Multiple Equilibria and International Monetary Coordination

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摘　要

由於一個具有轉折點的總體理論的提出，解決了 Lucas 兩難抉擇的問題。一個非線性的反饋政策用以衡量國際政策協調。最適化的福利函數包含了任何非平滑和不被確知的目標函數。此一途徑被用於最適化和競局策略，它拒斥了 Fama 認爲實質利率是不變的假說，也拒斥了 Kinal 和 Lahiri 所提出的隨機漫步模型，並具有理論和政策的重要性。

ABSTRACT

A macro theory with turning points is provided; and Lucas' dilemma (1976) is solved. The nonlinear feedback policy is estimated for international policy coordination. The welfare functions to be optimized include any nonsmooth and not exactly known objective function. This approach is applicable for both optimization and game strategies. It rejects the hypothesis that the real interest rate is constant (Fama, 1975) or follows a random walks model (Kinal and Lahiri, 1988). This has theoretial and policy relevance (Tobin, 1983) for turning points.

1. Introduction

Many conflicting policy recommendations were made because policymakers have different true macro theories and models in mind (Frankel *et al.*, 1988; Turnovsky, 1988). Conflicts between the Keynesian theory and monetarism, for example, can be caused by the turning point of a polynomial or nonlinear model (Lucas, 1981; Tobin, 1983). According to such a turning point, either an expansionary policy may be adopted for demand-sided management; or a contractionary policy or a tax cut is recommended for supply-side stimulation.

Here, we solve the dilemmas of repeated policy games among government, labor unions, and capitalists (Barry, 1985 and Fischer, 1985).

As the chaos theory implies, the relationships among macro variables are not stationary (Nelson *et al.*, 1982; Frank *et al.*, 1988), may show an explosion and return mechanism, and can be a continuous, high-degree polynomial. The international coördination of macro policies can be evaluated, using the linear and polynomial macro-models. The countries under consideration here include the United States, Brazil, Mexico, Saudi Arabia, and Taiwan. Such an evaluation leads to a nonlinear macro theory and polynomial optimal feedback policies. In order to stabilize world economic growth and price levels, what we need to know is how to stabilize the world monetary growth and realign the relative prices of currencies (McKinnon, 1984; Williamson, 1983). As in a policy game, when the leader economy (i.e. the United States) chooses an expansionary policy to increase monetary growth, other countries need not cooperate with this Stackelberg leader. They may choose a noncooperative, contractionary policy and stabilize the world money supply as long as the exchange rates do not depreciate or appreciate too drastically. Such a policy coordination can be estimated through an easily verifiable nonsmooth optimization technique (Hsieh, 1987). The dynamic programming or the minimization of variances is not necessary (Fukada and Hamada, 1988) because the minimization of output variances does not necessarily lead to the maximal output growth.

As conflicting evidence from different macro theories, a nonlinear relationship may show that the real interest rate or the inflation rate sometimes has a positive impact and sometimes a negative impact on output, exchange rates and other real activities. For example, as Frankel (1979), and Keynesian economists have suggested, the impact of the real interest rate upon the output and exchange rates is negative. Tight monetary policy tends to raise the domestic interest rate, attract a capital inflow, increase savings, and cause domestic currency to appreciate. Thus exports and output will decline. Monetarism, however, has suggested the existence of a positive relationship between interest rates and output growth. When price is flexible, a rise in nominal interest rates often reflects an increase in domestic, relative to foreign, inflation. Thus domestic currency is expected to depreciate, increasing exports and the growth of output. Furthermore, with real money stocks held constant, a depreciation of domestic currency tends to increase domestic output and the domestic interest rate, and thus the inflation rate of domestic goods (Turnovsky *et al.*, 1988, p. 345 and p. 353). It is in this way that the contradictory (positive and negative) relationships between output and the real interest rate arise. Similarly, evidence on the impacts of the real interest rate or the inflation rate on stock returns (Benderly and Zwick, 1985; Friend and Tokytsu, 1987), consumption, investment, exchange rates, trade balances (Frenkel and Gylfason, 1980), and unemployment rates (Kimbrough, 1986; Brubb, 1986) show conflicting effects.

