Optimal Policies, Investment Environments and Country Risk

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摘 要

最適反應法則和平均法則是在最適的地方交叉，其結果是和一般的最適控制方法一樣。因爲總體變數之間的關係是非線型的，這種非線型性質就可能造成景氣循環和投資變動，以及貿易和外滙的變動，所以非線型的政策反應應當用來減低景氣循環、儲蓄的變動和貿易差額的變動。

這裏所謂的最適政策是，當經濟成長率降低時，貨幣供給量，政府支出和稅收都應增加，同時預算變成赤字。此外，實質外滙滙率的貨幣政策也可計算出來。

ABSTRACT

The optimal response rule intersects with the average response rule at the same fixed point, as is attained by the optimal control method. Since there exists a nonlinear relationship among macro variables, such nonlinearity may giver rise to the business cycles, and changes in investment and trade balances. A nonlinear policy response should be used for counter business cycles, changes in savings and trade deficits as well as Wagner's hypothesis testing. Optimal macro policies are such that when the growth rate of output declines, the growth rate of money supply, government spending and tax revenue should increase, while the budget shows a deficit. Optimal monetary response rules to the real exchange rate are also derived here.

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1. Introduction and summary

Investment environments often vary positively with output growth and market size, while country risk rises with increases in the trade deficit. The present paper probes into the basic reasons for these relationships. Macro policies on economic growth and golden rules are focused on mathematical models, and argued for an infinite, intertemporal and efficient output path and for the balanced growth of both consumption and capital goods sectors (Findlay, 1966; Samuelson, 1966).

What has been omitted has been to find empirically optimal response rules for balanced growth. Previous studies on optimal control yield a fixed point, however. The response rule of policies can be detected by either the trajectory of iterations under the Newton algorithm or by the analystical methods. For example, Abel (1987) suggests that the optimal money supply should be set at the zero nominal interest rate. The optimal tax transfer is set such that savings are just enough to cover the desired capital formation. The optimal level of inflation is deflation since the aim of the money supply is to satisfy the consumers' demand for real balances. Such arguments, however, neglect the impact path of the money supply on savings, production and the transactions demand for money. Similarly, as shown by Turnovsky (1987), both too high or too low inflation will cause recessions and reduce output growth. The objective of optimal monetary policy is to maintain a stable price level. The estimation of the optimal response path remains largely ununswered, however. It is still controversial whether it should increase money supply, government expenditure and tax revenues during recessions.

The nonlinearity instead of the linearity of macro relationships may be the issue. Only moderate inflation can increase effective demand and output. When high inflation is a cost-push inflation, overinvestment leads to the adoption of capital-intensive techniques; profits tend to decline, and wages become price-indexed. At such high levels of inflation, aggregate wages was grow faster than aggregate profits, and production will be discouraged. Overinvestment, which is induced by high price levels, tends to rely on debt-financing instead of savings or equity-financing. Suppose the real interest rate is the market rate minus inflation. Since under loan contracts, market interest rates are sticky and does not rise as fast as the inflation rate, high inflation will reduce the real interest rate, discourage savings, and increase imports.

When inflation is low, any further rise in the market interest rate may raise the real interest rate and savings, thus reducing effective demand and output. In this sense, when recessions come, inflation has a positive impact on consumption, investment and output while a rise in market interest rates may reduce output growth. Such nonlinearity holds for most macro relationships. Ferson and Merrick (1987), for example, find that consumption growth is lower relative to treasury bill returns in recession periods than in non-recession periods. Boskin (1978) also finds that when inflation is low, inflation has a positive impact on consumption and vice versa when the inflation rate is high. Other nonlinear responses may also yield either a positive or a negative policy multiplier (Looney and Frederiksen, 1987; Kormendi and Mequire, 1985; Lothian, 1985; Gupta, 1987).

The response of a macro policy differs slightly among countries. In Brazil and Taiwan, for example, low inflation may increase savings and the trade surplus whereas high inflation will reduce savings, and increase domestic investment and consumption. In Saudi Arabia, however, output growth can sustain a slightly high rate of inflation, since inflation was primarily caused by an increase in oil exports instead of by monetary expansion. Oil exports increase the holdings of foreign assets, the money supply, and inflation. Such high inflation will increase wages, and reduce domestic investment but will increase consumption. Such a wage-push inflation causes declines in the manufacturing sector. It is called "Dutch Disease", a remedy for it is to reduce oil exports and inflation. In the Islamic world, the interest-free system, however, may discourage equity-financing in favor of debt-financing and credit-rationing, stimulating inefficient investment and inflation.

When the inflation rate is high, say higher than say 4%, as Zeira (1987) has argued, increases in savings will raise the domestic investment by a larger amount than the foreign investment or trade surplus. When the inflation rate is low, however, increases in savings will often reduce effective demand and domestic investment, and increase in trade surplus because exports and the domestic manufacturing industries become more competitive at low levels of inflation.

Methodologically, linear regression cannot detect such relationships. For example, according to the study by Looney and Frederiksen (1987), a rise in the budget deficit will increase output and savings when the inflation rate is low. Such a budget deficit, however, will reduce savings if output does not rise while the inflation rate is high, as shown in the following linear regression analysis in Mexico:

$$\underset{}{dS} = \underset{(9.8)}{0.46dY} - \underset{(-2.9)}{0.59dG} + \underset{(2.3)}{0.15dD} \qquad R^2 = 0.99 \qquad (1.1)$$

$$\underset{}{dS} = \underset{(9.8)}{0.46dY} - \underset{(-2.9)}{0.59dG} - \underset{(-14.3)}{1.78dD} \qquad R^2 = 0.89 \qquad (1.2)$$

where S is savings, dS is the change in savings, Y is output, and G government spendings, and D is deficit, i.e., government spendings minus tax revenues. All variables are expressed in real terms. The figures in parentheses are t statistics. A dummy variable, denoting the period 1954-63, is omitted in Eq. (1.1). In fact, Eqs. (1.1) and (1.2) can be expressed in the form:

$$S = a\, f(Y)\, D \qquad (1.3)$$

The coefficient, a f(Y), of the budget deficit is a nonlinear function of income and can be expanded as follows:

$$S = a\,D \quad \text{if } Y < \dot{Y}^* \quad \text{or} \quad p < p^* \tag{1.3}$$

$$S = -a\,D \text{ if } \dot{Y} > \dot{Y}^* \quad \text{or} \quad p > p^* \tag{1.4}$$

where $\dot{Y}$ is the growth rate of output, and $\dot{Y}^*$ and p^* denote an inflection point of output growth and the inflation rate respectively.

