

國立政治大學

1. Two firms simultaneously decide whether to enter a market. Firm  $i$  incurs an entry cost of  $c_i$  where  $i = 1, 2$ . If firm  $i$  is the only entrant, it earns a monopoly revenue of 1. If both firms enter, each earns a duopoly revenue of  $\frac{1}{3}$ . The payoff matrix is shown as follows.

		Firm 2	
		Enter	Not Enter
Firm 1	Enter	$\frac{1}{3} - c_1, \frac{1}{3} - c_2$	$1 - c_1, 0$
	Not Enter	$0, 1 - c_2$	$0, 0$

(i) (10 points) Suppose that  $c_1 = c_2 = \frac{1}{2}$ . Find all the Nash equilibria of the game. Draw the best response curves to verify that you have indeed identified all the Nash equilibria.

(ii) (15 points) Now suppose that  $c_1$  and  $c_2$  are independently and uniformly distributed on the interval  $[0, 1]$ , and are private information, unknown to the opponent. (a) Show that each firm's action is monotonic. That is, if firm  $i$  were to enter when  $c_i = c$ , it will enter if  $c_i = c' < c$ . (b) Find a symmetric Bayesian Nash equilibrium.

2. There are two goods and two consumers.

(i) (10 points) Preferences and endowments are given by

$$u^1(x_1, x_2) = \min\{x_1, x_2\} \quad \text{and} \quad \omega^1 = (1, 4)$$

$$u^2(x_1, x_2) = \min\{x_1, x_2\} \quad \text{and} \quad \omega^2 = (4, 1).$$

Illustrate Pareto optima, Core and Walrasian equilibrium allocations in an Edgeworth box and explain briefly.

(ii) (15 points) Now suppose that preferences and endowments are given by

$$u^1(x_1, x_2) = \max\{x_1, x_2\} \quad \text{and} \quad \omega^1 = (1, 4)$$

$$u^2(x_1, x_2) = \max\{x_1, x_2\} \quad \text{and} \quad \omega^2 = (4, 1).$$

Again illustrate Pareto optima, Core and Walrasian equilibrium allocations in an Edgeworth box and explain briefly.

◎ 以下 3~6 題任選 2 題作答 每題 25 分。

國立政治大學圖書館

3. You may have read the following excerpt from Wall Street Journal:

“According to the theory, the interest rate is one of the primary determinants of investment spending. Yet the major movements in investment spending during business cycles are not accompanied by changes in interest rates.”

Use the economic theory learned in class to reconcile these two statements. Can you give an example of positive correlation between interest rate and investment spending.

4. 假設家戶所面臨的問題如下:

$$\max \sum_{t=0}^{\infty} \beta^t u(c_t, m_t)$$

$$b_{t+1} - b_t = w_t n_t - c_t + \pi_t + r_t b_t$$

- (i) 從  $t = 0$  開始到  $t = \infty$ , 人們變得比較沒有耐性。請問均衡的產出、消費、勞動、工資率、利率會如何變化?
- (ii) 從  $t = 0$  開始到  $t = T$ , 人們變得比較沒耐性, 但從  $t = T + 1$  到  $t = \infty$ , 人們的耐性又回到原狀, 請問均衡又如何變化?
- (iii) 從  $t = 0$  開始到  $t = T$ , 人們的耐性維持不變, 但從  $t = T + 1$  到  $t = \infty$ , 人們變得比較沒有耐性。請問均衡如何變化?
- (iv) 若是在偶數期人們維持原來的耐性, 在奇數期則比較沒耐性, 請問均衡如何變化?

5. 假設家戶所面臨的問題如下:

$$\max \sum_{t=0}^{\infty} \beta^t u(c_t)$$

$$M_{t+1} - M_t = P_t y - P_t c_t + T_t$$

$$M_t \geq P_t c_t$$

其中  $y$  是秉賦 (endowment), 每期皆相同, 令

$$u(c) = \frac{c^{1-\sigma}}{1-\sigma}, \quad \sigma = 0.5, \quad \beta = 0.9$$

請計算當通貨膨脹率由 0% 升高到 10% 時的社會福利損失 (Social Welfare Loss)?

