1. (25%) Consider the utility function, $u(x_1, x_2) = (x_1)^{1/2} + (x_2)^{1/2}$.
   (a) (9%) Compute the demand functions, $x_i(p_1, p_2, y)$, $i = 1, 2$.
   (b) (8%) Compute the substitution term in the Slutsky equation for the effects on x_1 of changes in p_2.
   (c) (8%) Classify x_1 and x_2 as (gross) complements or substitutes.

2. (25%) A firm produces electricity to meet the demands of a city. The price it can charge for electricity is fixed and it must meet all demand at that price. It turns out that the amount of electricity demanded is always the same over every 24-hour period, but demand differs from day (6:00A.M. to 6:00 P.M.) to night (6:00 P.M. to 6:00 A.M.). During the day, 4 units are demanded, whereas during the night only 3 units are demanded. Total output for each 24-hour period is thus always equal to 7 units. The firm produces electricity according to the production function

$$y_i = (KF_i)^{1/2}, \qquad i = \text{day, night},$$

where K is the size of the generating plant, and F_i is tons of fuel. The firm must build a single plant; it cannot change plant size from day to night. If a unit of plant size costs w_k per 24-hour period and a ton of fuel costs w_f, what size plant will the firm build?

3. (25%) A per-unit tax, $t > 0$, is levied on the output of a monopoly. The monopolist faces demand, $q = p^{-\varepsilon}$, where $\varepsilon > 1$, and has constant average costs. Show that the monopolist will increase price by more than the amount of the per-unit tax.

4. (25%) In a two-good, two-consumer economy, utility functions are

$$u^1(x_1, x_2) = x_1(x_2)^2,$$
$$u^2(x_1, x_2) = (x_1)^2 x_2.$$

Total endowments are (10, 20).
   (a) (12%) A social planner wants to allocate goods to maximize consumer 1's utility while holding consumer 2's utility at $u^2 = 8000/27$. Find the assignment of goods to consumers that solves the planner's problem and show that the solution is Pareto efficient.
   (b) (13%) Suppose, instead, that the planner just divides the endowments so that $e^1 = (10, 0)$ and $e^2 = (0, 20)$ and then lets the consumers transact through perfectly competitive markets. Find the Walrasian equilibrium and show that the Walrasian equilibrium allocation is the same as the solution in part (a).

國立政治大學九十三學年度研究所博士班入學考試命題

第　頁

考試科目	總體經濟	所別	經濟學系261	考試時間	5月22日星期六下午第二節

1. Supposed the real interest rate is 3%, the output growth rate is 7%, the debt-income ratio is 50%, and the primary debt shows a deficit of 5% of GDP. Is the debt-income ratio rising or falling? Why? (10%)

2. The accompanying table presents data from the period 1973-1975, the first oil shock. Are these data are consistent with the aggregate demand-supply model? Why? (10%)

	1974	1975
Real fuel price, 1973 = 100	122.6	138.75
GNP deflator, 1973 = 100	108.8	118.9
Real GNP growth, % per year	-0.6	-1.2
Real wage change % per year	-2.8	0.8

3. In October 1979, the U.S. central bank announced that it would play less active role in limiting fluctuations in dollar interest rates. After this new policy was put into effect, the dollar's exchange rates against foreign currencies become more volatile. Please analyze the connection between these two events with the aid of diagram where the exchange rate is the price of dollars in terms of New Taiwan dollar (for example, NT 33.3 per dollar). (20%)

4. One view of technological progress is that the productivity of capital goods built at t depends on the state of technology at t and is unaffected by subsequent technological progress. This is known as *embodied technological progress*. This problem asks you to investigate its effects.

   (a) Now consider embodied technological progress. Specifically, let the production function be $Y(t) = J(t)^{\alpha} L(t)^{1-\alpha}$, where $J(t)$ is the effective capital stock. The dynamics of $J(t)$ are given by $\dot{J}(t) = sA(t)Y(t) - \delta J(t)$. The presence of the $A(t)$ term in this expression means that the productivity of investment at t depends on the technology at t.

   Show that the economy converges to a balanced growth path. (10%)

   (b) What are the growth rates of Y and J on the balanced growth path? (10%)

   (c) What is the elasticity of output on the balanced growth path with respect to s? (10%)

考試科目	總體經濟	所別	經濟學系261	考試時間	5月22日上下午第二節 星期六

5.

Bejing Looks to Apply Breaks to Investment Craze

--- Taipei Times, May 1, 2004

"China reported 9.7 percent growth in the first quarter of this year but faces resurgent inflation due to unchecked expansion in money supply and bank credit. In order to cool the economy, the government has been trying to curb economic growth through a variety of measures, which curb bank lending, a move interpreted by the markets as preparation for an interest rate hike."

The following is a model of aggregate demand, which allows roles for both money and credit (bank loans).