Similarly, in Chile and Brazil, Sheehey (1980) finds that inflation offsets the efforts of the monetary authorities' sterilization. The trade deficit will reduce the domestic money base if the government cannot finance a payment deficit. In his linear regression, the coefficients change in magnitude or in sign after the variables are deflated by the price level. Thus a decrease in foreign assets could reduce the domestic money supply and the price inflation rate, theregy appreciating the domestic currency, making exports competitive, and promoting output growth. Thus, savings and gross national product are a nonlinear fuction of the fiscal policy and the inflation rate. Since the relationship between income and government expenditure is nonlinear, sometimes positive and at other times negative (Ram, 1987), such nonlinearity rejects Wagner's hypothesis that government expenditures vary positively with income or economic development.

Such nonlinearity and policy noises can cause business cycles. According to Classical economics, business cycles are real cycles and reflect the time-lag interactions among output, employment, and prices, while according to Keynesian economics, business cycles are caused by sticky prices and ineffective demand. During a period of recession, firms in the manufacturing sector may cut down on output instead of prices, enhancing the unemployment rate. The time lag pattern of cycles may be caused by imperfect information and exogenous shocks. Policies may have no real effect whenever they are anticipated by the private sector (Lucas, 1980) Cycles can also be caused by endogenous factors, such as changes in the real interest rate whenever there occur changes in the wealth effect and in the intertemporal substitution effect on consumption. Business cycles often take the form of the time-lag or a high order autoregressive form. Recessions can lead to excessive savings and a trade surplus. Such a time-delay effect of cycles can be transformed into a high-degree polynomial. Thus we require nonlinear counter-cyclical responses of policies. In our study, each commodity has different supply and demand, elasticities, affecting each other. Several cyclical periods may coincide

These cycles are reflected in a time-delay autoregressive pattern because job search, investment search, and the implementation of innovations all take time. Thus, all kinds of exogenous and endogenous, real and financial shocks may lead to business cycles and affect each other.

In the present paper, Section 2 presents a general theory on open economy macroeconomics. Section 3 describes some empirical studies. Concluding remarks are contained in Section 4.

2. A new macro theory for an open economy

In times of inflation, people shift their holdings of real balances into investment in real assets. Inflation also reduces purchasing power, inhibiting consumption. However, when inflation is low, inflation stimulates profits, investment and consumption as well as output growth. So aggregate profits grow faster than aggregate wages when there is excess capacity. The price level overshoots due to the time-delay effects of production costs on prices or profits. Such nonlinearity reflects non-proportional changes among the money supply, prices, wages, and the exchange rate subject to the constraints imposed by wage contracts and the loan contract. However, as inflation stays high, wages become price-indexed; and the costs overshoot the prices and reduce profits and discourage production. When inflation is high, contractionary policies are often introduced, but recessions may come unexpectedly due to reductions in both the demand and supply of output.

The policies are noneutral because not all prices change proportionally. Prices of manufactured goods and services are stickier than agricultural or nondurable consumers' goods. The producers of manufactured goods often adjusted output more than prices during the great depression (Bosworth, 1982) probably due to the monopolistic power. This reflects non-proportional or noneutral changes in prices over business cycles. According to the data of *International Financial Statistics*, during the period 1950–1985, though the price index of consumers' goods in the world has risen equi-proportionally with the money supply, the income velocity of money and quasi-money has declined, and commodity prices have not risen as fast as world GDP in real terms. In the period 1980–1986, as the U.S. monetary expansion slowed down, commodity prices fell, since the nominal commodity prices quoted in dollars declined when the dollar appreciated. Although the consumers' prices and the money supply rose faster in the developing countries than in the industrial countries, the growth rates of real GDP rose, on average, only slightly faster in developing countries than in industrial countries.

Let C be consumption, I private investment, T tax revenues, and Y GDP. All variables are expressed in real terms. M denotes the money supply, P the price level, p the inflation rate, r the market interest rate, u the unemployment rate, and e the foreign exchange rate. The subscript -1, as say in C_{-1}, denotes the lagged variable. $\dot{Y}$ is the growth rate of output. The nonlinear relationships among macro variables appear as follows:

When inflation is low, inflation stimulates economic growth, and vice versa when inflation is high, as in Eq.2.1.1.

(2.1.1a) when inflation is low:

$$\dot{Y} = a_1 \, p \text{ if } p < p^*$$

(2.1.1b) when inflation is high:

$$\dot{Y} = -a_2 \, p \text{ if } p > p^*$$

$$\dot{Y} = -a_1 (r-p) \quad \text{if } p < p^* \tag{2.1.2a}$$

$$\dot{Y} = a_2 (r-p) \quad \text{if } p > p^* \tag{2.1.2b}$$

where p^* is an inflection point of the inflation rate. $r-p$ is the real interest rate.

In Eq. (2.1.2a), when inflation is low, it denotes a high real interest rate and a high rate of return. People tend to substitute future for current leisure, and increase work hours and real output. As shown in Eq. (2.1.2b), when inflation is high, a rise in real interest rates will increase savings and output growth.