The causality of such changes in structural relations have not been predicted satisfactorily nor explored previously. For example, stock price sometimes has a positive and sometimes a negative impact upon the demand for money (M. Friendman, 1988). Although it is assumed to have negative price-substitution effects when stock price has a negative impact on the demand for real balances, it is alleged to have wealth effects when it shows a positive impact on money demand. Alternatively, such changes have been explained as stochastic coefficients due to changes in time trends or to autoregressive relationships of the coefficients (Chiang, 1988). In our study, however, such variations can be explained over time and vary with changes in the constraints of resources. For example, when a budget deficit is monetized, the consequence is a high inflation rate, which reduces the real interest rate, increases imports and trade deficits, resulting in a lower rate of output growth. High inflation has a negative impact on production under resource constraints. The expansionary fiscal or monetary policies then give rise to a negative output multiplier. And vice versa for an economy with a high real interest rate or low inflation.

When the objective function or the constraints of the models are polynomial, the optimization process could be nonsmooth regardless of whether the horizon is one or multi-periods and whether we use dynamic programming or not. The sequence of optimal policy reactions is normally reflected in the iterative trajectories of computing optimization. However, when the objective function is nonsmooth, the trajectories under the Newton-like algorithms may show a local solution instead of a global solution. Another source of contradictory policy recommendations may arise because, under the Gauss-Siedel algorithm, a nonlinear model may yield a different solution when different initial values of policies are used in simulation.

This study is organized as follows: Section Two describes the macro theory under structural changes. The macro theory is expressed through the piecewise and polynomial equations and models, thereby yielding polynomial feedback policies. Our policy optimizations is an improved double graphic method and applicable for nonsmooth optimization. Section Three provides a numerical example and verifies such a polynomial model through the Goldfeld-Quandt

F test and Spectral frequency analysis. Finally, some empirical estimates of such polynomial policy reaction rules are provided based on these models. Section Four condlues with our remarks.

2. A Macro Theory under Structural Changes

In the following dynamic model, each equation involves both supply and demand sides. The money market is primarily determined by the nominal interest rate while the investment decisions are affected by the real interest rate and output.

In the money demand and supply functions (the LM curve), the demand for real balances increases positively with output (or income) and negatively with nominal interest rates. Interest rates denote the opportunity cost of holding money. However, when the inflation rate is low or when the real interest rate is high, the opportunity cost of holding money is negligible because during a recession there is a lack of investment opportunities. The demand for real balances may increase as the output increases and the interest rate rises. The money supply, including credits, is partly endogenous and could increase with the output or exports. Suppose that the money supply, M^s, is at least equal to money demand, M^d:

$$M^s > M^d = M$$

The real money balances increase with output sales and vary nonlinearly with the nominal interest rate:

$$(1) \quad M/P = a_1Y - a_2i + a_3i^2$$

where Y is real output; i is the nominal interest rate, M the money stock, P the price level, and a_i denotes a constant coefficient. After decomposing eq. (1), when inflation is high, the real interest falls since the market interest rate is bound by the loan contracts. There is an opportunity cost for holding real balances. The market interest rate has a negative impact on demand for money:

$$M^d/P = a_1Y - a_2i$$

When the inflation rate is very low, the real interest rate is sticky and high, the relationship becomes positive (Gibson, 1970; and Poole, 1987):

$$M^s/P = a_1Y + a_2i$$

This nonlinear money demand implies that we need to control both the money supply and interest rates for financial and real shocks, because a money stock may correspond to two interest rates.

In the investment function, investment usually declines as the real interest rate rises since the real interest rate reflects the cost of production. However, when the inflation rate is high or when the real interest rate is low, the investment will exceed savings because debt-financing leverage is more profitable than savings. There are resource constraints, however. For further investment, it is necessary to induce savings through raising the real interest rate or reducing the inflation rate. Thus the investment function can be written as a function of real interest rates, $r = i - p$, and output sales, Y:

$$(2) \quad I = b_1Y - b_2r + b_3r^2$$

where I is real private investment; both the supply of and demand for capital goods can be altered by the macro policies:

$$I^s > I^d = I$$

Equation (2) can also be decomposed. When the inflation rate is low or the real interest rate is high, a decrease in interest rates will stimulate investment, consumption, and aggregate demand. Investment varies negatively with real interest rates:

$$I^d = b_1Y - b_2r$$

When the inflation rate is high, i.e., the real interest rate is low, a reversal of the trend to a rise in real interest rates reduces inflation and encourage savings and investment (Dolley and Frankel, 1987). Indeed, both investment and consumption may increase as the real interest rate rises (Rossi, 1988). That is