Credit Market : $L(\underset{-}{\rho}, \underset{+}{i}, \underset{+}{y}) = \lambda(\underset{+}{\rho}, \underset{-}{i}, \underset{-}{v})(1-\tau)D$

Money Market : $D(\underset{-}{i}, \underset{+}{y}) = m(\underset{+}{i})R$

Goods Market : $y = y(\underset{-}{\rho}, \underset{-}{i})$

where

ρ : the interest rate on loan;
i : the interest rate on bond;
y :the real income;
$L^D = L(\underset{-}{\rho}, \underset{+}{i}, \underset{+}{y})$: loan demand;
$L^S = \lambda(\underset{+}{\rho}, \underset{-}{i}, \underset{-}{v})(1-\tau)D$: loan supply;
D : deposits;
R : high-powered money;
τ : the required reserve-deposit ratio;
v : the shock to curb bank lending.

Please analyze the effects of the shock to curb bank lending on the interest rate on bond, the loan rate and real income. (30%)

備　考	試題隨卷繳交

以下共有五題, 每題均需回答, 計算題需附計算過程, 否則不予計分.

1. (10%) 假設從二項分配抽取一組觀察值 $X_1, \ldots, X_n$, 其參數爲 θ, $\theta \in (0,1)$. θ 的 Maximum Likelihood Estimator 爲何?

2. (20%) 令 $X_1, X_2, \ldots$ 爲 uncorrelated 隨機變數且 $\mathrm{E}(X_i) = \mu$, $\mathrm{Var}(X_i) \leq K < \infty$ for all i. 如果定義 $Y_t = X_1 + \ldots + X_t$, 証明當 $t \to \infty$ 時, Y_t/t 以 mean square 形式收斂到 μ. 並且 $Y_t/t \xrightarrow{p} \mu$, $\xrightarrow{p}$ 代表 converge in probability.

3. 迴歸式中自變數和誤差項如果相關,
   3.1 (15%) 此時使用最小平方估計法有何影響, 並應如何解決.
   3.2 (15%) 舉一實例說明發生自變數和誤差項相關的情形, 以及你解決的方式. 答案正確的情況下, 愈詳盡愈好.

4. cross-sectional 資料, 迴歸式爲 $y_t = x_t'\beta + \epsilon_t$, 其中 y_t, x_t, ϵ_t 均爲 column vector, 並滿足一般之 regularity conditions.
   4.1 (10%) 若 ϵ_t 具有異質性, 影響爲何, 如何解決?
   4.2 (10%) 如果自變數應有一不隨時間改變的自變數, 但卻錯誤地不列在 x_t 中, 請問有何問題產生? 並建議解決之道。(同上, 愈詳盡愈好, 若能舉例更佳)

5. $Y_t = \sum_{j=0}^{\infty} \psi_j X_{t-j}$, X_t 爲 uncorrelated 隨機變數且 $\mathrm{E}(X_t) = 0$, $\mathrm{Var}(X_t) = K < \infty$ for all t.
   5.1 (5%) 計算 $\mathrm{E}(Y_t Y_{t-j})$.
   5.2 (15%) 定義 $\overline{Y}_T = T^{-1}(Y_1 + Y_2 + \ldots + Y_T)$, $\mathrm{Var}(\overline{Y}_T) < \infty$ 的條件爲何?

NOTE: Answer ONE question from question 1 to question 2; ONE question from question 3 to question 4; ONE question from question 5 to question 6; ONE question from question 7 to question 8; ONE question from question 9 to question 10; (20% for each). Only ONE question in each category will be counted. Thus, try your best to finish each question you choose rather than answer too many questions.

(Answer ONE question from question 1 to question 2)

1.

(i) Find the eigenvalues of

(a) $A = \begin{bmatrix} 2 & 3 \\ 3 & -6 \end{bmatrix}$ (b) $B = \begin{bmatrix} 2 & 7 \\ 7 & 2 \end{bmatrix}$

(ii) A consumer has the utility function

$$u = x_1 x_2$$

where x_1 is meat and x_2 are potatoes. The price of meat is \$10 per pound and that of potatoes is \$1 per pound. She has an income of \$80. In addition to her budget constraint, she has a *subsistence-calorie constraint*: she must consume at least 1,000 calories. One pound of potatoes yields 20 calories, one pound of meat yields 60 calories. Find her optimal consumption bundle. Now suppose that the price of potatoes rises to \$1.60 per pound. Find the new optimal bundle. Explain your results, and discuss their significance for the concept of a Giffen good.

2.

(i) Consider the following general CES production function

$$y = A\left[\delta x_1^{-r} + (1-\delta)x_2^{-r}\right]^{-1/r}, \qquad A > 0,\ 0 < \delta < 1,\ r > -1$$

defined on $x_1 > 0$ and $x_2 > 0$.

(a) Find an expression for the MRTS, and show that isoquants are strictly convex to the origin.

(b) Show that f is homogeneous, and find its degree of homogeneity.

(c) Show that the following result (from Euler's theorem) applies to f

$$f_1 x_1 + f_2 x_2 = kf(x_1, x_2)$$

where k is the degree of homogeneity of f.

(ii) (a) Obtain the orthogonal decomposition of

$$A = \begin{bmatrix} 7 & 2 \\ 2 & 4 \end{bmatrix}$$

(Answer ONE question from question 3 to question 4)

3.