(2.2) Consumption tends to be discouraged by inflation when inflation is high, wherease too low inflation may also reflect ineffective demand. Only modest inflation yields high returns and encourages work instead of leisure. Consumption declines while savings increases. When inflation is very high, future inflation is high and variable, savings will increase so as to insure the future stream of income (Gupta, 1987), and consumption will fall again. Eq. (2.2a) shows that consumption increases with inflation, and vice versa in Eq. 2.2b:

$$C = a_1 (Y-T) - a_2 (r-p) + a_3 C_{-1} \tag{2.2a}$$

$$C = a_1 (Y-T) + a_2 (r-p) \tag{2.2b}$$

(2.3) Investment is discouraged by inflation when inflation is high, because the purchasing power of money is reduced (Stockman, 1981). Low inflation induces the substitution of investment and physical capital for money holdings. Eq. (2.3a) shows when inflation is low, inflation stimulates investment, and a rise in real interest rates will reduce investment by increasing the interest cost. And vice versa when inflation is high, as shown in Eq. 2.3b)

$$I = b_1 Y - b_2 (r-p) \quad (2.3a)$$

$$I = b_1 Y + b_2 (r-p) \quad (2.3b)$$

(2.4) Demand for real balances will be reduced when inflation is high and the market interest rate rises, because the interest rate denotes an opportunity cost of holding money, and vice versa when inflation is low: (Gibson, 1970). Eq. 2 4a shows that a rise in interest rates will increase the demand for money balances, causing ineffective demand and recession when inflation is already low:

$$\text{Log } M/P = c_1 \text{ Log } Y + c_2 r \quad (2.4a)$$

$$\text{Log } M/P = c_1 \text{ Log } Y - c_2 r$$

(2.5) The Phillips curve is positively sloped when inflation is high (Kimbrough, 1986), and vice versa when inflation is low, as shown in Eq. 2.5a.

$$u = -d_1 p \quad (2.5a)$$

$$u = d_1 p \quad (2.5b)$$

(2.6) The exchange rate, e, are determined by the purchasing power parity, P/P*, and the interest rate parity, r-r*. A fall in the domestic price, P, or a rise in the domestic interest rate, r, will appreciate the domestic currency. However, when the output growth rate increases, there will be structural changes in the equation of the exchange rate. Eq.2.6a shows that when the output growth is low, a depreciation will promote trade surplus and enhance the output growth. In Eq.2.6b, when the output growth is high, depreciation not only cannot promote output growth but may have an adverse effect.

$$\text{Log } e = \text{Log } P - \text{Log } r \tag{2.6a}$$

$$\text{Log } e = -\text{Log } P + \text{Log } r \tag{2.6b}$$

where the exchange rate, e, denotes the domestic currency price per unit of foreign currency.

To close the model, an identity of income and expenditure is

$$Y = C + I + G + (X - e X^m)$$

where Y is output, G government spending, X exports and X^m imports. All variables are expressed in real terms.

The above model can be detected by the piecewise regression between two variables, Y and p (Tishler and Israel, 1981; Wecker and Ansley, 1983):

$$\begin{aligned} Y &= a_1 p + v \quad \text{If } p < \bar{p} \\ Y &= -a_2 p + v \quad \text{If } p \geqslant \bar{p} \end{aligned} \tag{3}$$

These relationships between Y and p constitute a nonlinear model, $Y = F(p) + v$ where the v are stochastic errors. For example,

$$Y = a_1 p + a_2 p^2 + a_3 p^3 + v \tag{4}$$

As time t goes to infinity, $\lim p(t) = p$ according to the definition of the boundary value problems. Eq. (4) is a nonsteady state of cyclical equations of output Y and inflation p because the second-degree polynomial is not equal to zero, $f(p) \neq 0$.

The response model, (4), is close to an autoregressive moving average model:

$$\begin{aligned} Y(t) &= b_1 p(t) + b_2 p(t-1) + b_3 p(t-2) + v(t) \\ &= (b_1 + b_2 L + b_3 L^2)\, p(t) + v(t) \end{aligned} \tag{5}$$

Both Eqs. (4) and (5) are equivalent if both minimize the squared errors. Here, L denotes the lag operator. The length of lags denotes the half-cycle period. In (4) changes in the sign of the coefficients are necessary for the existence of real roots and cycles. The disturbance, v(t), denotes the impact of imperfect information or exogenous variables, and could be a cause of financial and real shocks in the process of business cycles. The autoregressive relation of output, employment and prices constitutes the cyclical patterns (Otani, 1985). Equivalently, the polynomial of degree two or more may cause cyclical movements between Y and p and may have a pair of complex roots. Thus the counter-cyclical policies are analogues of adaptive control rules (5) or (4).

In what follows, we derive a new optimal response rule with a nondifferential optimization technique. Snyman and Fatti (1987)'s nonsmooth optimization method is applicable to a single global optimality only but cannot detect the common paths of multiple global optima. Previous optimal control methods are hardly applicable when there are multiple state and multiple policy variables or when the objective function is not monotone convex (Norman, Lasdon and Hsin, 1983; Nishimura, 1981).

Suppose that Y and X are vectors of state variables and control variables our problem is to minimize a cost function or to maximize the consumers' utility, U:

$$\begin{aligned}&\text{maximize } U(Y, X)\\&\text{subject to the model constraint } G(Y, X)\end{aligned} \tag{6.1}$$

Eq. (6.1) can be written as a uncontrained maximization:

$$\text{maximize } F(Y(X)) = E\int_0^{\infty} e^{-(r-p)t}\, U(X, Y(X))dt \tag{6.2}$$

where E denotes the expected state space over an infinite time horizon, and r-p is a discount factor r-p $\geqslant$ 0. The control problem is to steer the strategy X which maximizes the utility, U. When there are inequality constraints, we may use the exact penalty technique and measure the penalty due to violations of the constriants (Bartels and Mahdavi-Amiri, 1986).

