the child has to hypothesize, test, and explore. Play is a one way through which children learn by controlling their own little worlds. In this sense, Logo provides a language for exploration and learning concepts through microworlds.

IV. THE EVALUATION OF LOGO ENVIRONMENT ON CHILDREN'S LEARNING

As mentioned earlier, Logo is a language with specialized features that give it great potential to be an ideal medium for helping children explore mathematics, particularly geometry and for enhancing their problem-solving skills. Further, when combined with the relevant educational philosophy, working with Logo is fitted to a learning environment which is in accordance with piaget's discovery learning theory.

To date a variety of important claims therefore have been made about Logo's effectiveness in education, and yet few systemative and experimental studies have been conducted to support the claims. Further, the results of research are inconclusive and conflicting. Generally speaking, the evaluation of effect of Logo on children's learning has been controversial among Logo's proponents and critics on two areas, those being mathematical achievements and problem-solving skills (Rieber, 1987, p. 14).

Among the research in favor of Logo, most of them provide anecdotal evidence that Logo programming provides children with an opportunity to create their own learning environment and consequently problem-solving skills and mathematical achievements are improved. For the most part, these research focus on case studies, observations, and the creation of curriculum materials, rather than experimental findings. Although they assert that the powerful ideas and problem-solving skills learned from Logo programming can be transferred to other domains, most of the available evidence is related to Logo learning rather than the transfer of Logo learning to other contexts (Krasnor & Mitterer, 1984, p. 136).

The research of the M.I.T. Logo group led by Papert has been typical of this research approach. Papert and his colleagues assume that children experiencing with Logo programming will develop powerful cognitive skills such as mathematical abilities, creative thinking, and metacognitive skills (Vaidya, 1985, p. 222).

In the large-scale Franklin Logo project, Papert et al. provided detailed anecdotal descriptions of the learning of 16 children, such as children's interesting in Logo, the turtle's effect on children's ability of measure, etc. (Papert, et al., 1979). In the study, positive improvement with angles and distance was noted as a result of experience with Logo, even though no direct teaching about those topics took place (Kelly, et al., 1986/87, p. 24). However, the very open-ended learning environment in which Logo was used resulted in the difficult to have any systematic and objective statistical analysis of success (Martin A. 1975, p. 12). Hence, the result of problem-solving tests was inconclusive. Watt admitted: "the problem of developing objective tests in such areas as problem solving or procedural thinking is still an open question for educational researchers" (Tetenbaum, et al., 1984, p. 18).

In addition, several other researcheres, on the basis of their case studies, also provided anecdotal evidence to support the claim that children's exposure to Logo programming can produce favorable results in mathematical or problem-solving skills. For example, Hill et al. concluded in their study that third graders acquired geometric concepts of length, angles, and curves, after playing with Logo (Clements, 1985, p. 61). Weir found that working with Logo offered children an opportunity to improve their ability of doing spatial problem solving (Shively, 1984, p. 28). It is worthly to note that most of their support has derived from observations of positive student reaction to Logo rather than from empirical research.

Only few studies have provided empirical evidence to the claim that Logo has positive effects on childrens' mathematics achievement and problem-solving skills. In the Edinburgh Logo project, the 11-year-old children with low-achieving in mathematics were provided with Logo experience over 2 year duration period. The results of the study showed that the students in the Logo group were more willing to argue or raise mathematical issues. Howe, et al. concluded that Logo could be useful in learning mathematical concepts (Howe, et al., 1980, pp. 86-100; Martin,

A. 1985, p. 13). As a result of Reiber's experimental study, the students in Logo group performed significantly better on problem-solving measures than the students in non-Logo group (Reiber, 1987, pp. 14-15). Clements' study also demonstrated that Logo programming can increase students' performance in specific cognitive, creativity and metacognitive skills such as classification and seriation (Clements, 1986, p. 316). In summarizing his research results, Clements believed that Logo programming holds potential to facilitate the development of certain problem-solving skills. However, how Logo experience develops these skills is still an open issue (Clements, 1985, pp. 67-68).

However, Statz's research showed the different opinion. In her study, 4 problem-solving tests were administered to two groups. She predicted that working with Logo would improve all of these problem-solving skills (organization, a systematic solution strategy, permutation, and classification), but the results indicated that only permutation and classification reached slightly significance. Chait's study also failed to show the effect of Logo programming on the children's general problem-solving skills (Krasnor & Mitterer, 1984, pp. 136-137). The weakness of his study is that no control group was used.