$$I^s = b_1Y + b_2r$$

Similarly, in an open economy, according to the Keynesian theory, an increase in interest rates implies an increase in foreign assets and an appreciation in exchange rates. According to the monetarism, a rise in the interest rate implies a rise in domestic inflation and an depreciation of the domestic currency. The current account surplus, the foreign assets and real exchange rate, e P*/P, are all determined nonlinearly by output productivity and indirectly by the real interest rate differential, r:

$$(3) \quad (d/dt)(e P^*/P) = r - r^*$$

$$(4) \quad (X_x - X_m) = c_0 Y + c_1(r - r^*) - c_2 r^2$$

where In Eq. (3), the arbitrage tends to raise the real exchange rate, e P*/P, when there exist a real interest rate differential. (d/dt) denotes a change in exchange rates over time. In Eq. (4), trade balances, $(X_x - X_m)$, increase with output but vary with the purchasing power parity and the interest rate parity. e is the nominal exchange rate (the foreign currency price of the domestic currency). P and P* are, respectively, the domestic price and the foreign price levels. Suppose, the foreign interest rate r*, is constant and ommitted here. To complete the model, we introduce the identity of output (income) and expenditure:

$$(5.1) \quad PY^d = PC + PI + PG + P X_x - e P^* X_m$$

$$(5.2) \quad PY^s = PC + PS + PT$$

where the output supply, Y^s, is at least equal to output demand or expenditure, Y^d:

$$Y^s > Y^d = Y$$

where C is consumption, I is investment, G is government spending, X_x and X_m are, respectively, exports and imports, respectively. T is tax revenues; and S denotes savings and All variables are expressed in real terms.

We can solve the system of the equations (1) – (5) by Cramer's rule or the Gaussian elimination method. There are five endogenous variables, Y, P, I, C, $X_x - X_m$, and five exogenous or policy variables, G, M, i, T and e. According to monetarism, the trade surplus, the exchange rate, e, should freely float so as to balance foreign trade, $X_x - X_m$. It is assumed that the price levels are primarily affected by money supply. As a response to both the possibilities of monetary and real shocks and the nonlinear relationship among variables, both the money supply and the nominal interest rate should be controlled. Keynesian theory predicts that changes in the real interest rate have a real effect on investment, output and employment. Savings, S, are regarded as residual and determined by the identities (5.1) and (5.2). The Walransian theorem states that when the money and commodity markets are in equilibrium, the bond market will be in equilibrium.

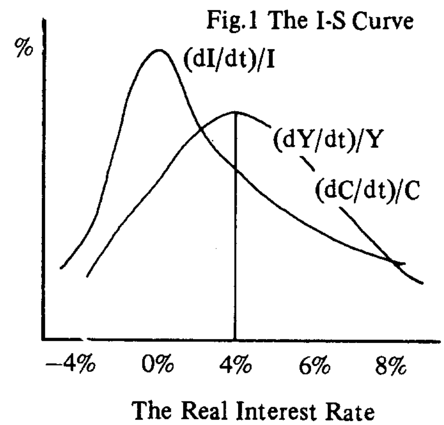
By combining equations (2) through (5), we can derive the investment-savings function (the IS curve) as follows:

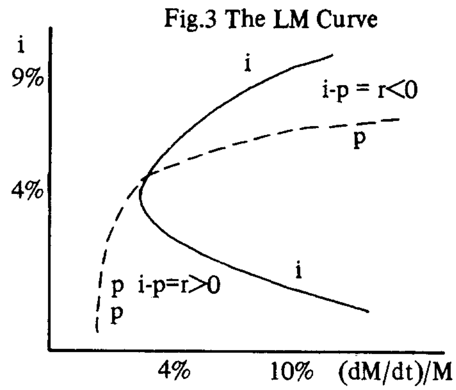
$$(6) \quad Y = (1/(s + m)(A_0 - d_1 r + d_2 r^2)$$