(20 points for each question)

1. Let  $\Omega = \mathbb{R}^2$ . Let  $A_n$  be the interior of the circle with center at  $(\frac{(-1)^n}{n}, 0)$  and radius 1.  
Find  $\limsup_n A_n$  and  $\liminf_n A_n$ .
  
2. (i) Let  $I$  be an open interval and the function  $f: I \rightarrow \mathbb{R}$  be differentiable. Prove that if  $f'(x) > 0$  for all  $x$  in  $I$ , then  $f: I \rightarrow \mathbb{R}$  is strictly increasing.  
 (ii) Define  $f(x) = \begin{cases} x - x^2 & x \in \mathbb{Q} \\ x + x^2 & x \notin \mathbb{Q} \end{cases}$ . Show that  $f'(0) > 0$  but for which there is no open interval  $I$  containing 0 on which  $f: I \rightarrow \mathbb{R}$  is increasing.  
 (iii) Can you explain why we can't apply the result of (i) to (ii).
  
3. Give a detailed definition for Riemann-Integrability. Is it true that any Riemann integrable function is Lebesgue Integrable (if not, give an countable example).
  
4. (i) Define  $f(x,y) = e^{xy} x$  for  $(x,y)$  in  $I = [1,2] \times [0,1]$ . Find  $\int_I f(x,y) dx dy$ .  
 (ii) If that the function  $f: \mathbb{R} \rightarrow \mathbb{R}$  is continuous. Define  $G(x) = \int_0^x (x-t)f(t)dt$ , calculate  $G''(x)$ .  
 (iii) Find the range of  $p$  such that  $\sum_{n=2}^{\infty} \frac{1}{n(\log n)^p}$  converges.
  
5. Let  $I_0 = [0,1]$ . Remove the segment  $(1/3, 2/3)$ , and let  $I_1 = [0, 1/3] \cup [2/3, 1]$ . Then remove the middle thirds of these intervals, and let  $I_2 = [0, 1/9] \cup [2/9, 3/9] \cup [6/9, 7/9] \cup [8/9, 1]$ .  
 Continuing in this way, we obtain a sequence of compact set  $I_n$ . The set  $C = \bigcap_{n=1}^{\infty} I_n$  is called Cantor set. Show that
  - (i) The Cantor set  $C$  is compact.
  - (ii) The Cantor set  $C$  has no isolated points. (If  $p \in E$  and  $p$  is not a limit point of  $E$ , then  $p$  is called an isolated point of  $E$ .)
  - (iii) The Lebesgue measure (length) of  $C$  is zero. i.e.  $\mu(C) = 0$ .
  - (iv) The element of Cantor set  $C$  is not countable.

前 4 題佔 55%，後 2 題佔 45%

以下符號

$y$ : dependent variable

$x$ : independent variable

$\varepsilon$ : white noise  $\sim iid N(0, \sigma^2)$

$a, b, \rho, \theta$  等等未知

參數,  $T$  為時間總數,  $N$  為

公司數目

請寫下下列 1 ~ 4 題的 likelihood function

$$1. \quad y_t = a_0 + a_1 x_t + \varepsilon_t, \quad t=1, \dots, T$$

$$2. \quad y_t = a_0 + a_1 x_t + u_t$$

$$u_t = \rho u_{t-1} + \varepsilon_t$$

$$3. \quad y_t = a_0 + a_1 x_t + \varepsilon_t \quad t \leq t_0$$

$$y_t = b_0 + b_1 x_t + \varepsilon_t \quad t > t_0$$

$$t = 1, 2, \dots, t_0, t_0+1, \dots, T$$

$$4. \quad y_t = a_0 + a_1 x_t + \varepsilon_t, \quad x_t \leq \theta$$

$$y_t = b_0 + b_1 x_t + \varepsilon_t, \quad x_t > \theta$$

5. 考慮 - panel data 模型

$$y_{it} = a_i + b x_{it} + \varepsilon_{it}$$

$$i = 1, \dots, N, \quad t = 1, \dots, T$$

$a_i$ : 表示每家公司有不同截距項

請說明各種可能估計方法  
以考慮不同截距項的特性

6.

