科目 urse	個體經濟學	系級	經濟所	日 期 Date, Period	6 月 10 日 第 1 節	試 C

1. In a Rubinson Crusoe economy, only 2 products, x and y, are produced with technology $x = (L_x)^{\frac{1}{2}}$ and $y = \frac{1}{2}(L_y)^{\frac{1}{2}}$ respectively, and $L_x + L_y = 100$. Suppose a utility function $U(x,y) = (xy)^{\frac{1}{2}}$.

   (1) Check if there exist equilibrium prices (hint: by fixed point theorem). Find out the equilibrium prices and outputs if your answer is positive; explain your reason if your answer is negative. (20%)

   (2) Show that Pareto allocation efficiency will be achieved in the equilibrium. (10%)

2. Consider a infinitely repeated game, where in each stage the following game is played:

	b_1	b_2
a_1	3,3	7,1
a_2	1,7	6,6

   . Denote the discount factor as δ.

   (1) Find the Nash equilibrium or equilibria, and the dominant equilibrium for the stage game.(6%)

   (2) Find a subgame perfect equilibrium that results in an average payoff of (2,5). (20%)

3. Consider the following utility function: $u(x) = \frac{B_1 x_1}{A_1 + x_1} + \frac{B_2 x_2}{A_2 + x_2} + \dots + \frac{B_n x_n}{A_n + x_n}$, where A_i, B_i are constants. Show that the income expansion curves are straight lines (except when $x_i = 0$ for some i). (19%)

4. In Akerlof's lemon market, assume that there are both new cars and used cars in the market. There is half chance for both new and used cars to be good or lemon. Let N^G, N^L, U^G, U^L be a buyer's utility from new good cars, new lemon cars, used good cars and used lemon cars. Assume $N^L = U^L = 0$ and $N^G > U^G$. There are four kinds of players in this market: new car seller, good used car seller, lemon used car seller and plain consumer. Explain Akerlof's proposition that there is no good used car in equilibrium. (hint: find the equilibria and depict in a diagram)(25%)

科目 Course	總體經濟學	開課系級 Dept. & Class	經濟所	日期 Date, Period	6 月 10 日 第 二 節	試題編號 CourseNo.

1. Suppose the production function is: $Y = K^{\alpha}L^{1-\alpha}$. Show that α is capital's share of income and $(1-\alpha)$ is labor's share of income. (25%)

2. Let in a basic classical Model as follows:

GME: $c^d(y^* - \tau) + i^d(r) + g - y^* = 0$

FME: $\frac{1}{g_m}\left[y^* - \tau - c^d(y^* - \tau) - f^s(r)\right] - \left(\frac{M}{P}\right)^d(y^*, r) + \frac{M}{P} = 0$

MME: $\left(\frac{M}{P}\right)^d(y^*, r) - \frac{M}{P} - \frac{1}{g_m}\cdot \dot{m}^s = 0$

Try to use the Cramer's rule and diagrams to analyse what the effect of a temporary tax cut financed by a temporary increase in the rate of expansion of the money supply ($d\dot{m}^s = -d\tau$) in r and P? (25%)

3. Suppose the government faces the following problem. $\max \sum_{i=0}^{\infty} \beta^i U(y_{t+i}, \pi_{t+i})$, $0<\beta<1$ (1), where the government's utility function is given by $U(y_{t+i}, \pi_{t+i}) = -(y_{t+i} - Ky^*)^2 - s(\pi_{t+i} - \pi^*)^2$, $K>1$, $s>0$ (2), and $Ky^*(>y^*)$ and π^* are the government's ideals for output and inflation respectively. Suppose that output in each period is determined by the philips curve: $y_{t+i} = y^* + b(\pi_{t+i} - E_{t-1+i}\pi_{t+i})$, $b>0$ (3) where $E_{t-1+i}\pi_{t+i}$ are agent's rational expectation of the rate of inflation between period $t-1+i$ and $t+i$ conditional of information available at time $t-1+i$.

(a) Suppose the government can commit itself irrevocably to a desired path of current and future inflation rates. Derive the optimal choice of the inflation rate for the government.

(b) Suppose instead that the government cannot commit itself irrevocably to a desired path of current and future inflation rate. Derive the optimal choice of the inflation rate for the government.

(c) How do the equilibrium inflation rate and level of output in part (a) compare to those in part (b)? Explain your answer. (25%)

4. For the IS-LM Model, a common "adjustment" Scheme Specification is as follows:

$\dot{y} = k_1(y^d - y)$, $k_1 > 0$;

$\dot{r} = k_2(m^d - m^s)$, $k_2 > 0$.

where $y^d = c^d(y-\tau) + i^d(r, y) + g$ and $m^d = L(r, y)$, $m^s = \bar{m}$.