To optimize F(Y) is to take the first derivatives of F(Y(X)) with respect to Y and X:

$$d\ F/d\ Y = 0 \quad \text{and} \quad d\ F/d\ X = 0 \tag{7.1}$$

With a probability density $P(Y > y) > 0$, the solutions of the simultaneous equations (7.1) are, in special cases, a linear response rule:

$$d\ F/d\ X = X^* - a\,Y^* = 0 \qquad \text{if } Y > y \tag{7.2}$$

$$d\ F/d\ Y = Y^* - b\ X^* = 0 \qquad \text{if } Y > y$$

where a random variable Y is such that the probability $P(Y < y) = 0$. $Y > y$ is a lower bound of optimality conditions. $X^* = b\ Y^*$ is a trajectory of the optimal response X^*. This gives a piecewise response rule in the probability sense. The Lagrangian multiplier is not entered here because the product of the multiplier and the discount rate $e^{-(r-p)t}$ tends to zero as the time, t, goes to infinity. This serves as a transversality condition. We consider a control policy with an unbounded horizon where both finite and infinite terminal times are feasible.

If both optimal solutions X^* and Y^* maximize the utilities of $F(Y(X))$, they should be within the feasible region, and should satisfy the negativity of the second-order necessary condition, i.e. Pontryagin's maximum principle:

$$F(Y^*, X^*) \geqslant F(Y, X) \qquad X^* \subset X \text{ and } Y^* \subset Y \tag{8.1}$$

This principle is applicable to both continuous data and discrete data.

An admissible rule need not refer to any specific observations. The optimal response rule, sometimes called the Golden rule, is an intertemporally efficient output path since it is an estimator of the efficient or good experiences of policy responses. A sample is good when the observations do not deviate far away from the best one. Here the optimal response rule is estimated using the good subsample of the period under consideration. The category of the good population can be defined by the policy makers. By using the sample as a whole, the ordinary regression equation can be regarded as an average response rule, which contains both the good and bad responses. The optimal response rule can reflect those good policy path while optimal control often yields only an optimal fixed point. The nonlinear response rule of (4) is time-invariant but can reflect the time-delay effect, (5), of policies.

Let control variables be government expenditure, G, the money supply, M, tax revenue, T, and the inflation rate, p, i.e. $X = (G, M, T, p)$. The solutions of Eq. (7) may yield the nonlinear optimal response paths if the objective function is nonlinear:

$$X^* = f(Y^*) \text{ and } Y^* = g(X^*) \qquad X^* \subset X \text{ and } Y^* \subset Y \tag{8.2}$$

which should satisfy Pontryagin's principle. In game theory, these equations (8.2) denote the strategies of two parties X and Y.

In a certainty case, one state variable should be steered by one policy. However, when one control policy cannot perfectly control one state variable, we need to use multiple control policies to steer one state variable. In such cases, instead of using the Gauss-Seidel algorithm, it is more efficient to use the reduced form along with the Newton algorithm than to use the whole model. If the functions U(Y, X) or F(Y(X)) are nonlinear, the reduced form, $X^* = f(Y^*)$, is difficult to derive. Instead of using analytical solutions, the differential equations of (7) are often approximated by the difference equations:

$$F'' = \frac{d F'}{d X} = \frac{F'(X(i+1)) - F'(X(i))}{X(i+1) - X(i)} \tag{8.3}$$

where i = 1, 2 . . . denotes the ith iteration. At a root of the equation $X^* = X$, $F'(X(i+1)) = 0$. A solution under the Newton-Raphson algorithm is, say

$$X^* = X(i) + F'(X(i))/F''(X(i)) \tag{8.4}$$

According to Taylor's expansion series, the maximal utility is

$$F(X^*) = F(X(i)) + F'(X(i))(X^* - X(i)) + R \tag{8.5}$$

where the remainder, R, is assumed to be negligible. If the solution (8.4) is nonlinear, it can be written as a time-delay model (5) or a time-invariant equation, (4).

When the concave function of utility, F, is smooth and has a unique solution this fixed point can be obtained by solving the Eqs. in (7.1) simultaneously. When the objective function is nonlinear and not smooth, the solutions in (7.1) may take the form (4), (5) or (8.2) instead of (7.2), since there could be multiple locally optimal roots but one or several globally optimal solutions.

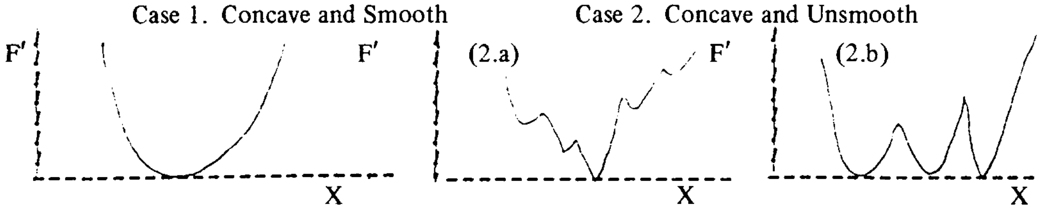

Fig.1 Objective functions and their derivatives

When a system of simultaneous equations is not smooth, as in Fig. 1 Case 2, the Newton algorithm cum with the Gauss-Seidel algorithm can not yield the global solution of case 2.b and cannot provide a confidence interval and an optimal response path with time invariance.

