考試科目	基礎數學	所別	統計學系	考試時間	2月23日(六) 第 一 節

(13) (5%) Let matrix $A=\begin{bmatrix} 1 & 2 & 7 & 5 \\ -2 & -1 & -8 & -7 \\ -1 & 3 & 3 & 0 \end{bmatrix}$, the rank of A = _____.

(14) (5%) Let $A=\begin{bmatrix} 4 & -4 & 2 \\ 2 & -2 & 2 \\ 0 & 0 & 1 \end{bmatrix}$, what is the eigenvector $\mathbf{v}$ corresponding to the eigenvalue $\lambda = 0$?

$\mathbf{v}$ = _____.

(15) (5%) Let $A=\begin{bmatrix} 0 & -2 & 1 \\ 1 & 3 & -1 \\ 0 & 0 & 1 \end{bmatrix}$, find a diagonal matrix B similar to A. B = _____

(16) (5%) Let $A=\begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & \sin\theta \\ 0 & -\sin\theta & \cos\theta \end{bmatrix}$, is A an orthogonal matrix ? (Yes/No) _____

(17) (5%) Continue with problem (16), $\det(A^{-1})$ = _____.

(18) (5%) Let $L: R^3 \to R^3$ be defined by

$$L(x,y,z)=(x-y,x+2y,z).$$

Find a basis for the kernel of L. _____

(19) (5%) Continue with problem (18), find a basis for the range of L. _____

(20) (5%) Let V be the vector space consisting of all functions of the form

$$ae^{2x}\cos x + be^{2x}\sin x.$$

Consider a linear transformation $L: V \to V$ such that $L(f)=f'+f$.

Find the matrix representing L with respect to the basis $\{e^{2x}\cos x,\ e^{2x}\sin x\}$. _____

備註	試題隨卷繳交

考試科目	基礎數學	所別	統計學系	考試時間	2 月 23 日(六) 第 一 節

Part I: 微積分 (填充題)

(1) (5%) $\lim_{x\to\infty} x-\sqrt{x^2+7} =$ _____.

(2) (5%) Assume $\lim_{x\to -1^-} f(x)$ exists and $\frac{x^2+x-2}{x+3} \le \frac{f(x)}{x^2} \le \frac{x^2+2x-1}{x+3}$, then $\lim_{x\to -1^-} f(x) =$ _____.

(3) (5%) For what values of x is the function $f(x)=\sqrt{x^2-2x}$ continuous? _____.

(4) (5%) Let $y=\log(4+\cos x)$, $\frac{dy}{dx}=$ _____.

(5) (5%) Assume that y is a function of x such that $x^3+y^3=4$, then $\frac{dy}{dx}=$ _____.

(6) (5%) Let $f(x)=\int_x^{2x} \sin(t^2)\,dt$, then $f'(x) =$ _____.

(7) (5%) $\lim_{n\to\infty}\sum_{i=1}^{n}\left(\frac{10}{n}+\frac{9i}{n^2}\right)=$ _____.

(8) (5%) $\int \frac{x}{x^2+4}\,dx =$ _____.

(9) (5%) Write down the first three nonzero terms of Taylor series for $f(x)=\log\cos x$ at $x=0$.

_____.

(10) (5%) Let $f(x)=2x^3-9x^2+12x-3$, the maximum value of $f(x)=$ _____.

Part II. 線性代數 (填充題)

(11) (5%) Consider the linear system

$$\begin{aligned} x+ \quad & y= 4 \\ x+(a^2-15)y &= a \end{aligned}$$

What are the value(s) of a so that the linear system has a unique solution: _____.

(12) (5%) For what value(s) of λ is the set of vectors $\{(\lambda^2-5,\ 1,\ 0),\ (2,-2,\ 3),\ (2,\ 3,-3)\}$ linearly dependent ? _____

備註	試題隨卷繳交

考試科目	數理統計學	所別	統計系	考試時間	2月23日(六) 第 3 節

1. If X is a random variable, h(x) is a non-negative real-valued function, and c is any positive constant . Prove that

   (a) (10pts)$P(u(X) \geqq c) \leqq \dfrac{E(u(X))}{c}$

   (b) (5 Pts)Use the result in (a) to prove Chebychev inequality

   $P[\,|X-\mu| \geqq k\sigma\,] \leqq \dfrac{1}{k^2}$, for any $k > 0$, where $E(X)=\mu$ and $V(X)=\sigma^2$.