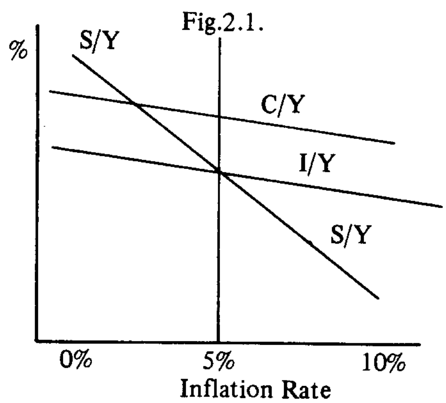
where A_0 is the autonomous absorption component of the expenditures, s is the marginal propensity to save and m is the marginal propensity for imports. The points of intersection between the LM and the IS curves, (1) and (6), are the full equilibrium point for the money market and the commodity market. Fig. 1 indicates that when the real interest rate rises from, say, –5% to 1%, it stimulates saving, investment, output growth and indirectly consumption. Inversely, when the real interest rate rises too high, output growth will fall due to declines in effective demand. In contrast, as Dornbusch (1976), among others, suggested, a rise in the real interest rate or the depreciation of domestic currency will always raise savings and the growth of output. In Figs. 2.1 and 2.2, domestic inflation is higher than the world average inflation rate, say around 3%, it tends to encourage imports and increase the trade deficits, and reduce output in accord with purchasing power parity. Fig. 3 shows the LM curve. Since the market (nominal) interest rate is less flexible under loan contracts, inflation tends to reduce the real interest rate. However, when the nominal interest rate rises faster than inflation, the inflation rate will decline. Thus the LM curve is nonlinear. There could exist several intersected points between the IS and LM curves, leading to multiple equilibria. The role of policy is to achieve the full-employment equilibrium (Shleifer, 1986).

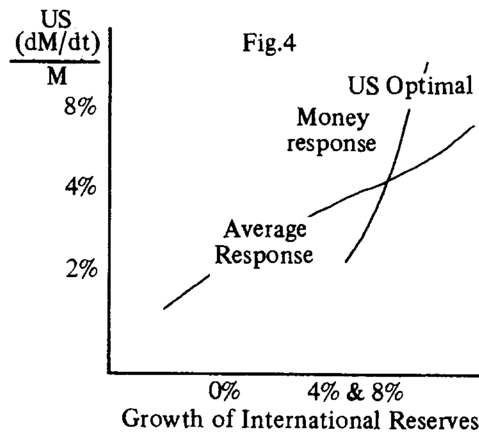
In order to estimate the optimal reaction policy for growth rates, we use the reduced forms and take the first difference of the logarithms for all the variables. For example,

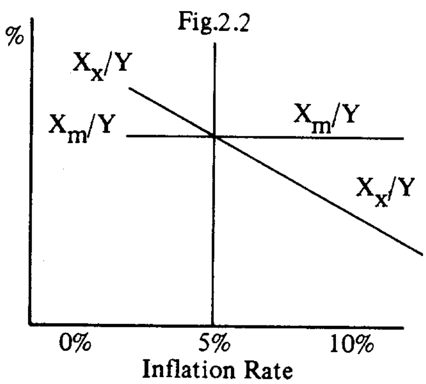
$$y_t = \log Y(t) - \log Y(t-1)$$

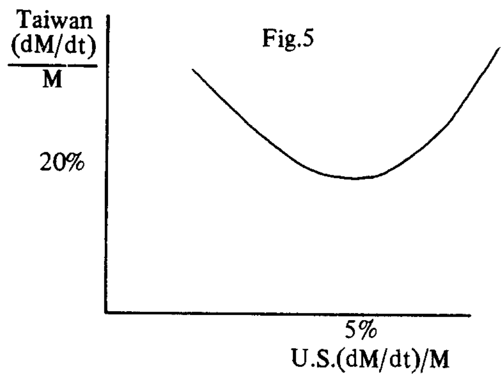












(Assuming that the world real interest rate is 4% and the world inflation rate is 5%).

The model becomes dynamic. By maximizing the output growth and minimizing the inflation, the objective functions for both country, say, Taiwan and the United States, is to maximize a quadractic function of social welfare or utility. The optimization problem is to maximize output growth and minimize the variation of output growth and price inflation:

$$\text{maximize} \quad U = E\, d^t (y^2 - p^2)$$

$$\text{subject to} \quad y = y(m_{us}, m_{taiwan})$$

$$p = p(m_{us}, m_{taiwan})$$

where $d = (1/(1+r)$ is the discount rate over time; y_t, the output growth; p_t is the inflation rate. Both y and p are affected by money growth in the United States and Taiwan, m_t is money supply growth. The time subscript is omitted for the sake of convenience. A time-consistence policy requires that the inflation rate and the real interest rate (i.e., the discount rate) be constant over time. By assuming that the targets are linearly related to the international policy instruments, our focus is limited to the domestic money growth, m_{us}, and foreign money growth, m_{taiwan}, respectively. Since both output and the price levels are a function of the domestic and foreign money supply, welfare can be written as