The Logo literature rewards considerable criticisms from an experimental point of view. The lack of an experimental methodology by the Logo enthusiastiasts has drawn some criticisms. They argued that there was no good evidence to support the claims that powerful ideas generalize or transfer. In 1983, the Bank Street researchers provided strong empirical evidence to raise serious doubts about the current optimism concerning the cognitive benefits of Logo programming. Pea, in his Logo project, attempted to present Logo to children in the context of free or open discovery learning as advocated by Papert. However, the findings of the study indicated that after a year's experience in Logo programming, the students did not display greater problem-solving skills than the students without doing Logo programming (Pea, 1983, pp. 2-3). He also found that there is little, if any, transfer of learning from the Logo situation to similar non-Logo tasks. Pea and his colleague believe that the transfer of problem-solving strategies between dissimilar problems, or problems of different content, is quite difficult to achieve even for adult, let alone for children'' (Pea, 1983, pp. 6-7). Pea criticized that case-study methods are inappropriate to evaluate the transfer problem. That is the development of thinking skills that transcend the programming context (Pea, 1983, p. 3).

The results of Pea's study has brought about a great deal of controversy among Logo enthusiasts. Some argue that Logo should not be viewed as a necessary and generally good for the childrens' cognitive development and mathematical achievement unless it has been proved. They prefer to use Logo as a manipulative tool for exploring geometric concepts previously taught by the teacher. In this respect, Logo still can be contributory to childrens' mathematical achievement (Battista, 1987, p. 295). Other Logo's supporters, however, argue that the methods used by the Bank Street researchers were not adequate for testing the effectiveness of Logo. They argue that "traditional experimental methodologies are not appropriate for measuring Logo-facilitated changes" (Krasnor & Mitterer, 1984, p. 136). Papert made it clear that "because 'objective' educational researchers seek out what can be applied, they are obliged in doing this to ignore all that cannot easily be measured. And it is in this latter area that the real fruits of education lie" (Martin, A. 1985, p. 14). Obviously, the results of research on the effects of Logo programming on children's mathematical and problem-solving skills are inconclusive and conflicting. Papert and his supporters believe that by working with Logo programming even young children will develop powerful cognitive skills, such as creative thinking, mathematical abilities and metacognitive skills. Such claims definitely suggest the transfer of important abilities from Logo experience to another domain. However, very little objective and systematic experimentation or statistical analysis is provided in their studies. Actually much of the evidence providing the support for their claims of Logo's benefits in education is principally anecdotal accounts.

On the contrary, the results of experimental research conducted by the Bank Street Researchers showed no significant effect of Logo programming on the development of children's mathematical or problem-solving skills. While the Bank Street researchers acknowledged that it would be premature to abandon the use of Logo in schools, their findings suggest the need for a moratorium on the implementation of Logo programming as a generalized problem-solving model until further research can be done.

Several conclusions can be drawn from these diverse studies. In the first place, usually the results of case studies are suggestive and not conclusive. The anecdotal descriptions of case studies, such as the Brooklin Logo proojeet, fail to show "the specific nature of changes a child goes through as a result of learning Logo" and "often mislead educators into thinking that all children can explore microworlds and develop new cognitive structures merely by having accessibility to computers" (Vaidya, 1985, p. 225). This fact limits the generalizability of the research. As Watt remarked: "The Brookline Logo Project was not very successful in obtaining 'objective' data about learning gains made by the students. Standardized tests had been rejected as irrelevant to the goals of the project. The problem-solving tests and mathematical tests devised and administered by the project staff had inconclusive resuts. The problem of developing objective tests in such areas as problem solving or procedural thinking is still an open question for educational researchers" (Watt, 1982, p. 120). The MIT Logo group assume that all children, even young children, working with Logo will naturally and spontaneously develop their mathematical and creative ability. While, Vaidya's study suggested that these skills were not naturally improved through experiencing with Logo; and that individual difference among childen may have its implications on children's these abilities (1985, p. 222).