The only new wrinkle here is the assumption that investment depends on y, show under what conditions this system is stable. (25%)

考試科目 Course	數理方法	系級 經濟	博士班	日期 Date, Period	6月10日 第3節	試題編號 Course No.

（共四題，每題配分不同，請把握時效）

I. 30% Let the production function and the utility function be

$$Y = AK^{\alpha}L^{1-\alpha} \qquad (0 < \alpha < 1)$$

$$U = U - \frac{1}{b}c^{-b} \qquad (b > 0)$$

(a) Find the $y = \phi(k)$ function and the $U'(c)$ function.
(b) Write the specific optimal control problem.
(c) Apply the maximum principle, using the current-value Hamiltonian.
(d) Derive the differential-equation system in the variables k and c. Solve for the steady-state values $(\bar{k}, \bar{c})$.

II. 20% 1. Show that the price index $P(p^1, p^0; u^R)$ satisfies the following propositions and interpret each of them:

（每小題各佔 2%）

(a) $P(\lambda p^0, p^0; u^R) = \lambda$
(b) $P(p^0, p^0; u^R) = 1$
(c) $P(p^1, p^0; u^R) = 1/P(p^0, p^1; u^R)$
(d) $P(p^0, p^1; u)\, P(p^1, p^2; u) = P(p^0, p^2; u)$
(e) $\min(p_i^1/p_i^0) \leq P(p^1, p^0; u^R) \leq \max(p_i^1/p_i^0)$
(f) Hence, show that upper and lower bounds can be derived for both true indices, that is,

$$\min(p_i^1/p_i^0) \leq P(p^1, p^0; u^0) \leq P(p^1, p^0; q^0)$$

$$P(p^1, p^0; q^1) \leq P(p^1, p^0; u^1) \leq \max(p_i^1/p_i^0)$$

(8%) 2. Show that the cost-of-living index number $\log P = \Sigma w_k \log(p_k^1/p_k^0)$ is equal to $\log P(p^1, p^0; u)$ if preferences are Cobb-Douglas.

III. 30% Consider the simple autoregressive model with autocorrelated errors.

$$y_t = \alpha y_{t-1} + u_t, \qquad u_t = \rho u_{t-1} + \varepsilon_t,$$

where the ε_t are independent $N(0, \sigma^2)$ random variables. This can be written as

$$(*) \qquad y_t = (\alpha + \rho)y_{t-1} - \alpha\rho y_{t-2} + \varepsilon_t.$$

1. Assuming (without proof) that $\text{plim}(1/T)\Sigma y_t^2$ and $\text{plim}(1/T)\Sigma y_t y_{t-1}$ exist, find the probability limit of the least squares estimator $a = \Sigma yy_{-1}/\Sigma y_{-1}^2$. [*Hint:* Multiply (*) by y_{t-1} and sum.]
2. Using (*), write the likelihood function for a sample of size T. Find the asymptotic information matrix for α and ρ (assuming that σ is known). [*Hint:* Let $A = Ey_t^2 = \text{plim}(1/T)\Sigma y_t^2$ and use part 1 to express $E(yy_{-1}) = \text{plim}(1/T)\Sigma yy_{-1}$ in terms of A; write your answer in terms of A without explicitly evaluating it.] Inverting the information matrix, give an expression (in terms of α, ρ, and A) for the asymptotic variance of an efficient estimator of α.
3. Suppose that ρ were known. What is an efficient estimator for α?

IV. 20% Consider the estimation of elasticities as

$$\hat{\delta} = \frac{\sum p_i q_i}{\sum p_i^2},$$

where p_i and q_i here are the logarithms of price and quantity, measured as deviations from the means of the logarithms. According to the model:

$$q_i^D = \beta p_i + u_i \qquad \text{where} \quad u_i \sim N(0, \sigma_u^2),$$

$$q_i^S = \gamma p_i + v_i \qquad \text{where} \quad v_i \sim N(0, \sigma_v^2),$$

$$E(u_i u_{i'}) = E(v_i v_{i'}) = 0 \qquad \text{for } i \neq i',\ E(u_i v_{i'}) = 0, \quad \text{all } i, i'.$$

show $E(\hat{\delta}) = ?$ and indicate those circumstances in which $\hat{\delta}$ is an acceptable estimate of the price elasticity of demand.