(i) A consumer has the utility function

$$u = u(x_{11}, x_{12}) + \beta u(x_{21}, x_{22}), \qquad 0 < \beta < 1$$

where x_{ti} is the amount of good $i = 1,2$ consumed in period $t = 1,2$. The prices of the goods are p_1 and p_2, and are the same in each period. The consumer's income in period t is m_t and not necessarily equal in both periods.

Assume first that the consumer has separate budget constraints in each period. Derive the indirect utility function and comment on its form. Interpret the Lagrange multipliers in this problem. Under what conditions are they equal?

(ii) Evaluate the following integrals:

(a) $\int (2x^3 + 5x^2 + x + 5)dx$

(b) $\int \left(\sum_{i=0}^{n} a_i x^i \right) dx = \int (a_0 + a_1 x + a_2 x^2 + \cdots + a_n x^n) dx$

4.

(i) A consumer has the strictly quasiconcave indirect utility function

$$u = v(m_1) + \beta v(m_2), \qquad 0 < \beta < 1$$

where m_t is income available for expenditure on consumption in period $t = 1,2$. Her *wealth constraint* is

$$m_1 + \frac{m_2}{1+r} = \overline{m}_1 + \frac{\overline{m}_2}{1+r}$$

where r is the interest rate at which she can borrow or lend and $\overline{m}_t$ is the exogenously *endowed income* in period t. Derive and interpret the Slutsky equations for the effect of changes in the interest rate on the choice of income in period $t = 1,2$.

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(ii) Use the substitution rule to find the integral

$$\int (x^3 + 4x^2 + 3)^4 (3x^2 + 8x) dx$$

(Answer ONE question from question 5 to question 6)

5.

(i) For the following difference equation

$$y_{t+1} = (-1)^t y_t + 1, \quad y_0 = 2$$

(a) Obtain the unique solution that satisfies the given value for y_0.

(b) Calculate the values for y_1, y_2, y_3, y_4, and y_5 directly from the difference equation, and observe the speed of convergence to the steady state, if it exists.

(c) Calculate the value for y_5 from the solution obtained in (a) and check that it matches the value obtained in (b).

(ii) In the growth model, the differential equation for the economy's capital-labor ratio is

$$\dot{k} = sf(k) - nk$$

where $f(k)$ gives output per worker, y, as a function of the capital-labor ratio, k; s is the saving rate in the economy; and n is the growth rate of the labor force. Let

$$f(k) = k^{1/2}$$

and obtain and explicit solution for k as a function of t. Show that $k(t)$ converges to the steady-state equilibrium point as long as n is positive.

6.

(i) Two firms share the market for a product. Firm 1's output is x; firm 2's output is y. The two reaction functions of the firms are

$$x_{t+1} + \beta y_t = b, \qquad \beta \neq 1$$
$$y_{t+1} + \alpha x_t = b, \qquad \alpha \neq 1$$

Derive and solve the second-order difference equation for x implied by this model.

(ii) Solve the following linear, second-order differential equations:

(a) $\ddot{y} - \dot{y} - 2y = 10$

(b) $\ddot{y} + 4\dot{y} + 5y = 10, \quad y(0) = 2, \quad \dot{y}(0) = 1$

(Answer ONE question from question 7 to question 8)

7. In the following nonlinear differential equation system, $I(t)$ is a firm's investment at time t, $K(t)$ is its capital stock at time t, δ is the rate of depreciation of capital, and α is a parameter of the firm's production function with $0 < \alpha < 1$.

$$\dot{I} = \delta I - \frac{\alpha K^{\alpha-1}}{2}$$

$$\dot{K} = I - \delta K$$

Find the steady-state point, show that it is a saddle point, and construct the phase diagram.

8. Analyze the solution to the following optimal growth model using a phase diagram drawn in (k, c) space:

$$\max \int_0^\infty e^{-\rho t} \ln c\, dt$$

$$\begin{aligned} \text{subject to } \dot{k} &= \frac{k^{1-\alpha}}{1-\alpha} - c - \delta k \\ k(0) &= k_0 \\ k(t) &\geq 0 \end{aligned}$$

(Answer ONE question from question 9 to question 10)

9. Consider the following linear, first-order differential equations:

$$\dot{y} = a_{11} y + a_{12} x$$

$$\dot{x} = a_{21} y$$

Use these equations to derive a linear second-order differential equation for y. Solve for $y(t)$ and derive the conditions on the parameters that must be satisfied for $y(t)$ to converge to its steady-state value of $y = 0$.

10. Suppose that the demand for wheat is given by $q^D = A + Bp$ but the supply is given by

$$q^S = F + Gp + H\left(1 - e^{-\mu t}\right), \qquad \mu > 0$$

where the last term reflects productivity growth over time. That is, the supply curve for wheat shifts (smoothly) out over time because of continuous technological improvements in production. Assume that price adjusts if there is excess demand or supply according to $\dot{p} = \alpha\left(q^D - q^S\right)$. Solve this differential equation for $p(t)$ given $p(0) = p_0$.