

1. You want to make an investment and you are offered two financial assets. These assets are fully characterized by their random return X and Y . Both assets promise the same average return $E[X] = E[Y]$ and have the same price. But X is at least as risky and perhaps riskier than Y . You are risk-averse. Denote the variance of X to be σ_x^2 and the variance of Y to be σ_y^2 , and their correlation by ρ . And define a new random variable Z as $Z = \alpha X + (1 - \alpha)Y$, with $\alpha \in [0, 1]$.

- (1) Under what circumstances are mean and variance a satisfactory measure of comparative risk? (10%)
- (2) Suppose the conditions in (1) are satisfied so we can only focus on mean-variance analysis. Then under what conditions you should diversify, that is, to have $\alpha \in (0, 1)$? (15%)

2. An individual's consumption set is \mathbb{R}_+^2 . The individual has income I and pays unit price $p_1 > 0$ for good 1. The price of each unit of good 2 is $p_2 = b_2 x_2$ where x_2 is the number of units of good 2 purchased and $b_2 > 0$. The individual's utility is $U(x_1, x_2) = x_1 x_2$. Determine the optimal consumption of both goods. What price will the individual pay for each unit of good 2 at his optimal consumption of good 2? (25%)

3. Use the open-economy IS-LM model under floating exchange rate to discuss the effects of expansionary fiscal and monetary policies, respectively, on output, interest rate, exchange rate, and trade balance. (25%)

4. Compare the "costs of disinflation" under New Classical and New Keynesian models. (25%)

高等微積分

國立政治大學金融學系九十三年度博士班入學考

Instructions: Answer Four Question Out of Five.

1. [25 Points] Show that the following limit exists and is finite

$$\lim_{\lambda \rightarrow 0^+} \left(\int_0^1 \frac{dx}{(x^4 + \lambda^4)^{1/4}} + \log \lambda \right)$$

2. [25 Points] The fundamental Theorem of Calculus tells us that, if a function f is continuous on $[a, b]$, then for any antiderivative F of f ,

$$\int_a^b f = F(b) - F(a)$$

This equation clearly holds for the particular antiderivative $G(x) = \int_a^x f$ because $G(b) - G(a) = \int_a^b f - 0$, but why does it hold for ANY antiderivative F ?

3. [25 Points] Recall that a function f from a subset A of \mathbb{R} into \mathbb{R} is said to be contractive if and only if there is a constant $K \in (0, 1)$ such that, for all $x_1, x_2 \in A$, we have $|f(x_1) - f(x_2)| \leq K|x_1 - x_2|$.
- (a) Prove that a contractive function must be uniformly continuous.
 - (b) Prove that if A is a compact interval $[a, b]$, $f'(x)$ is continuous on $[a, b]$, and $-1 < f'(x) < 1$ for all $x \in [a, b]$, then f is contractive.
 - (c) Give an example to show that, if the interval $[a, b]$ is replaced by the interval $[a, \infty)$ (still a closed interval but no longer bounded) in (b), then the conclusion fails.

4. [25 Points] Let $\{f_n\}$ be a sequence of twice differentiable functions on $[0, 1]$ such that

$$f_n(0) = f_n'(0) = 0$$

for all n . In addition, suppose that

$$|f_n''(x)| \leq 1$$

for all n and all $x \in [0, 1]$. Prove that there is a subsequence of $\{f_n\}$ which converges uniformly on $[0, 1]$.

5. [25 Points] Let $A^{1/3}$ be the set of real valued functions on the closed interval $[0, 1]$ such that: (i), $f(0) = 0$; (ii), $\|f\|$ defined as

$$\|f\| = \sup \left\{ \frac{|f(x) - f(y)|}{|x - y|^{1/3}} \mid x \neq y \right\}$$

is finite.

Verify that $\|\cdot\|$ is a norm for the space $A^{1/3}$, and prove that $A^{1/3}$ is complete with respect to this norm.

考試科目	計量經濟學	所別	金融所	考試時間	5月22日 星期六	13:20 第5節 16:40
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1. (25%) Assume x has the distribution

$$f(x) = \alpha \beta x^{\beta-1} e^{-\alpha x^{\beta}}, \quad x \geq 0, \alpha, \beta > 0$$

We want to test the hypothesis that $\beta = 1$.

- (a) Describe the rationale for the (i) likelihood ratio test (ii) Wald test
(iii) Lagrange multiplier test.

(b) Write down the test statistic for the three tests in (a).

2. (25%) Consider the following model:

$$y_1 = \alpha_1 y_2 + \beta_{11} x_1 + \varepsilon_1$$

$$y_2 = \alpha_2 y_1 + \beta_{22} x_2 + \beta_{23} x_3 + \varepsilon_2$$

All variables are measured as deviations from their means. The sample of 25 observations produces the following matrix of sums of squares and

cross products (i.e. $\sum_t x_i x_j t$, etc.):

	y_1	y_2	x_1	x_2	x_3
y_1	20	6	4	3	5
y_2	6	10	3	6	7
x_1	4	3	5	2	3
x_2	3	6	2	10	8
x_3	5	7	3	8	15

- Estimate the parameters of the first equation by 2SLS and LIML.
- Estimate the reduced-form coefficients by OLS and ILS (Indirect Least Squares) for the first equation.