Secondly, on the other hand, the methods used by the Bank Street researchers, in a sense, are also not adequate to test the benefits of Logo. Because there is little agreement on what exactly problem-solving or planning skills involve in cognitive terms, how to measue these skills becomes a problem. Probably Bank Street researchers chose a number of tasks which seemed to them appropriate, and compared the progress of children in Logo and in no — Logo groups. They were also criticized for not providing the children with sufficient time to work with Logo in order to develop general skills (Martin, 1985, pp. 13-14). Further, the almost uninvolved role of the teachers in their Logo classrooms is neither a typical nor a fair one (Martin, A., 1985, pp. 13-13). The results of their research therefore are also suggestive and not conclusive.

Truly, Papert remarks that Pea and Kurland are negative, Clement and Gullo positive about the effect of Logo programming on children's learning. Papert said:

> Pea and Kurland approach their experiment with a very specific idea of what cognitive effect to look for; they are checking for an improvement in a very narrow and specific form of planning activity, so they use a focused and hoc test. Clement approach the problem with a relatively open mind about what the cognitive effects of doing Logo might be: they apply a broad spectrum of well known, standard tests of cognitive function. Even before one sees the results, it is obvious that the Kent State experiment stands a much higher chance of coming out positive as, indeed, it does (Papert, 1987, pp. 26-27).

Thirdly, some steps should be taken to reconcile the conflict betwen those who support observational approaches and those who place emphasis on experimental approaches, if Logo is to become more widely acceptable. If the case is as what the MIT Group claims, traditional experimental approaches are not appropriate for testing Logo effectiveness, then this argument should be made clear, and the emphasis should shift away from traditional psychological testing methods, statistics, and standardized procedures. This issue should be seriously taken into account and not be ignored (Michayluk, 1986, p. 39).

The fourth, whether skills learned in Logo programming can be transferred to other domains is still an open issue. There are many factors which may influence transfer, such as the completeness of originial learning, exposure to a variety of situations in which the specific skill is useful (Krasnor & Mitterer, 1984, pp. 134-135), individual difference among learners. To date, no valid evidence is able to verify this particular effort. In a summary of the research into the "transfer problem," Erik de Corte remarked: "At present there is no convincing evidence that learning to program results in the acquisiton of generalizable and transferable conceptual knowledge and thinking methods. . . . On the other hand the transfer hypothesis can be retained for further study in more systematic and better designed experiments" (Corte, 1983, p. 10).

Finally, although satisfactory objective data appears to be lacking, observation and personal positions are not without any merit. Observational data suggests that Logo can be effective with most populations. Pea, in fact, has already suggested that instruction in thinking skills in conjunction with Logo might be beneficial. However, a question should be addressed about whether teaching problem solving through teaching programming is the most efficient way to achieve the objective.

V. THE IMPLICATIONS OF LOGO ENVIRONMENT ON THE TEACHER'S ROLE

Logo is not just a computer language but more importantly also a philosophy of education. As an educational philosophy Logo makes the assumption that if learning is to be meaningful it must involve intellectual exploration and discovery learning. A sound theoretical rationale seems to exist for the use of Logo programming in mathematics teaching. In the Logo experience children develop strategies for purposeful exploration. Turtle geometry therefóre has a great potential for being a successful tool in helping children to understand mathematical concepts and processes and also a foundation for exploring and building mathematical skills. It is increasingly becoming an integral part of the mathematics curriculum in schools throughout the country. The rush to teach children this new curriculum has resulted in an important issue: What is the role of the teacher in Logo-based learning? This question boils down to asking what a teach should do when teaching children, what approach would best enhance learning utilizing Logo as an aid.

Although the introduction of Logo into the classroom does not necessarily force teachers to adopt a different style of teaching, it does indeed, in some cases, call for a modification in technique. Intrinsically, Logo's features only become a truly valuable aid of teachers and learners when it is allowed a freedom for application, a looser rein and less constraints. After all, discovery and construction is the mainline of the Piaget theory. Logo-based instruction is fundamentally based on discovery learning which has been extensively researched and has substantial merit.

There is an ongoing debate in education about the teacher's role in the process of discovery learning. The point at issue is how guided should discovery be? In Papert's view, learning with Logo, children must learn from the materials of the world without being taught (Papert, 1980, p. 7). Namely, it is sufficient for acceptable learning to occur when children are allowed to explore. However, many teachers and researchers who have used Logo in the classroom have argue that it probably takes a lot of time and many diverse efforts to acquire knowledge of a subject being undertaken completely and solely by discovery techniques. Moreover, very often the discovery would not take place, if the teacher can not give the student the necessary guidance at the proper time (Martin, 1985, p. 27). Admittedly, Logo learning is a process of discovery through the achievement of insight into the nature of a problem. Changes in approach come into existence only over the extent to which the process should be guided.