2. (10pts)Show that when the population size N approaches infinity, and M/N is kept fixed when N approaches infinity, a Hypergeometric distribution Hyp(N, M, n) can be approximated by a binomial distribution.
3. (10pts)Let X_1, X_2 be independent standard normal random variables, derive the distribution of X_1/X_2.
4. (10pts)Prove that: If X_n converge in probability to X and Y_n converge in probability to Y, then $X_n + Y_n$ converge in probability to X+Y.
5. Let $X_1, \cdots, X_n$ be a random sample from a population with pdf $f(x;\theta)=\theta^{-1}\exp(-x/\theta)$, for x, $\theta>0$

   (a) (5pts)Find MLE of θ.

   (b) (5pts)Find MLE of θ^2 and the asymptotic distribution of this estimator.

   (c) (10pts)Find the UMVUE of θ.

   (d) (10pts)Find the MP size α test for testing H_0: $\theta=\theta_0$ against H_1: $\theta=\theta_1(>\theta_0)$.
   (5pts)Is this test UMP for testing H_0: $\theta=\theta_0$ against H_1: $\theta=>\theta_0$? Why or Why not?

   (e) (10pts)Find the size α LR(Likelihood Ratio) test for testing H_0: $\theta=\theta_0$ against H_1: $\theta\neq\theta_0$.
6. (10pts)Let $X_1, \cdots, X_n$ be a random sample from a Bernoulli distribution, $f(x\,|\,\theta)=\theta^x(1-\theta)^{1-x}$, x=0,1, and let θ~UNIF(0, 1). Find the Bayes estimator with respect to a weighted squared error loss; $L(t;\theta)=(t-\theta)^2/\theta(1-\theta)$.

備註	試題隨卷繳交

考 試 科 目	統計方法	系 別	統計學系	考試時間	2月23日(六) 第4節

1. **(15%)** A headache pattern was called "episodic", if the headaches occurred lese often than 15 times a month; otherwise, the headaches were called "chronic." Of the 1600 women in a survey aged 18 to 29, a total of 653 said that they had experienced episodic headaches in the last year. Of the 2122 women in the 30- to 39-year-old age group, the number having experienced episodic headaches was 995.

   (1) Compute a 95% confidence interval for the difference between the proportions in the population for these two age groups who experienced an episodic headache in the previous year. (7%)

   (2) Test to determine whether age group and the risk of episodic headaches are independent at the 0.05 level of significance. (8%)

2. **(65%)** Following are data for 20 individuals on resting pulse rate and whether or not the individual regularly exercises.

Person	1	2	3	4	5	6	7	8	9	10
Pulse	72	62	72	84	60	63	66	72	75	64
Exercises	No	Yes	Yes	No	Yes	Yes	No	No	Yes	Yes
Person	11	12	13	14	15	16	17	18	19	20
Pulse	62	84	76	60	52	60	64	80	68	64
Exercises	No	No	No	Yes	Yes	No	Yes	Yes	Yes	Yes

   (1) Test the equality of the variances of the pulse rate for the two groups defined by whether or not the individual regularly exercises at the 0.1 level of significance. (6%)

   (2) Calculate a 95 percent confidence interval estimate for the difference in means of the pulse rate for the two groups defined by whether or not the individual regularly exercises. (7%)

   (3) Test the equality of the means of the pulse rate for the two groups defined by whether or not the individual regularly exercises using a t-test at the 0.05 level of significance. (7%)

   (4) Verify necessary conditions and state any assumptions for carrying out part (3). (5%)

   (5) Use a one-way analysis of variance method to test the same hypothesis for part (3). (10%)

   (6) Consider a simple linear regression for the relationship between y = resting pulse rate and x = whether or not the individual regularly exercises. Estimate the the slop of the linear regression model. (10%)

備 註	試 題 隨 卷 繳 交

考試科目	統計方法	系別	統計學系	考試時間	2月23日(六)-4

(7) Estimate the standard error of regression. (10%)

(8) Test whether the slope is zero. (10%)

3. **(20%)** Suppose that a simple random sample of $n = 2$ numbers will be selected from the list of values 1, 2, 3, 4, and 5.

