

考試科目	微積分	所別	應用數學系 811b, 8111	考試時間	2月23日(日) 第一節
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★ Problems:

1. Evaluate the following limits if they exists.

(a) (5 points) $\lim_{x \rightarrow 0} \frac{(e^{2x^2} - 1 - 2x^2)(\cos x - 1)}{(\sin 3x - \ln(1 + 3x))x^4}$.

(b) (5 points) $\lim_{x \rightarrow 0} \frac{x \cos x - xe^{-x^2}}{\sin^3 x}$.

(c) (5 points) $\lim_{x \rightarrow 0} \frac{e^x - e^{\sin x}}{x - \sin x}$.

2. (10 points) Let $0 \leq \theta \leq 2\pi$ and consider the series

$$\sum_{n=1}^{\infty} (-1)^{n+1} \tan^{2n}(\theta).$$

Determine the values of θ for which the series converges and compute the sum.

3. (15 points) Suppose that f is a continuous real-valued function defined on the closed interval $[0, 1]$. Which of the following must be true?

- I) There is a constant $C > 0$ such that $|f(x) - f(y)| \leq 1$ for all x and y in $[0, 1]$ that satisfy $|x - y| \leq C$.
 - II) There is a constant $D > 0$ such that $|f(x) - f(y)| \leq D$ for all x and y in $[0, 1]$.
 - III) There is a constant $E > 0$ such that $|f(x) - f(y)| \leq E|x - y|$ for all x and y in $[0, 1]$.
4. The intersection of the two surfaces $x^2 + \frac{y^2}{2} = 1$ and $z^2 + \frac{y^2}{2} = 1$ consists of two curves.
- (a) (4 points) Parameterize each curve in the form $r(t) = (x(t), y(t), z(t))$.
 - (b) (3 points) Set up the integral for the arc length of one of the curves.
 - (c) (3 points) What is the arc length of this curve?
5. (10 points) Find all real solutions to the differential equation $\cos^2 x \frac{dy}{dx} + y = e^{\tan x}$.
6. (10 points) Discuss the convergence and divergence of

$$\int_0^\infty x^p e^{-x} dx$$

for $p < \infty$.

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7. Let S be the part of the spherical surface $x^2 + y^2 + z^2 = 2$ lying in $z > 1$. Orient S upwards and give its bounding circle, C , lying in $z = 1$ the compatible orientation.

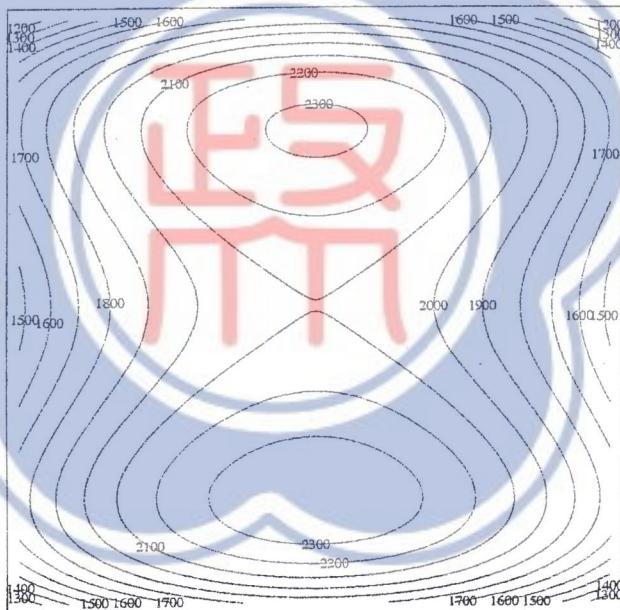
(a) (6 points) Parametrize C and use the parametrization to evaluate the line integral

$$I = \oint_C xz \, dx + y \, dy + y \, dz.$$

(b) (6 points) Compute the curl of the vector field $\mathbf{F} = xz\vec{i} + y\vec{j} + y\vec{k}$.

(c) (8 points) Write down a flux integral through S which can be computed using the value of I .

8. (10 points) On the contour plot below, mark the portion of the level curve $f = 2000$ on which $\frac{\partial f}{\partial y} \geq 0$.



備註	試題隨卷繳交
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Please show all your work.

- Prove that if W_1 and W_2 are finite-dimensional subspaces of a vector space V , then the subspace $W_1 + W_2$ is finite-dimensional, and $\dim(W_1 + W_2) = \dim(W_1) + \dim(W_2) - \dim(W_1 \cap W_2)$.
- Prove that the set of all $n \times n$ symmetric matrices forms a subspace of $M_n(\mathbb{R})$.
Find the basis for this subspace.
- Let T be a linear transformation from $M_n(\mathbb{R}) \rightarrow M_n(\mathbb{R})$ defined by

$$T(A) = \frac{A + A^T}{2}, A \in M_n(\mathbb{R}).$$

Can T be diagonalized?

- Find the eigenvalues, eigenvectors, the algebraic multiplicity and geometric multiplicity of each eigenvalue of the matrix

$$\begin{bmatrix} 3 & 0 & 4 & 4 \\ 0 & -1 & 0 & 0 \\ 0 & -4 & -1 & -4 \\ 0 & 4 & 0 & 3 \end{bmatrix}$$

- Let $V = P_4(\mathbb{R})$, and $W = \text{span}\{1, x\}$. Consider the inner product $\langle f, g \rangle$ on V defined by

$$\langle f, g \rangle = \int_0^1 f(x)g(x)dx, f, g \in V.$$

What is the orthogonal projection of x^2 in W ?

p.s. 20 points for each problem.

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