It seems wrong to expect that merely working with Logo will automatically enhance students' mathematical achievements. A pedagogy of complete non-intervention will very likely leave the children in a structureless anarchy in which there is little profit. As Noss warns: "transfer of understanding to mathematical concepts tends not to occur automatically. Such effects tend to be found when explicit linkages between mathematical ideas and programming are made" (Battista, 1987, p. 7). The effective implementation of Logo-based instruction depends on well-trained, experienced, committed teachers. Logo has the potential to be a powerful supplement to learning. However, without the help of a well-trained and knowledgeable teacher and the appropriate software, little can be done by Logo itself for most students. It is impossible for Logo itself to create "the teacher-related parts of sound educational environment" (Moursund, 1983-84, p. 3). Doubtless there may be rare exceptions in which students can learn Logo and explore concepts on their own efforts to create interesting and challenging projects. But for the most part, students need not only the guidance from knowledgable and experienced teachers, but also the help of high quality of curriculum materials (Moursund, 1983-84, p. 3).

A strongly teacher-centred approach on the other hand will leave the children little room for their activities of exploring. A Logo teacher is different from a master teacher. A master teacher in mathematics is well familiar with the knowledge of the subject matter and also knows how to learn this subject. With this knowledge, he/she guides students to acquire certain mathematical concepts and skills. Logo on the other hand provides children with a very good environment for learning how to learn. This is to say how best to process information. The child interacts with the Logo environment according to his ability and learning style. Students often take the lead while exploring a problem. Learning activities within the Logo learning environment is open-ended and exploratory; therefore, a minimum of teacher intervention is required. It is especially important for teachers not to intervene in such situation as this, even though the student's approach might not produce the desire results. They can learn also from their mistakes. "The Logo teacher's interaction with students eventually takes on a guidance and co-learning flavor," said Streibel (1983, p. 482). The principle is to help students gain an increasing control over their own learning process.

It therefore seems clear that the proper timing of intervention on the part of the teacher sometimes is required and necessary to facilitate children's learning. In this context, the teacher's role is to set goals and limits where needed, and to guide discreetly if necessary. The issue of great concern for the teacher should be how to decide the frequency, timing, and nature of interventions. Generally speaking, how often the teacher should come to aid a student at the computer will depend on several factors, such as the personality of the child, the task on which the child is engaged, etc. The timing of the teacher's interventions is very important. In an attempt to assess the timing of interventions, the teacher's awareness of the children's activities is important. The intervention of the teacher at an incorrect time is not helpful for the child who is trying to solve a problem. They will only become dependent on their teacher and not develop their own confidence. As to the nature of teacher's interventions, it should not be "teacher-centered and dismissive of the children's activities;" conversely, the intervention should embody "an acceptance of the children's activities and seek to draw from the children some statement of understanding of the processes involved" (Martin, 1985, pp. 37-39).

Logo offers teachers an opportunity to gain new insights into learning and to provide students with constructive and discovery learning experiences. Clearly that Logo can have a great effect on geometry curricula. Logo can make the learning of geometry more alive for the student by changing geometry from a static subject involving rote memorization of definitions and theorems to a dynamic subject with stress on visualization and spatial imagination and more emphasis on original proofs by promoting inquiry through interaction. In order to fully realize the potential of Logo-based instruction, teachers must accept the challenge of fundamental change in their own perspectivè on children, education, and computers. A Logo curriculum with appropriate instruction does not happen automatically and immediately. It grows and matures with increased sophistication of teachers and of students.