(1) How many possible equally likely samples of $n = 2$ numbers that could be selected as sampling without replacement? List all the possible samples, and give the value of the range R for each sample. (5%)

(2) Summarize the results of part (1) into a table showing the sampling distribution of R. (5%)

(3) Find the mean of R. (4%)

(4) Find the variance of R. (6%)

t-table (right tail)

For each row (degrees of freedom k) and column (right tail probability α), the table entry e satisfies $\Pr(t_k \geq e) = \alpha$. Note that the t-distribution is symmetric about 0.

degrees of freedom	right tail probability 0.25	0.10	0.05	0.025	0.01
1	1.000	3.078	6.314	12.706	31.821
2	0.816	1.886	2.920	4.303	6.965
3	0.765	1.638	2.353	3.182	4.541
4	0.741	1.533	2.132	2.776	3.747
5	0.727	1.476	2.015	2.571	3.365
6	0.718	1.440	1.943	2.447	3.143
7	0.711	1.415	1.895	2.365	2.998
8	0.706	1.397	1.860	2.306	2.896
9	0.703	1.383	1.833	2.262	2.821
10	0.700	1.372	1.812	2.228	2.764
11	0.697	1.363	1.796	2.201	2.718
12	0.695	1.356	1.782	2.179	2.681
13	0.694	1.350	1.771	2.160	2.650
14	0.692	1.345	1.761	2.145	2.624
15	0.691	1.341	1.753	2.131	2.602
16	0.690	1.337	1.746	2.120	2.583
17	0.689	1.333	1.740	2.110	2.567
18	0.688	1.330	1.734	2.101	2.552
19	0.688	1.328	1.729	2.093	2.539
20	0.687	1.325	1.725	2.086	2.528
21	0.686	1.323	1.721	2.080	2.518
22	0.686	1.321	1.717	2.074	2.508
23	0.685	1.319	1.714	2.069	2.500
24	0.685	1.318	1.711	2.064	2.492
25	0.684	1.316	1.708	2.060	2.485
26	0.684	1.315	1.706	2.056	2.479
27	0.684	1.314	1.703	2.052	2.473
28	0.683	1.313	1.701	2.048	2.467
29	0.683	1.311	1.699	2.045	2.462
30	0.683	1.310	1.697	2.042	2.457
35	0.682	1.306	1.690	2.030	2.438
40	0.681	1.303	1.684	2.021	2.423
45	0.680	1.301	1.679	2.014	2.412
50	0.679	1.299	1.676	2.009	2.403
gaussian	0.675	1.282	1.646	1.962	2.330

備註	試題隨卷繳交

考試科目	統計方法	系別	統計學系	考試時間	2月23日 第4節

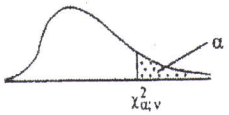
F - Distribution ($\alpha = 0.05$ in the Right Tail)