請說明 Seemingly Unrelated  
Regression 的估計式與 OLS  
有何不同？且二者何時會  
相等？

Let random variables  $X$  and  $Y$  have joint pdf

$$f(x, y) = e^{-y} \quad 0 < x < y < \infty$$

- (a) Compute  $P(X+Y \geq 1)$   
 (b) Compute  $E(X|Y=2)$

Let  $X$  and  $Y$  be independent random variables with  $X \sim \text{Gamma}(r, 1)$  and  $Y \sim \text{Gamma}(s, 1)$ . Show that  $Z_1 = X+Y$  and  $Z_2 = X/(X+Y)$  are independent and find the distribution of each.

Let  $X_1, X_2, \dots, X_n$  be a random sample of size  $n$  from a distribution with mean  $\mu$  and variance  $\sigma^2 < \infty$ . Show that (a)  $S^2$  converges in probability to  $\sigma^2$

(b)  $\frac{\sqrt{n}(\bar{X} - \mu)}{\sigma}$  converges in distribution to  $N(0, 1)$

Let  $X_1, X_2, \dots, X_n$  be a random sample from  $U(0, \theta)$ .

(a) Show that  $Y_n = \max(X_1, X_2, \dots, X_n)$  is a complete and sufficient statistic for  $\theta$ .

(b) Find the best unbiased estimator (i.e. UMVUE) of  $\theta$

Suppose that we have two independent random samples.  
 $X_1, X_2, \dots, X_n$  are exponential ( $\theta$ ) and  $Y_1, \dots, Y_m$  are exponential ( $\lambda$ )

- (a) Find the Likelihood Ratio Test of  $H_0: \theta = \lambda$  vs.  $H_a: \theta < \lambda$   
 (b) Show that the test in (a) can be based on the statistic

$$T = \frac{\sum X_i}{\sum X_i + \sum Y_i}$$

Let  $X_1, X_2, \dots, X_n$  be a random sample from  $N(\theta, 1)$  and let  $\theta_0$  be a specified value of  $\theta$ .

Find the UMP size  $\alpha$  test for testing  $H_0: \theta = \theta_0$  vs.  $H_a: \theta < \theta_0$

1. 試簡單推論貨幣政策、金融監理和總體經濟活動的關係。 (25分)
2. 對貨幣與實質產出間的關係，Real Business Cycle Model, New Classical Model 及 New Keynesian Model 有何不同的論點。 (25分)
3. 訊息不完全(imperfect information)在金融借貸活動中造成了很多問題，如事前的 adverse selection 問題及事後的 moral hazard 問題。借貸契約關係中有那些特色或機制是來改善或減緩這些問題？它們如何能改善或減緩這些問題？ (25分)
4. 1988 年 BIS 公佈了 Basel Accord，對於銀行的經營有『資本適足率』的規範。在 2001 年 BIS 公佈了該協定的第二次草案，在資本適足率的計算過程中加強了風險因素的考量。資本適足率有何重要功能？為何需要加強風險因素的考量？ (25分)



## 壹. (30%)

- a) 甲生批評 B 生的論文：「你的訂價模型都是假設風險中立的。可是實際上，大部分的經濟個體都是風險趨避的，所以你的訂價公式是有問題的！」B 生回答：「...，大家都這樣用，文獻上也很普遍。」請你評論上述之對話。
- b) 以風險中立評價法推導你所知的衍生性產品價格；可假設利率是固定的或隨機的兩種情況之一。
- c) 說明風險中立評價法與 Partial differential equation approach 之關係。

## 貳. (25%) 試說明

- a) 利率的期間結構理論
- b) 零息債券的評價原理
- c) 舉例說明及評論利率模型

## 參. (25%) 試說明

- a) 最適資本結構的理論
- b) 公司債的評價與最適資本結構如何聯繫起來？

## 肆. (20%)

試以你最強的專長、最深的程度自設題目並詳細分析之。