Thus, for Logo to be an effective instructional medium for teaching geometry, mathematics teachers must pay great attention to creating explicit connections between Logo activities and geometric concepts found in the curriculum (Battista, 1987, p. 7). The great challenge to a teacher using Logo in teaching geometry is how to use Logo programming to achieve his main teaching objectives: to improve children's understanding of basic geometric concepts and skills; and to increase their self-confidence in geometry. To some extent, the most important thing the teacher needs to do is to provide an interesting environment in which children can explore their ideas. Piaget suggested that teachers offer students subject matters to explore that make them understand the nature of the problems and encourage them to find out answers for themselves. Because real understanding involves reinvention by children, teacher in Logo-based instruction should be "less the giver of lessons and more the organizer of engaging problematic situations" (Clements, 1986, p. 173). Students studying geometry with Logo often take the initiative while exploring a particular geometrical concept or skill. It seems that once children are able to understand how Logo procedures are executed, programming provides a basis for making concrete explanations of a variety of basic geometrical concepts and skills. The logo teacher's interaction with students eventually takes on a guidance and co-learning stance. These quidance and co-learning sessions are far more effective for the student's mastery of an idea than leaving the student totally alone with Logo. Basically, the teacher acts as a facilitator, allowing the student to discover how to create a solution to his problem in Logo. To do this, the teacher must estimate the student's understanding of how to create a solution to a Logo project by listening to the questions that the child raises, observing the solutions the child proposes, and being aware of the level of success, as well as frustration of the student (Vaidya & McKeeby, 1985, pp. 83-84). As a whole, this author agrees with Rubens, et al. in advocating the following roles for the teacher using Logo in his mathematics classroom: 1) To serve as a model for using Logo in new and innovative ways; 2) To encourage and promote an environment of noncritical sharing; 3) To stimulate and encourage the student to take part in discussion; 4) To allow the student to have sufficient time for meaningful exploration; 5) To provide the student with a variety of experiences; 6) To be a good listener to students' questions (Vaidya & McKeeby, 1985, pp. 84-85).

Since the teacher plays a crucial role in the integration of Logo learning environment into mathematics curriculum, a long-range, continuous teacher training development is required and critical. At the very least, teachers are required to understand the value of discovery learning and student interaction. It is important to help teachers to have positive attitudes toward Logo. Further, at all sites where Logo is introduced into the classroom, teachers need training to become skillful at teaching the turtle to do things. This is a stage of skill building. The final and the most important thing is that the teacher needs to be able to change his stance from "how to teach Logo" to "what to teach with Logo" (Tipps & Bull, 1985, pp. 276-277). To be specific, this is the adoption and implementation of Logo in the classroom. Using Logo as a medium of mathematical instruction is the most complex and difficult task to engage in. More time is needed to develop this kind of activity.

VI. CONCLUSION

To summarize, geometry is the science of space. According to Piaget, children's geometric concepts change with the intellectual development and the evolution of these concepts is in anti-historical order. The child, at his first stage, is able only to conceive space in terms of topological relationship, such as neighborhood, order, between and closure. Subsequently, he learns to construct space by the 'point of view' of an observer and to describe space in terms of coordination, such as left-right, before-behind, and above-below. At the final stage, he develops his concepts of distance and begins to conceive space in metric terms. These three stages of the development of the child's spacial notions correspond to spatial relationships constituting three branches of geometry — topological space, projective space, and Euclidean space.

Learning-by-doing is one of the central parts of Piaget's approach to education. Piaget believes that young children learn best through working their environment, dealing with the concrete things that can be seen, felt, touched, and manipulated. According to Piaget, the development of the child's spatial concepts proceeds at two distinct levels: at perceptual level and at conceptual level (Piaget & Inhelder, 1967, p. 3). As is usual in the Piagetian scheme of things, the construction of the child's geometric concepts occurs through his own active exploration of, and interaction with, his perceived environments and is intimately related to the cognitive development.

Papert, on the basis of this assumption, designed Logo to provide children with a learning environment in which they can explore geometric concepts and improve their problem-solving abilities, by actually manipulating objects in the environment. It is a "mathland" where mathematics is the spoken language and children can learn mathematics as easily and successfully as they learn to speak.

Due to its features of turtle graphics, structured programming, exploratory learning, interactive learning, etc. We indeed have reason to argue that Logo can be an effective learning environment for geometry study and, to a certain extent, to be an ideal device for enhancing children's certain specific problem-solving skills. Unfortunately, to date there is little definitive research and evidence to support that claim. This is mainly because existing research reports do not convincingly test for generalization or becuse the relevant research has not yet been done.