$$U = U(m_{us}, m_{taiwan})$$

The first order condition of optimization is

$$(6) \quad dU / dm_{us} + (dU / dm_{taiwan})(dU / dm_{us}) = 0$$

where the second terms of the derivative imply an interaction of the domestic and foreign money supply. Let m* denote the optimal solution, regardless of whether the case is a noncooperative Nash equilibrium, a Stackelberg game, or a cooperative solution. The second order condition is approximated by the Pontryagin maximal principle:

$$(7) \quad U(m) - U(m^*) \leqslant (dU / dm)(m - m^*)$$

Here m is a vector, $m = (m_{us}, m_{taiwan})$, and the transitionary condition is that the discounted first order condition at a far distant future time becomes negligible.

The optimal solution of equation (6) is often iterated by means of the Quasi-Newton algorithm and dynamic programming. Since the objective function, U, could be nonsmooth, the iterative solution may not be globally optimal. However, the optimal policy reaction, m*, for the United States can be approximated through a regression equation as long as it satisfies the maximal principle. For example, the second order condition can be approximated by differential equations or by a polynomial:

$$d^2 U / d m^2 \leqslant 0$$

after discretization, it yields

$$(1/h)^2 (U(t) - 2U(t-1) + U(t-2)) \leqslant 0$$

or

$$(1/h)^2 (a_1 U(t) - a_2 U^2(t) + a_3 U^3(t)) \leqslant 0$$

where $h = d m / dt$ is the step size of the computer iteration.

Since the U.S. money supply accounts for about 70% of international reserves, its policy rule would assume an appropriate relationship with world money supply. Thus the U.S. monetary reaction function can be written as

$$m_{us} = f(\text{international reserves})$$

subject to the condition that the U.S. money supply will maximize U.S. welfare, such as output growth and the stability of the price level.

For a small open economy, such as Taiwan, the monetary policy should maximize its output growth, stabilize inflation, and insulate the small open economy from external shocks. For example, the United States is Taiwan's major trade partner. Monetary policy in Taiwan should respond to the U.S. money supply:

$$m_{taiwan} = f(m_{us})$$

subject to its optimality constraint, $U(m) \leq U(m^*)$, as shown in eq. (7). Such an optimization method holds for any economy.

3. Numerical Examples

With the following examples, it rejects the hypothesis that the real interest rate is constant or follows a random walk model. Since the real interest rate is systematically low when inflation is high. A rise in interest rates will stimulate the real activities (Wilcox, 1983). The parameters in a nonlinear model is robust to the policy changes. This solves Locus' (1976) dilemma in which the parameters of the model will change as the policy changes.

3.1 Hypotheses of Parameter-Constancy

By using the data from Benderly *et al.* (1985) over 1955-1981, and the three month treasury bill rate as the interest rate, as reported in the International Financial Statistics, we estimate the simultaneous equations in Section 2. The estimation method used is the three stage least squares of the SHAZAM computer package. The real interest rate under consideration includes both the market interest rate minus the inflation rate, $r = i - p$, and the expected real interest rate $r = i - (0.5(p(t) + P(t-1))$.

To verify that structural changes should be estimated by a polynomial of the third degree rather than by simple linear regression, we use the Goldfeld-Quandt F test. First, we test the impact of the real interest rate upon output growth. When the real interest rate is greater than 1.2%,

$$(8) \quad y = \underset{(2.65)}{0.04} - \underset{(-3.08)}{0.05r} \qquad R^2 = 0.0005$$

$$(9) \quad y = \underset{(2.65)}{0.04} - \underset{(-3.08)}{0.05r} + \underset{(2.33)}{346r^2} - \underset{(-1.66)}{2209r^3} \qquad R^2 = 0.19$$

where y is the output growth and r the real interest rate. The nonstationary coefficients are not within the unit circle because the equation implies an explosion and return mechanism.