df_2 \ df_1	Numerator Degrees of Freedom								
Denominator Degrees of Freedom	1	2	3	4	5	6	7	8	9
1	161.45	199.50	215.71	224.58	230.16	233.99	236.77	238.88	240.54
2	18.513	19.000	19.164	19.247	19.296	19.330	19.353	19.371	19.385
3	10.128	9.5521	9.2766	9.1172	9.0135	8.9406	8.8867	8.8452	8.8123
4	7.7086	9.9443	6.5914	6.3882	6.2561	6.1631	6.0942	6.0410	6.9988
5	6.6079	5.7861	5.4095	5.1922	5.0503	4.9503	4.8759	4.8183	4.7725
6	5.9874	5.1433	4.7571	4.5337	4.3874	4.2839	4.2067	4.1468	4.0990
7	5.5914	4.7374	4.3468	4.1203	3.9715	3.8660	3.7870	3.7257	3.6767
8	5.3177	4.4590	4.0662	3.8379	3.6875	3.5806	3.5005	3.4381	3.3881
9	5.1174	4.2565	3.8625	3.6331	3.4817	3.3738	3.2927	3.2296	3.1789
10	4.9646	4.1028	3.7083	3.4780	3.3258	3.2172	3.1355	3.0717	3.0204
11	4.8443	3.9823	3.5874	3.3567	3.2039	3.0946	3.0123	2.9480	2.8962
12	4.7472	3.8853	3.4903	3.2592	3.1059	2.9961	2.9134	2.8486	2.7964
13	4.6672	3.8056	3.4105	3.1791	3.0254	2.9153	2.8321	2.7669	2.7144
14	4.6001	3.7389	3.3439	3.1122	2.9582	2.8477	2.7642	2.6987	2.6458
15	4.5431	3.6823	3.2874	3.0556	2.9013	2.7905	2.7066	2.6408	2.5876
16	4.4940	3.6337	3.2389	3.0069	2.8524	2.7413	2.6572	2.5911	2.5377
17	4.4513	3.5915	3.1968	2.9647	2.8100	2.6987	2.6143	2.5480	2.4943
18	4.4139	3.5546	3.1599	2.9277	2.7729	2.6613	2.5767	2.5102	2.4563
19	4.3807	3.5219	3.1274	2.8951	2.7401	2.6283	2.5435	2.4768	2.4227
20	4.3512	3.4928	3.0984	2.8661	2.7109	2.5990	2.5140	2.4471	2.3928
21	4.3248	3.4668	3.0725	2.8401	2.6848	2.5727	2.4876	2.4205	2.3660
22	4.3009	3.4434	3.0491	2.8167	2.6613	2.5491	2.4638	2.3965	2.3419
23	4.2793	3.4221	3.0280	2.7955	2.6400	2.5277	2.4422	2.3748	2.3201
24	4.2597	3.4028	3.0088	2.7763	2.6207	2.5082	2.4226	2.3551	2.3002
25	4.2417	3.3852	2.9912	2.7587	2.6030	2.4904	2.4047	2.3371	2.2821
26	4.2252	3.3690	2.9752	2.7426	2.5868	2.4741	2.3883	2.3205	2.2655
27	4.2100	3.3541	2.9604	2.7278	2.5719	2.4591	2.3732	2.3053	2.2501
28	4.1960	3.3404	2.9467	2.7141	2.5581	2.4453	2.3593	2.2913	2.2360
29	4.1830	3.3277	2.9340	2.7014	2.5454	2.4324	2.3463	2.2783	2.2229
30	4.1709	3.3158	2.9223	2.6896	2.5336	2.4205	2.3343	2.2662	2.2107
40	4.0847	3.2317	2.8387	2.6060	2.4495	2.3359	2.2490	2.1802	2.1240
60	4.0012	3.1504	2.7581	2.5252	2.3683	2.2541	2.1665	2.0970	2.0401
120	3.9201	3.0718	2.6802	2.4472	2.2899	2.1750	2.0868	2.0164	1.9588
∞	3.8415	2.9957	2.6049	2.3719	2.2141	2.0986	2.0096	1.9384	1.8799