Currently, most criticisms of Logo are concerned with problems of using or misusing Logo, rather than problems inherent in Logo itself. Used in appropriate ways, Logo will have its definite value in helping children's learning. Proponents of Logo often overexagerate that Logo programming will not only improve children's mathematics achievement but also necessarily enhance children's general thinking and problem-solving skills.

The application of Logo and Piaget's ideas to geometry teaching and learning is by no means an educational panacea. It has generated a new set of unanswered questions. For instance, one of the most important issues is what kind of teaching model would be most consistent with the use of Logo in classes. Teacher-training and availability of resource materials are important to make sure the success of any Logo project. In this respect, the suggestion from the Bank Street researchers of the need for a moratorium on the implementation of Logo programming as a generalized problem-solving model until further research can be conducted is a sound recommendation. Perhaps, logo will prove to be an effective medium to enhance children's cognitive development under a given set of circumstance that have not yet be accomplished (Tetenbaum, 1984, p. 18).

Papert's claims about what Logo can accomplish may be overly optimistic. However, it is also true that today it is in no case that the full potential of Logo has been realized. Much work and research, such as on the effectiveness of Logo, or about how to effectively integrate Logo into mathematics classroom, and so on, need to be done. Educators should cautiously weigh the claims made for Logo and the findings of research related to Logo. Any overemphasis on either the positive support or negative research findings may bring about a potential danger to the effective use of Logo. Just as Horner and Maddux warn: "If advocates of Logo promise more than can be delivered, educators and parents may become disenchanted and decide Logo has no real educational value. Similarly, the lack of empirical research to support the claims for Logo should not be considered proof that Logo is not educationally useful'' (Horner & Maddux, 1985, p. 46).

REFERENCES

Barnes, B. J. & Hill, S. ''Should Young Children Work with Microcomputers — Logo Before Lego?'' *The Computing Teacher*, May 1983, pp. 11-14.

Bass, J. E. ''The Roots of Logo's Educational Theory: An Analysis,'' *Computers in the Schools*, Summer/Falll 1985, pp. 107-115.

Battista, M. T. ''Mathstuff Logo Procedures: Bridging the Gap between Logo and School Geometry,'' *Arithmetic Teacher*, September 1987, pp. 7-11.

Battista, M. T. ''The Effectiveness of Using Logo to Teach Geometry to Preservice Elementary Teachers,'' *School Science and Mathematics*, April 1987, pp. 286-296.

Berlin, D. & White, A. ''Computer Simulations and the Transition from Concrete Manipulation of Objects to Abstract Thinking in Elementary School Mathematics,'' *School Science and Mathematics*, October 1986, pp. 468-479.

Birch, L. ''A Turtle in the Classroom,'' *Computers in the Schools*, Summer/Fall 1985, pp. 91-94.

Chernysheva, L. Y. ''Teaching Geometry in the USSR,'' *Studies in Mathematics Education*, Vo. 5, 1986, pp. 97-106.

Clark, V. A. ''The Impact of Computers on Mathematics Abilities and Attitudes: A Pilot Study Using Logo,'' *Journal of Computers in Mathematics and Science Teaching*, Winter 1985/86, pp. 32-33.

Clements, D. H. ''Logo and the Nature of Learning,'' *Educational Horizons*, Summer 1986, pp. 173-176.

Clements, D. H. ''Effects of Logo and CAI Environments on Cognition and Creativity,'' *Journal of Educational Psychology*, Vol. 78, No. 4, 1986, pp. 309-318.

Clements, D. H. ''Research on Logo in Education: Is the Turtle Slow but Steady, or Not Even in the Race?'' *Computers in the Schools*, Summer/Fall 1985, pp. 55-71.

Cohen, H. G. ''A Longitudinal Study of the development of Spatial Conceptual Ability,'' *Journal of Genetic Psychology*, March 1987, pp. 71-78.

Copeland, R. W. *How Children Learn Mathematics: Teaching Implications of Piaget's Research* (New York: Macmillan Publishing Co., Inc., 1974).

Corte, E. D. ''Logo and Learning to Think,'' *Logo Almanack*, Vol. 1, 1983, pp. 10-11.

Dodwell, P. C. ''Children's Perception and Their Understanding of Geometrical Ideas,'' in Rosskopf, M. F. Steffe, L. P. & Taback, S. eds., *Piagetian Cognitive-Development Research and Mathematical Education* (Reston, VA: National Council of Teachers of Mathematics, 1971).