When the real interest rate is less than 1.2%,

$$(10)\quad u = \underset{(2.06)}{0.03} + \underset{(2.28)}{0.85\,r} \qquad R^2 = 0.08$$

$$(11)\quad y = \underset{(2.06)}{0.03} + \underset{(2.28)}{0.85\,r} + \underset{(2.06)}{498r^2} + \underset{(2.03)}{8569r^3} \qquad R^2 = 0.53$$

To test the homoskedasticity of the explanatory powers between eqs. (8) and (9), for example, the F test is

$$F = (R^2/4)/(R^2/2) = (0.19/4)/(0.0005/2) = 0.05/0.00025 = 200$$

where the F value exceeds the critical F value at the five percent level of significance, $P(F > 19) = 0.05$ with degrees of freedom 4 in the numerator and 2 in the denominator.

Most Goldfeld-Quandt F tests significantly reject the homoskedasticity hypotheses of the residuals or the explanatory powers, when we either compare equations (8) and (10), or equations (8) and (9) or equations (10) and (11). Equations (9) and (11) are estimated through Tschebychef polynomials. It is an orthogonal regression (Dodes, 1977).

Then the causality test also confirms that when the real interest is very high, say $r > 1.2\%$, the real interest rate has a negative impact on output growth;

$$(12)\quad y = \underset{(-1.71)}{-0.23y_{-1}} - \underset{(-1.46)}{1.46\,r}$$

When all observations, including low real interest rates, are estimated, the output growth increases as the interest rate rises. A rise in the real interest rate has a positive impact on output growth:

$$y = \underset{(-0.67)}{-.05y_{-1}} + \underset{(1.48)}{0.63\,r}$$

where the values in the paranethese are t statistics and are significant at the 80 percentage of probability.

Next we will explain why the real exchange rate deviates from purchasing power parity, $Dex = ex + p^* - p$. Here ex is the rate of change in nominal exchange rates, p^* and p the inflation rates in West Germany and the United States, respectively. According to causality tests,

$$\underset{(2.14)}{Dex = 0.75\,Dex_{-1}} \;-\; \underset{(-1.39)}{4.90\,r} \qquad \text{for } r < 0.9\%$$

and

$$\underset{(1.53)}{Dex = 1.03\,Dex_{-1}} \;+\; \underset{(0.36)}{1.24\,r} \qquad \text{for } r > 1.1\%$$

Further, under the Goldfeld and Quandt F test, when the real interest rate is greater than 1.2%,

$$Dex = \underset{-1.74)}{-0.07} + \underset{(2.26)}{4.22\,r} \qquad R^2 = 0.29$$

$$Dex = \underset{(-1.74)}{-0.07} + \underset{(2.26)}{4.22r} - \underset{(1.57)}{997\,r^2} + \underset{(-1.61)}{6334r^3} \qquad R^2 = 0.90$$

When the real interest rate is less than 1.2%

$$Dex = \underset{(-2.1)}{-0.4} + \underset{(5.2)}{1.21\,r} \qquad R^2 = 0.03$$

$$Dex = \underset{(-2.1)}{-0.4} + \underset{(5.2)}{1.21\,r} - \underset{(-5.87)}{2192\,r^2} - \underset{(-5.90)}{34287\,r^3} \qquad R^2 = 0.81$$

To test the homoskedasticity of residuals, the F test is,

$$F = (1 - R^2)/(T - k)/(1 - R^2)/(T - k).$$

where T is the number of observations, and k is the number of explanatory variables. Thus

$$F = (1 - 0.03)(14 - 4)/(1 - 0.81)(14 - 2) = (0.97 \times 10)/(0.19) \times 12 = 4.5$$

The F value, 4.5, exceeds the critical F value 2.9 with ninety five percent probability. This indicates the rediduals under the linear regression are not homoskedastic. The F tests are statistically significant. Therefore, a polynomial macro theory is more able to explain the data.

In the equation (12), on output growth, the negative first order autoregressive coefficient, −0.23, indicates a higher cyclical frequency of output growth when the real interest rate is high. Further, under the causality test, if $r > 1.1\%$, a rise in the real interest rate has a negative impact on all real activities, including consumption, investment, the real rate of stock return, but has a positive impact on unemployment rates, wages, and the deviation of the exchange rate from purchasing power parity. The relationship is reversed if $r < 1.1\%$ (Hsieh, 1988).