A nonlinear function, F(Y, X), may yield various reduced forms, i.e., X = f(Y). The average response function, X = f(Y), is estimated using the whole sample. The optimization of utility, F, yields say

$$F'(X^*, Y^*) = X^* - g(Y^*) = 0 \qquad \text{for } Y^* > y$$

An optimal response function is $X^* = g(Y^*)$. This optimal path would be the same as the trajectory of iterations (8.4). By substituting some initial values of Y and X, X = g(Y) can be iterated and lead to the same fixed point as the Newton algorithm does. The fixed point X^* lies at the intersection of the two lines X = f(Y) and X = $f(Y^*)$ because $U' = X^* - g(Y^*) = 0$, $X = f(Y) = g(Y^*)$ when $X^* = X$ and $Y^* = Y$. Thus the optimal values X^* and Y^* lie within the feasible region of X and Y, i.e., $X^* \subset X$ and $Y^* \subset Y$. In numerical analysis, this optimal response function sometimes is called the X = g(X) method or the double graphed method, and is often used to locate the solution. With this initial global estimate at the intersected point of the average response and the optimal response path, a refinment can further be made by the Newton-Raphson algorithm.

When we consider the Type I error of including the wrong population and the Type II error of excluding the desired population, a sample can be classified into several categories,

$$\bar{X}_1 < \ldots < \bar{X}_n \text{ and } \bar{Y}_1 < \ldots < \bar{Y}_n$$

$\bar{X}_n$ and $\bar{Y}_n$ are the lower bound of the n-th category. The optimal response rule, $X^* = g(Y^*)$, is estimated by ordinary least squares subject to the categorical control, $x_i > \bar{X}_n$' and $y_i > \bar{Y}_n$. The n-th category is a set of good observations or a concave set from the sample, and can be estimated to yield the optimal response functions. If the objective utility declines over the business cycles, the optimal response rule should be counter-cyclical.

The optimal response rule is estimated using the good observations. Such an optimal rule is time-consistent since it is applicable to the whole sample period. To estimate the optimal response rule subject to constraints (8.1) we use commands such as "SKIPIF (Y.LE.0.10)" and "SELECTIF (Y.GE.0.055) in the computer packages. Finally, since no single policy has a perfect controllability, several policies are used to steer a state variable, say economic growth. Similarly, one policy could also steer multiple targets in probability sense.

3. Empirical study

A comparison of the average response and the optimal response functions is made by using the Saudi Arabia data, as published in *International Financial Statistics* by the *International Monetary Fund.* We first estimate the nonlinear response of output and components of output to varying inflation rates. Since the interest charged is regarded as return without effort, and is prohibited by the Islamic religion, data on interest rates are unavailable in Saudi Arabia. Owing to loan contracts, however, the market interest rate or profit shares are sticky. As the inflation rate rises, the real interest rate and investors' profits may decline. In Table 1, the average response functions of (4) or (8.2) are estimated using the whole sample, and plotted in Fig. 1. Some of the regression equations are not listed in Table 1 but are available on request.

By using the sample over the period 1968-84, Fig. 1 denotes an average relationship. When the inflation rate deviates away from 4%, investment and consumption tend to fall. The inflation rate of 4% is most favorable for domestic investment. An inflation rate beyond 4% may increase imports, and speculative activities and crowd out private investment. In Saudi Arabia, more than half of domestic investment and exports are in the oil and petrochemical industries. Almost 70 percent of the tax revenues come from the exports of the crude oil. To reduce inflation, and encourage domestic investment, Saudi Arabia needs to reduce oil exports, the money supply, and government spending and should not appreciate its domestic currency.

Modest inflation often promotes exports and investment. When inflation is very low or very high, either the economy may suffer recessions or the capital may flight. Exports then decline. High inflation will increase imports and cause a trade deficit. In Saudi Arabia, inflation is caused primarily by increases in oil exports. Thus exports are not deterred by inflation, and the trade surplus has continued to expand. As a result, for example, modest inflation maybe generated by a trade surplus in Saudi Arabia, but a trade deficit in Brazil.