Dodwell, P. C. ''Children's Understanding of Spatial Concepts,'' *Canadian Journal of Psychology*, Vol. 17, 1963, pp. 141-161.

du Boulay, J. B. H. & Howe, J. A. M. ''Logo Building Blocks: Student Teachers Using Computer-Based Mathematics Apparatus,'' *Computers & Education*, Vol. 6, 1982, pp. 93-98.

Elliott, P. C. ''Mathematics Matters: Matters of Consequence or Doing something Consequential?'' *International Journal of Mathematics Education and Science Technology*,

Vol. 18, No. 1, 1987, pp. 127-138.
Feurzeig, W. & Lukas, G. "Logo — A Programming Language for Teaching Mathematics," *Educational Technology*, March 1972, pp. 39-46.
Forman, G. E. & Sigel, I.E. *Cognitive Development: A Life-Span View* (Monterey, California: Brooks/Cole Publishing Co., 1979).
Glenn, J. A. *Children Learning Geometry* (New York: Harper & Row, Publishers, 1977).
Gorman, H. Jr. "Learning to think by Learning Logo: Rule Learning in Third-Grade Computer Programmers," *Bulletin of the Psychonomic Society*, May 1983, pp. 165-167.
Hartfield, L. L. "A Case and Techniques for Computers: Using Computers in Middle School Mathematics," *Arithmetic Teacher*, February 1979, pp. 53-55.
Heddens, J. W. *Today's Mathematics: Concepts and Methods in Elementary School Mathematics*, 5th ed. (Chicago: Science Research Associates, Inc., 1984).
Horner, C. M. & Maddux, C. D. "The Effect of Logo on Attributions toward Success," *Computers in the Schools*, Summer-Fall 1985, pp. 45-54.
Howe, J. A. M., O'Shea, T. & Plane, F. "Teaching Mathematics through Logo Programming: An Evaluation Study," in Lewis, R. & Tagg, E. D. eds., *Computer Assisted Learning: Scope, Progress and Limits* (New York: North-Holland Publishing Co., 1980).
Huber, L. N. "Computer Learning Through Piaget's Eyes," *Classroom Computer Learning*, October 1985, pp. 39-42.
Johnson C. S. & Swoope, K. F. "Boy's and Girl's Interest in Using Computers: Implications for the Classroom," *Arithmetic Teacher*, September 1987, pp. 14-16.
Kerr, D. R. Jr. "A Case for Geometry: Geometry Is Important, It Is There, Teach It," *Arithmetic Teacher*, Feb. 1979, p. 14.
Klein, E. L. "Computer Graphics, Visual Imagery, and Spatial Thought," in *Children and Computers* (Washington, D. C.: Jossey-Bass Inc., Publishers, 1985), pp. 55-74.
Krasnor, L. R. & Mitterer, J. O. "Logo and the Development of General Problem-solving Skills," *The Alberta Journal of Educational Research*, June 1984, pp. 133-144.
Kuchemann, D. E. "Children's Difficulties with Single Reflections and Rotations," *Mathematics in School*, Vol. 9, No. 2, 1980, pp. 12-13.
Martin, A. *Teaching and Learning with Logo* (New York: Teachers College Press, 1985).
Martin, J. L. "A Test with Selected Topological Properties of Piaget's Hypothesis Concerning the Spatial Representation of the Young Child," *Journal for Research in Mathematics Education*, January 1976, pp. 26-38.
Michayluk, J. O. "Logo: More Than a Decade Later," *British Journal of Educational Technology*, January 1986, pp. 35-41.
Moore, M. L. *Geometry Problems for Logo Discoveries: Explorations in Turtle Geometry* (Palo Alto, CA: Creative Publications, 1984).
Moursund, D. "Logo Frightens Me," *The Computing Teacher*, December-January 1983-84, pp. 3-4.
Nolan, P. & Ryba, K. *Learning with Logo* (Eugene, O. R.: International Council for Computers in Education, 1986).
Papert, S. *Mindstorms: Children, Computers, and Powerful Ideas* (New York: Basic Books, Inc., Publishers, 1980).
Papert, S. "Computer Criticism vs. Technocentric Thinking," *Educational Research*, January/February 1987, pp. 22-30.
Papert, S., Watt, D., diSessa, A. & Weir, S. *Final Report of the Brookline Logo Project*

Part II: Project Summary and Data Analysis (M.I.T. Artificial Intelligence Laboratory, 1979).