備註	試題隨卷繳交

考試科目	統計方法	系別	統計學系	考試時間	2月23日 文-4

Table of the Chi-square Distribution



α =	0.995	0.99	0.98	0.975	0.95	0.90	0.80	0.20	0.10	0.05	0.025	0.02	0.01	0.005	0.001	= α
ν = 1	0.0000393	0.000157	0.000628	0.000982	0.00393	0.0158	0.0642	1.642	2.706	3.841	5.024	5.412	6.635	7.879	10.827	ν = 1
2	0.0100	0.0201	0.0404	0.0506	0.103	0.211	0.446	3.219	4.605	5.991	7.378	7.824	9.210	10.597	13.815	2
3	0.0717	0.115	0.185	0.216	0.352	0.584	1.005	4.642	6.251	7.815	9.348	9.837	11.345	12.838	16.268	3
4	0.207	0.297	0.429	0.484	0.711	1.064	1.649	5.989	7.779	9.488	11.143	11.668	13.277	14.860	18.465	4
5	0.412	0.554	0.752	0.831	1.145	1.610	2.343	7.289	9.236	11.070	12.832	13.388	15.086	16.750	20.517	5
6	0.676	0.872	1.134	1.237	1.635	2.204	3.070	8.558	10.645	12.592	14.449	15.033	16.812	18.548	22.457	6
7	0.989	1.239	1.564	1.690	2.167	2.833	3.822	9.803	12.017	14.067	16.013	16.622	18.475	20.278	24.322	7
8	1.344	1.646	2.032	2.180	2.733	3.490	4.594	11.030	13.362	15.507	17.535	18.168	20.090	21.955	26.125	8
9	1.735	2.088	2.532	2.700	3.325	4.168	5.380	12.242	14.684	16.919	19.023	19.679	21.666	23.589	27.877	9
10	2.156	2.558	3.059	3.247	3.940	4.865	6.179	13.442	15.987	18.307	20.483	21.161	23.209	25.188	29.588	10
11	2.603	3.053	3.609	3.816	4.575	5.578	6.989	14.631	17.275	19.675	21.920	22.618	24.725	26.757	31.264	11
12	3.074	3.571	4.178	4.404	5.226	6.304	7.807	15.812	18.549	21.026	23.337	24.054	26.217	28.300	32.909	12
13	3.565	4.107	4.765	5.009	5.892	7.042	8.634	16.985	19.812	22.362	24.736	25.472	27.688	29.819	34.528	13
14	4.075	4.660	5.368	5.629	6.571	7.790	9.467	18.151	21.064	23.685	26.119	26.873	29.141	31.319	36.123	14
15	4.601	5.229	5.985	6.262	7.261	8.547	10.307	19.311	22.307	24.996	27.488	28.259	30.578	32.801	37.697	15
16	5.142	5.812	6.614	6.908	7.962	9.312	11.152	20.465	23.542	26.296	28.845	29.633	32.000	34.267	39.252	16
17	5.697	6.408	7.255	7.564	8.672	10.085	12.002	21.615	24.769	27.587	30.191	30.995	33.409	35.718	40.790	17
18	6.265	7.015	7.906	8.231	9.390	10.865	12.857	22.760	25.989	28.869	31.526	32.346	34.805	37.156	42.312	18
19	6.844	7.633	8.567	8.907	10.117	11.651	13.716	23.900	27.204	30.144	32.852	33.687	36.191	38.582	43.820	19
20	7.434	8.260	9.237	9.591	10.851	12.443	14.578	25.038	28.412	31.410	34.170	35.020	37.566	39.997	45.315	20
21	8.034	8.897	9.915	10.283	11.591	13.240	15.445	26.171	29.615	32.671	35.479	36.343	38.932	41.401	46.797	21
22	8.643	9.542	10.600	10.982	12.338	14.041	16.314	27.301	30.813	33.924	36.781	37.659	40.289	42.796	48.268	22
23	9.260	10.196	11.293	11.688	13.091	14.848	17.187	28.429	32.007	35.172	38.076	38.968	41.638	44.181	49.728	23
24	9.886	10.856	11.992	12.401	13.848	15.659	18.062	29.553	33.196	36.415	39.364	40.270	42.980	45.558	51.179	24
25	10.520	11.524	12.697	13.120	14.611	16.473	18.940	30.675	34.382	37.652	40.646	41.566	44.314	46.928	52.620	25
26	11.160	12.198	13.409	13.844	15.379	17.292	19.820	31.795	35.563	38.885	41.923	42.856	45.642	48.290	54.052	26
27	11.808	12.879	14.125	14.573	16.151	18.114	20.703	32.912	36.741	40.113	43.194	44.140	46.963	49.645	55.476	27
28	12.461	13.565	14.847	15.308	16.928	18.939	21.588	34.027	37.916	41.337	44.461	45.419	48.278	50.993	56.893	28
29	13.121	14.256	15.574	16.047	17.708	19.768	22.475	35.139	39.087	42.557	45.722	46.693	49.588	52.336	58.302	29
30	13.787	14.953	16.306	16.791	18.493	20.599	23.364	36.250	40.256	43.773	46.979	47.962	50.892	53.672	59.703	30
40	20.706	22.164	23.838	24.433	26.509	29.051	32.345	47.269	51.805	55.759	59.342	60.436	63.691	66.766	73.402	40
50	27.991	29.707	31.664	32.357	34.764	37.689	41.449	58.164	63.167	67.505	71.420	72.613	76.154	79.490	86.661	50
60	35.535	37.485	39.699	40.482	43.188	46.459	50.641	68.972	74.397	79.082	83.298	84.580	88.379	91.952	99.607	60
70	43.275	45.442	47.893	48.758	51.739	55.329	59.898	79.715	85.527	90.531	95.023	96.388	100.425	104.215	112.317	70
80	51.171	53.539	56.213	57.153	60.391	64.278	69.207	90.405	96.578	101.880	106.629	108.069	112.329	116.321	124.839	80
90	59.196	61.754	64.634	65.646	69.126	73.291	78.558	101.054	107.565	113.145	118.136	119.648	124.116	128.299	137.208	90
100	67.327	70.065	73.142	74.222	77.929	82.358	87.945	111.667	118.498	124.342	129.561	131.142	135.807	140.170	149.449	100

備註	試題隨卷繳交