考試科目	計量經濟學	所別	金融所	考試時間	420	星期	日	上午	第	節
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3. (25%) Consider the following two specifications of the GARCH model:

$$y_t = \beta_1 + \beta_2 x_{2t} + \beta_3 x_{3t} + u_t, \quad u_t \sim N(0, \sigma_t^2)$$

Model I:

$$\sigma_t^2 = \alpha_0 + \alpha_1 u_{t-1}^2 + \beta \sigma_{t-1}^2 + \delta u_{t-1}^2 I_{t-1}$$

where $I_{t-1} = 1$ if $u_{t-1} < 0$
 $= 0$ otherwise.

Model II:

$$\ln \sigma_t^2 = \alpha_0 + \beta \ln \sigma_{t-1}^2 + \delta \frac{u_{t-1}}{\sqrt{\sigma_{t-1}^2}} + \alpha \left[\frac{|u_{t-1}|}{\sqrt{\sigma_{t-1}^2}} - \sqrt{\frac{2}{\pi}} \right]$$

(a) Discuss the purposes of the two models.

(b) Discuss the advantages of the two models relative to the pure GARCH specification.

(c) State briefly how to estimate the parameters under the two models.

4. (25%) Answer the following questions:

(a) What is a switching model? Describe and distinguish between Markov switching models and threshold autoregressive models.

Also, how would you decide which model is more appropriate for a particular application?

(b) state a model or methodology that can demonstrate your ability in econometrics.

備

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試 題 隨 卷 繳 交

-80-

考試科目	數學統計學	所別	金融所	考試時間	5月22日 星期六 13:20 下午 16:40
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1. (20%) Let X_1, \dots, X_n be iid rv's, and define

$$\bar{X} = \frac{1}{n} \sum_{i=1}^n X_i, \quad S^2 = \frac{1}{n-1} \sum_{i=1}^n (X_i - \bar{X})^2$$

Suppose that the common distribution is symmetric. Assuming the existence of moments of appropriate order, show that $\text{cov}(\bar{X}, S^2) = 0$.

2. (20%) Let X_1, \dots, X_n be independent normal rv's with $EX_i = \mu_i$ and $\text{Var}(X_i) = \sigma_i^2$, $i = 1, 2, \dots, n$. Let $Y = \sum_{i=1}^n \frac{X_i}{\sigma_i^2}$. Find $MY(t) = Ee^{tY}$, EY , and $\text{Var}(Y)$.

3. (20%) (a) Let X_1, \dots, X_n be a sample from uniform $U[\theta - \frac{1}{2}, \theta + \frac{1}{2}]$, Find MLE of θ (maximum likelihood estimator).

(b) Let $X \sim b(1, p)$, Bernoulli with parameter p , and $p \in [\frac{1}{4}, \frac{3}{4}]$, Find MLE of p . Is it unbiased?

4. (20%)

Let X_1, \dots, X_n be a random sample from pdf

$$f_{\theta}(x) = \frac{\theta}{x^2}, \quad 0 < \theta \leq x < \infty$$

Find an MP test of $\theta = \theta_0$ against $\theta = \theta_1$ ($\neq \theta_0$.)

5. (20%) Let $(A_n)_{n=1}^{\infty}$ be a sequence of events such that $\sum_{n=1}^{\infty} P(A_n) < \infty$.

Show that $P(\limsup_{n \rightarrow \infty} A_n) = P(\bigcap_{n=1}^{\infty} \bigcup_{k=n}^{\infty} A_k) = 0$; and if A_n are independent such that $\sum_{n=1}^{\infty} P(A_n) = \infty$, show that $P(\limsup_{n \rightarrow \infty} A_n) = 1$.

備 考

試 題 隨 卷 繳 交

1. 請討論理論上關於「權衡」(discretion)與「法則」(rule)的爭論。根據你的觀察，現在一般國家央行是採「權衡」或「法則」？為什麼？(25%)
2. Milton Friedman 曾說：「Inflation is always and everywhere a monetary phenomenon。」請解釋這句話。(25%)
3. 請從資訊不對稱(asymmetric information)問題的角度對「直接金融」與「間接金融」做比較。(25%)
4. 試述預期通貨膨脹(expected inflation)的上升對「貨幣供給創造過程」的影響。(25%)

問答題，每題 25 分

1. 請說明現金股利、庫藏股對於股東權益、股價的影響為何？
2. 說明惡意購併與 tender offer 之差別？再說明二者之經濟涵義
3. 請說明下列公式意義

$$Accruals = (\Delta CA - \Delta Cash) - (\Delta CL - \Delta STD - \Delta TP) - Dep$$

- where
- ΔCA = change in current assets
 - $\Delta Cash$ = change in cash/cash equivalents
 - ΔCL = change in current liabilities
 - ΔSTD = change in debt included in current liabilities
 - ΔTP = change in income taxes payable
 - Dep = depreciation and amortization expense

再說明 accrual 與 cash flow 對股價可能的影響

4. 請說明下列股權結構，甲對 Y 公司之現金流量請求權之值 (cash flow rights) 及控制權 (voting rights)，並說明其差距之意義