Pea, R. D. "Logo Programming and Problem Solving," *Technical Report No. 12* (New York: Bank Street College of Education Library 1983).

Pea, R. D. & Kurland, D. M. "On the Cognitive Effective Effects of Learning Computer Programming," *New Ideas Psychology*, Vol. 2, No. 2, 1984, pp. 137-168.

Piaget, J. & Inhelder, B. *The Child's Conception of Space*. 3rd. ed., Langdon, F. J. & Lunzer, J. L. trans. (London: Routledge & Kegan Paul Ltd., 1967).

Rahim, M. H. & Sawada, D. "Revitalizing School Geometry Through Dissection-Motion Operations," *School Science and Mathematics*, March 1986, pp. 235-246.

Reys, R. E. & Post, T. R. The Mathematics Laboratory: Theory to Practice (Boston, MA: Prindle, Weber & Schmidt, Incorporated, 1973).

Rieber, L. P. "Logo and Its Promise: A Research Report," *Educational Technology*, February 1987, pp. 12-16.

Riordon, T. "Creating a Logo Environment," *The Computing Teacher*, November 1982, pp. 46-50.

Robinson, E. "Mathematical Foundations of the Development of Spatial and Geometrical Concepts," in Martin, J. L. ed., *Space and Geometry* (Columbus, Ohio: Eric, 1976), pp. 7-29.

Rosser, R. A. & Campbell, K. P. "The Differential Salience of Spatial Information Features in the Geometric Reproductions of Young Children," *The Journal of Genetic Psychology*, December 1986, pp. 447-455).

Shively, J. E. "Computer Utilization in Education: Problems and Prerequisities," *AEDS Journal*, Spring 1984, pp. 24-34.

Smock, C. D. "Piaget's Thinking about the Development of Space Concepts and Geometry," in Martin, J. L. ed., *Space and Geometry*, 1976, pp. 31-73.

Streibel, M. J. "On First Encountering Logo; Some Questions for Further Research," *Educational Considerations*, Spring 1983, pp. 22-24.

Streibel, M. J. "The Educational Utility of Logo," *School Science and Mathematics*, October 1983, pp. 474-484.

Taylor, R. P. ed. *The Computer in the School: Tutor, Tool, Tutee* (New York: Teachers College Press, 1980).

Tetenbaum, T. J. & Mulkeen, T. A. "Logo and the Teaching of Problem Solving: A Call for a Moratorium," *Educational Technolgy*, November 1984, pp. 16-19.

Thompson, P. W. "A Piagetian Approach to Transformation Geometry via Microworlds," *Mathematics Teacher*, September 1985, pp. 465-471.

Tipps, S. & Bull, G. "Teachers Need Time for Turtles: Planning for Teacher Development with Logo," *Computers in the Schools*, Summer/Fall 1985, pp. 273-281.

Vaidya, S. R. "Individual Differences among Young Children in Logo Environments," *Computers and Education*, Vol. 9, No. 4, 1985, pp. 221-226.

Vaidya, S. & McKeeby, J. "Developing a Logo Environment in the Preschool," *Computers in the Schools*, Summer/Fall, 1985, pp. 81-89.

Watson, J. A., Calvert, S. L., & Popkin L. A. "Microworlds, Sprites, Logo, and Young Children: A Multipurpose Software Application," *Journal of Educational Technology Systems*, Vol. 15, No. 2, 1986-87, pp. 123-134.

Watt, D. "Logo in the Schools," *Byte*, August 1982, pp. 116-134.

Weir, S. *Cultivating Minds: A Logo Casebook*. (New York: Harper & Row, 1987).

Weir, S. & Watt, D. *Logo: A Computer Environment for Learning-Disabled Students*. (Cambridge, Mass.: Logo Group, 1980).

Willis, J. W. *Educational Computing: A Guide to Practical Applications* (Scottsdale, Arizona: Gorsuch Scarisbrick, Publishers, 1987).

Wirszup, I. "Breakthroughs in the Psychology of Learning and Teaching Geometry," in Martin J. L. ed. *Space and Geometry* (Columbus, Ohio: ERIC Center for Science, Mathematics, and Environmental Education, 1976), pp. 75-97.