3.2 The Nonlinear Phillips Curve

Since a decline in price or an increase in wages will be accompanied by a rise in unemployment in the following two or three years. The trade-off between inflation, p, and unemployment, u, is better depicted by a second order differential equation or a nonlinear differential equation (Grubb, 1986) or a nonlinear model as follows:

$$\frac{d^2p}{d^2t} = \frac{d^2u}{d^2t} = u(t+1) - 2u(t) + u(t-1)$$

or approximately

$$= a_1u^2 + a_2u + a_3$$

This equation implies that when inflation rates are either too low or too high, there will be ineffective demand or capital flight, and the unemployment rate will rise. When u = sin x or u = cos x, and x denotes the angles, the above equations will show the unemployment rates spiralling over time. By using the data from the U.S. president's Economic Reports, and the National Income and Expenditures, reported by the Government in Taiwan, China, we estimate the nonlinear model as shown in Table 1 below.

Table 1. The Nonlinear Phillips Curves

(The dependent variable is the inflation rate of the GDP price deflator)

	In the United States	In Taiwan	
Intercept	0.023 (1.63)	0.31 (2.91)	0.345 (3.36)
Inflation rate p(t–1) or p^2	1.33p(t–1) (9.17)	$2.61p^2$ (13.45)	
p(t–2) or p^3	–0.32p(t–2) (–1.85)		
Unemployment rate u(t) or u	–0.008u(t) (–2.97)	–0.31u (–2.28)	–0.23u (–2.60)
u(t–1) or u^2	–0.005u(t–1) (–1.68)	$0.104u^2$ (2.02)	$0.046u^2$ (2.55)
u(t–2) or u^3	0.01u(t–2) (4.29)	$-0.01u^3$ (–1.77)	
$\bar{R}^2$	0.87	0.88	0.14
D.W.	1.59	1.86	1.49
S.E.	0.013	0.021	0.06
N	30	30	30

Note: The figures in parentheses are t statistics. As the time t goes to infinity, lim u(t) = u according to the definition of the boundary value problems, u is the unemployment rate, and p the inflation rate.

3.3 The Nonlinear Feedback Rule

Now, we estimate the optimal monetary policy reaction for Taiwan. We select those annual observations in the sample period in which the annual output growth is high, say, $y > 8\%$, and in which the annual inflation rate is low, say $p < 7\%$, and estimate the polynomial regression:

$$m_{taiwan} = a_1 + a_2 m_{us} + a_3 m_{us} \qquad \text{for } y > 8\% \text{ and } p < 7\%$$

In Fig. 4, the optimal U.S. monetary policy is vertical and should maintain a stable international reserve. Fig. 5 indicates that the optimal monetary policy looks like a U shape for a small open economy, such as Taiwan. It implies that when U.S. money growth is very low, the money supply in Taiwan should increase faster and keep the world money supply stable. When U.S. money growth is moderate, the money growth in Taiwan should remain at a moderate rate in accord with U.S. monetary policy. When the U.S. monetary growth is very high, the money supply in Taiwan should expand as fast as the U.S. monetary growth, thereby keeping the foreign exchange rate relatively stable. The U.S.-Taiwan trade volume accounts for as high as 50% of the Taiwan global trade. A drastic appreciation of the bilateral exchange rate will cause recession for this small open economy. Thus, when the leader in the Stackelberg game adopts and expansionary macro policy, the follower sometimes should adopt the same expansionary policy but at other times should adopt a contractionary policy.

4. Remarks

A polynomial macro theory and policies are provided here. It rejects, among others', Dornbusch's (1976) hypothesis that a decline in real interest rates or a devaluation of currencies will always raise output growth and other real activities. At a very low real interest rate or a very high inflation rate, the expansionary monetary and fiscal policy will yield a negative income multiplier and worsen trade deficits. Our optimal policy rule is easier to verify than dynamic programming.

Appendix: An Alogorithm of Approximate Optimization

To find the approximate solutions of nonlinear equations, the double graphic method in numerical analysis is often used (Dodes, 1977). For example, $f(x) = 0$ can be writen as $f(x) = g(x) + h(x) = 0$. The intersection point of $g(x) = 0$ and $h(x) = 0$ will be one of the local solutions. In contrast, we estimate the regression equation, say, $g(x) = 0$, subject to the maximal principle. Such an equation $g(x) = 0$ is called the optimal reaction rule while the unconstrained equation, $f(x) = 0$, can be called the average reaction rule. The intersected point of the optimal response rule and the average response rule is close to an optimal feasible fixed point.

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