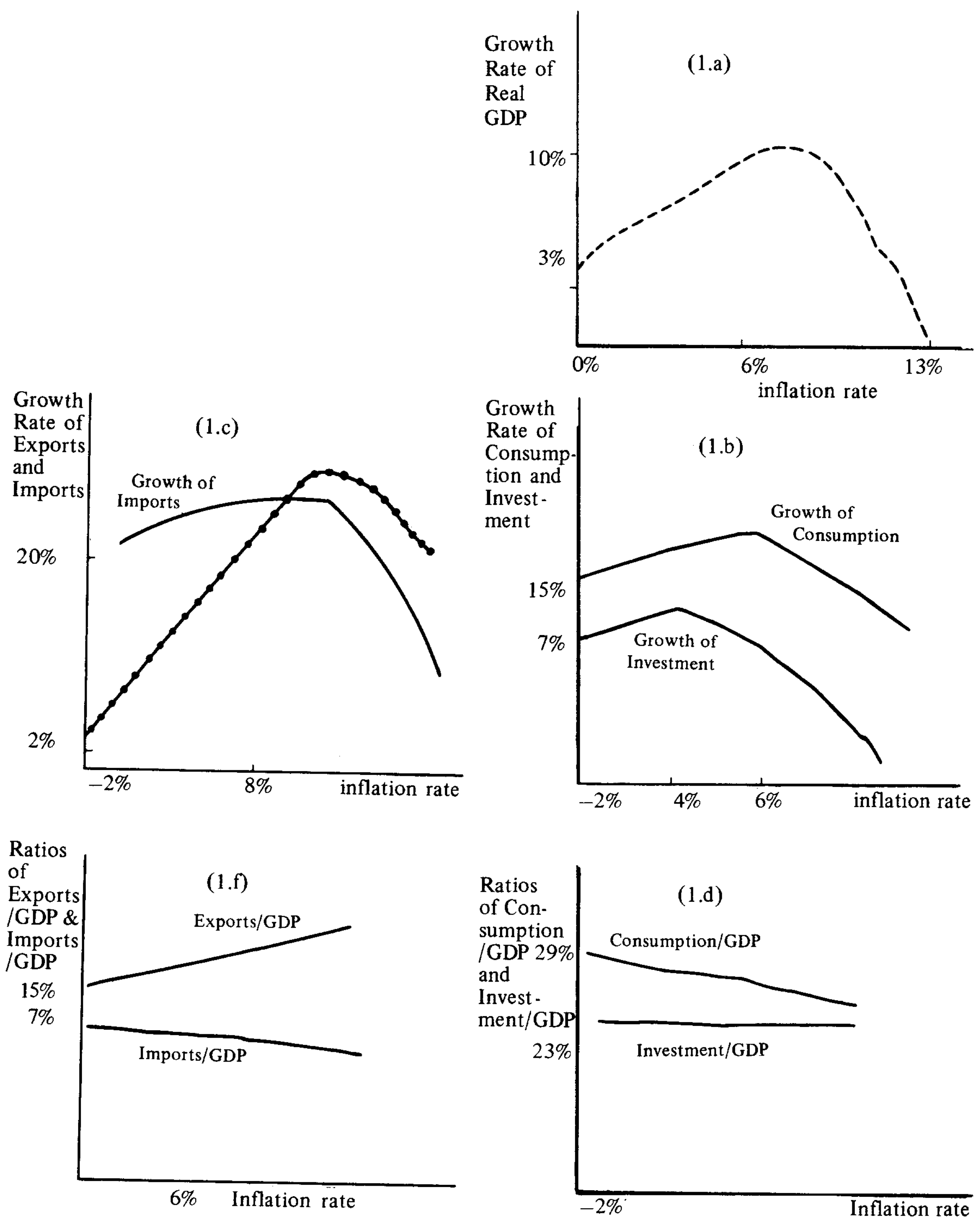

Fig. 1. Macro relationships in Saudi Arabia

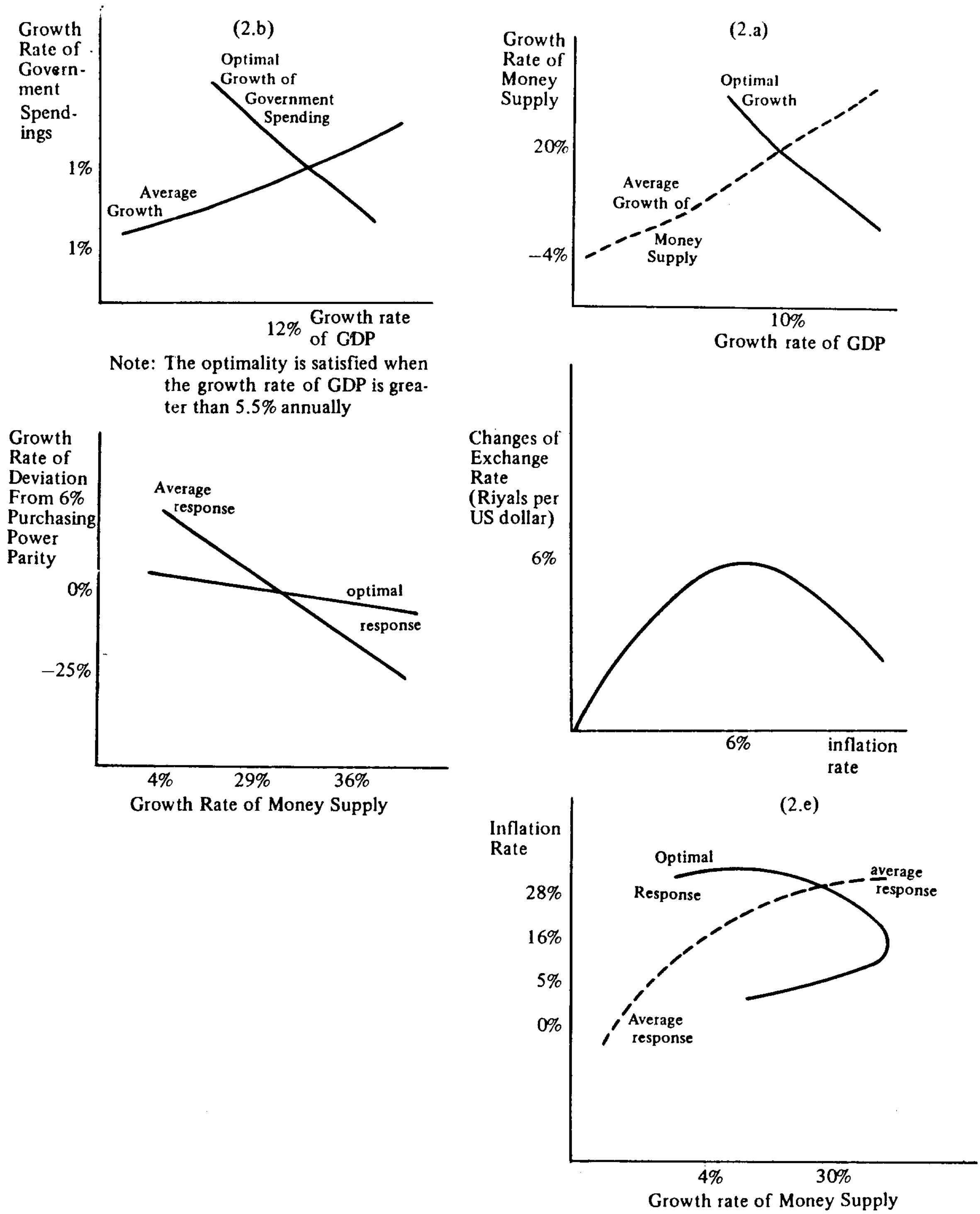

Fig. 2. Policy responses in Saudi Arabia

Table 1. Average response functions of macro variables in Saudi Arabia over the period 1968–1984

Explantory Variables \ Dependent Variables	Consumption Growth Eq. (9)	Investment Growth Eq. (10)	Exports Growth Eq. (11)	Imports Growth Eq. (12)	Output Growth Eq. (13)
Intercept	0.164 (2.27)	0.08 (0.787)	0.081 (0.973)	0.233 (3.13)	0.0059 (0.072)
Inflation p	0.790p (0.573)	0.327p (1.53)	3.118p (3.023)	1.609p (1.760)	1.423p (1.396)
p^2	$2.496p^2$ (−0.344)	$-3.51p^2$ (0.31)	$-9.992p^2$ (−2.447)	$-8.040p^2$ (−2.222)	$-7.200p^2$ (−2.354)
p^3	$-76.22p^3$ (−0.83)	$-168.6p^3$ (−1.20)	$8.838p^3$ (2.998)	$5.482p^3$ (2.098)	$6.631p^3$ (2.690)
p^4	$260.9p^4$ (1.10)	$463.56p^4$ (1.27)			
p^5	$-170.5p^5$ (−1.19)	$-283.2p^5$ (−1.28)			
R^2		0.34			
$\bar{R}^2$	0.401	0.04	0.901	0.284	0.317
D. W.	1.290	1.07	1.771	1.233	1.622
S. E.	0.162	0.25	0.279	0.247	0.174
N	17	17	17	17	17

Note: The figures in parentheses are the t statistics. Although some t statistics are not significant due to multicollinearity, the explanatory power of each equation is significant under the F test.

By comparing the average responses of consumption, investment, and exports and imports, as shown in Fig. 1, we find that the growth rate of consumption spending is higher in Saudi Arabia than in other countries, and its consumption is supported by oil exports rather than by domestic investment. A modest rate of inflation of 4% or less is most favorable for domestic capital formation. Increases in oil exports cause high inflation and capital flight instead of increases in investment.

Fig. 2. a and 2.b show the average responses of the money supply and government spending, which are estimated using the sample as a whole. In the estimates of nonlinear feedback equations, we found that a one percent growth in the money supply tends to change inflation by 0.16% and output growth by 0.44% according to R^2 values. The remaining impact, 40%, is neutral due to perhaps a reduction, in the velocity of money. In contrast, a one percent increase in government expenditure tends to change output by 0.64%. Both money and fiscal policies matter.

In Table 2, the optimal response functions are estimated with those observations in which the annual growth rates of gross national product are not less than 5.5% according to Eq. (8.2). An alternative optimality is defined in which the growth rate of output is greater than 5.5% and inflation is less than 10%. The latter optimality condition is not estimated here because of the small number of observations remaining. In Fig. 2.a, the optimal monetary response is that the growth rate of the money supply should be slowed down when the growth rate of output rises to avoid inflation pressure.

In Eq. 16 and Fig. 2.b, to maximize the growth rate of real GDP, the optimal government expenditure should increase to stimulate effective demand when the growth rate of output declines. Eq. (16) shows that the optimal government expenditure should decline as inflation increases. Thus the macro policies should be counter-cyclical. To avoid crowding out the investment in the private sector (Wells, 1986), the optimal growth rate of government expenditure can range from 11% to 18%, which is associated with the economic growth of 11%, while average government spending grows at 29% annually, as shown in Table 3.

Since oil exports could increase foreign assets, thereby inducing monetary expansion and inflation, the causality between inflation and the currency depreciation (riyals per U.S. dollar) may go in both directions. Fig. 2.c shows a J curve. When inflation is less than 6%, the exchange rate may overshoot the inflation rate and vice versa when the inflation rate rises higher than 6%. When inflation is low, the exports of manufactured products have a comparative advantage in the world market. This explains why low inflation will make the domestic manufactures more competitive. Thus the short-run effect of the depreciation in the domestic currency sometimes may not improve the balance of trade and payments, even though the prices of financial assets and exchange rates are more volatile than commodity prices (Arndt and Dorrance, 1987) because at high inflation the manufacturing industry will decline due to high production costs and wage levels and the overvalued exchange rate.

Empirically we find a significant optimal response rule of money supply to the real exchange rate but not to the nominal exchange rate. Let S be the spot exchange rate (riyals per U.S. dollar). P* and P are the price levels in the U.S. and in Saudi Arabia. The purchasing power parity is

$$S = P/P^*$$

Taking the logarithmic values and differentiating will yield

$$\log S = \log P - \log P^*$$

The growth rate of deviations from the purchasing power parity is

$$\dot{D} = \log S_t - \log S_{t-1} + \dot{P}^* - \dot{P}$$

As shown in Fig. 2.d, the average response of real exchange rate to money supply in Saudi Arabia is

$$\dot{D} = f(\dot{M}, \dot{M}^2, \dot{M}^3) \quad \text{for all } \dot{Y}$$

An increase in oil exports can raise the foreign assets, thereby increasing money supply. Thus increases in money supply will appreciate the nominal exchange rate. However, the optimal response rule of monetary growth to the deviation from the purchasing power parity is

$$\dot{M}^* = f(\dot{D}, \dot{D}^2, \dot{D}^3) \quad \text{for } \dot{Y} > 5.5\%$$

where the optimality condition is that the growth rate of GDP should not be less than 5.5%. Fig. 2.d shows that when the domestic currency appreciates by say 2%, the optimal money supply should increase and grow at 33% while the actual money supply on the average path grows at around 8%. On the optimal path the exports of the manufacturing industry can maintain their competitiveness along the purchasing power parity.

When we define the average response rule of inflation to monetary expansion as

$$\dot{P} = f(\dot{M}, \dot{M}^2, \dot{M}^3) \quad \text{for all } \dot{Y}$$

Fig. 2.e shows that as the money supply increases, the inflation rate also rises. However, as we estimate the optimal monetary response rule we find that the

$$\dot{M}^* = f(\dot{P}^*, \dot{P}^{*2}, \dot{P}^{*3}) \qquad \text{for } \dot{Y} > 5.5\%$$

money supply is allowed to increase as long as inflation is low. The maximum tolerable growth rate of money supply is 42% when the inflation rate attains 16%, beyond which money supply should decline. The optimal feasible money supply is around 28% where the optimal path intersects with the average path, as shown in Fig. 2.e.

In Saudi Arabia, the average growth rate of real GDP is 7%. To estimate the optimal policies, we select those annual observations in which the annual growth rate of real GDP is not less than 5.5%. The average economic growth rate is 11% during the selected 12 years. Under such an optimal condition, the optimal money supply ranges from 15% to 20% at which the economic growth could be maintained at 11% annually.

When an additional optimal criterion is added such that the inflation rate should not go beyond 10% annually, we find that the optimal growth rate of government spending ranges from 11% to 18%. In Fig. 2.a and 2.b, the optimal response rule of Eqs. (14) and (15) intersects at the fixed point with the average response. In Eq. (16) and Fig. (2.b), the optimal government spending should decrease with output growth and decrease with inflation, to avoid crowding out the private investment since the private sector is more efficient when the economic growth is high. Such optimal response rules (14) and (15) are also found in Taiwan and Brazil. Similarly, Kamas (1986) suggests that in order to protect the growth of the traded manufacturing sector, the economy with Dutch disease should take contractionary macro policies and not appreciate the exchange rate too much.

Finally to verify our result, we find a similar relationship between the optimal response and the average response paths in Taiwan, Brazil and Mexico.

Table 2. Optimal response functions of policies to output growth

Explanatory Variables	The Dependent Variables			
	Optimal Monetary Growth Eq. (14) (Note 1)	Optimal Growth of Government Spending Eq. (15) (Note 2)	Optimal Government Spending (G) Eq. (16) (Note 2)	Average Government Spending (G) Eq. (17) (Note 3)
Intercept	−22.16 (−2.36)	−14.28 (−2.31)	−0.079 (−1.98)	−0.094 (−1.43)
Output growth $\dot{Y}$ or output Y	$831.09\dot{Y}$ (2.37)	$436.49\dot{Y}$ (2.30)	0.00025Y (12.82)	0.00027Y (12.73)
Output growth $\dot{Y}^2$ or inflation p	$-10960\dot{Y}^2$	$-4092.0\dot{Y}^2$	−1.415p	−1.228p
$\dot{Y}^3$	$60911\dot{Y}^3$ (2.37)	$12157\dot{Y}^3$ (2.17)		$0.839p^2$ (4.21)
$\dot{Y}^4$	$-120760\dot{Y}^4$ (−2.36)			
$\bar{R}^2$	0.22	0.34	0.98	0.98
D. W.	1.25	2.06	2.89	2.08
S. E.	0.194	0.211	0.032	0.09
N	12	7	7	17

Note 1: The optimal criterion (1) is that the growth rate of real GDP should not be less than 5.5%.

Note 2: The optimal criterion (2) is that the growth rate of real GDP should not be less than 5.5% and that the inflation rate should not exceed 10%.

Note 3: The whole sample in the period 1968–84 is used.

Table 3. The average growth rates in Saudi Arabia over the period 1964–1984

	Optimal Growth Rate Notes (1)	(2)	Average Growth Rate		Optimal Growth Rate Notes (1)	(2)	Average Growth Rate
Real GDP	11%	10%	7.5%	! Money Supply	28%	31%	27%
Consumption	8%	22%	10%	! Inflation	20%	4%	15%
Investment	12%	24%	13%	! Government ! Spending	29%	27%	27%
Exports	58%	19%	35%				
Imports	20%	35%	15%	! Exchange rate (Note 3)	–0.02%	–1.8%	–1.2%

Note 1: The optimal criterion (1) is set at the annual growth rate of real GDP exceeding 5.5%. Our objective function is to maximize GDP growth. The optimal criterion of 5.5% used here is lower than the average growth rate such that the sample size will not be too small. The remaining sample size is 12 points.

Note 2: The alternative optimal criterion (2) used here is that the growth rate of real GDP should not be less than 5.5% and that the inflation rate should not go beyond 10%. Then there remain only seven data points.

Note 3: The exchange rate is riyals per US dollar.

4. Concluding remarks

To optimize an objective function, the Newton algorithm cannot provide a confidence interval or controllability, and cannot yield a global solution when the objective function is nonsmooth. For such optimal control problems, the present paper proposes an optimal response rule using the good experience of the policy responses within the observation period. The optimal response rule is based on the nonlinearity of relationships among macro variables. Methodologically, the linear regression may bias the nonlinear impacts of explanatory variables. An innovative concept is that policy variables could sometimes show a positive and at other times a negative impact on the dependent variable. Thus we control the categorical response rule.

In general, modest inflation is favorable for output growth, consumption and investment as well as the balance of payment. This conclusion holds in Saudi Arabia as well as other countries. The existence of the interest-free system in the

Islamic countries is a weakness of their monetary policy since the government is unable to raise the interest rate and reduce inflation rates when inflation is high. Finally, low inflation can encourage private investment and cure the Dutch disease, because the capital then will not flight abroad and the money has a greater purchasing power for investment.

For example, in Saudi Arabia, an increase in oil exports may raise the oil revenue and the real GDP. Exports also increase the receipt of the foreign assets and the money supply, thereby enhancing domestic inflation. To encourage domestic investment and cure the Dutch disease, Saudi Arabia needs to reduce oil exports, tax revenue, government expenditure and the money supply. An increase in government spending will raise inflation. High inflation reduces the growth rate of output while deflation also leads to recessions. The optimal inflation rate in Saudi Arabia, is around 4%, the optimal monetary growth rate is around 16%, and the optimal growth rate of government expenditure could range from 11% to 18%. Both fiscal and monetary policies should be counter-cyclical, increasing with output but decreasing with inflation.

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