

考試科目

個體經濟學

所別

經濟

考試時間

5月21日(六) 第一節

1. (36%) The director of public service employment for a small city funded two different programs last year, each with a different constant-returns-to-scale production function. The director was not sure of the specification of the production functions last year but hopes to allocate resources more wisely this year.

Production data were gathered for each program during three periods last year; they are listed in the accompanying table.

	Program A			Program B		
	$K_A$	$L_A$	$Q_A$	$K_B$	$L_B$	$Q_B$
1	24	26	48	25	25	50
2	24	28	48	25	36	60
3	24	22	44	25	16	40

- a. (8%) Program A operates with a fixed proportions production function. What is it? Explain.
- b. (8%) Program B operates with a Cobb-Douglas production function. What is it? Explain.
- c. (20%) Suppose in the third period that the capital was fixed in each program but you were free to allocate the 38 labor units between the two programs any way you wished. If each unit of  $Q_B$  is equal in value to each unit of  $Q_A$ , how would you allocate labor to maximize the total value of outputs?

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註 試題隨卷繳交

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2. (40%) Two firms produce a homogeneous product. Let  $p$  denote the product's price. The output level of firm 1 is denoted by  $q_1$ , and the output level of firm 2 by  $q_2$ . The aggregate industry output is denoted by  $Q$ ,  $Q = q_1 + q_2$ . The aggregate industry demand curve for this product is given by  $p = \alpha - Q$ .

Assume that the unit cost of firm 1 is  $c_1$  and the unit cost of firm 2 is  $c_2$ , where  $\alpha > c_2 > c_1 > 0$ .

- (15%) Solve for a sequential-moves equilibrium assuming that firm 1 sets its output level before firm 2 does.
- (25%) Solve for a sequential-moves equilibrium, assuming that firm 2 sets its output level before firm 1 does. Is there any difference in market shares and the price level between the present case and the case where firm 1 moves first? Explain.

3. (24%) Individual 1's utility function is  $U_1 = \alpha a + \beta b$ , where  $\alpha$  and  $\beta$  are positive constants,  $a$  is the amount of the good A consumed by person 1, and  $b$  is the amount of good B. Individual 2's utility function is  $U_2 = \rho x + \sigma y$ , where  $\rho$  and  $\sigma$  are positive constants, and  $x$  and  $y$  denote person 2's consumption of commodities A and B, respectively.

Show that if  $a$ ,  $b$ ,  $x$ , and  $y$  are all positive and the two individuals have different marginal rates of substitution then there is a trade between the two that raises the utility of each.

考試科目	總體經濟學	所別	經濟研究所	考試時間	5月21日(六)第二節
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1. (30%) Consider a basic Ramsey-Cass-Koopmans economy. Assume that the household solves the problem:

$$\text{Max } U = \int_{t=0}^{\infty} e^{-\rho t} \frac{C(t)^{1-\theta}}{1-\theta}, \quad \theta > 0$$

$$\text{s.t. } c_t + \dot{k}_t + nk_t = w_t + r_t k_t$$

- (8%) Obtain the law of motion of  $c$  and  $k$  in this model.
- (8%) Is the decentralized equilibrium in this model socially optimal? Explain.
- (6%) Show the phase diagram of  $c$  and  $k$ .
- (8%) Assume that the economy is at the steady state. Show the effects of an increase in the government expenditure on the economy with the phase diagram.

2. (20%) Consider an economy consisting of a constant population of infinitely lived individuals. The representative individual maximizes the expected value of

$$\sum_{t=0}^{\infty} (1+\rho)^{-t} u(C_t), \quad \rho > 0.$$

The instantaneous utility function,  $u(C_t)$  is  $u(C_t) = C_t - \theta C_t^2$ ,  $\theta > 0$ .

Assume that  $C$  is always in the range where  $u'(C)$  is positive. Output is linear in capital, plus an additive disturbance:  $Y_t = AK_t + e_t$ .

We assume that there is no depreciation, thus  $K_{t+1} = K_t + Y_t - C_t$ , and the interest rate is  $A$ . Assume  $A = \rho$ .

The disturbance follows a first-order autoregressive process:  $e_t = \phi e_{t-1} + \varepsilon_t$ , where  $-1 < \phi < 1$  and where the  $\varepsilon_t$ 's are mean-zero, i.i.d. shocks.

- (8%) Find the Euler equation.
- (12%) Guess that consumption takes the form  $C_t = \alpha + \beta K_t + \gamma e_t$ . Obtain the value of the parameters  $\alpha$ ,  $\beta$ , and  $\gamma$  for the Euler equation to be satisfied for all values of  $K_t$  and  $e_t$ .

3. (15%) How do the permanent income hypothesis and rational expectation generate the random walk of consumption in Hall's model? Illustrate the model to explain.

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4. (20%) Construct a model with adjustment costs of investment:

(i) The accumulation of capital is subject to adjustment costs. Expenditure on a given increase in capital,

$\dot{k} = I$ . The investment requires the adjustment cost which is assumed to be  $C(I) = I + hI^2/2$ , where  $h$  is an exogenously given, positive variable.

(ii) The production function is characterized by  $Y = F(k)$ , where  $F_k > 0$  and  $F_{kk} < 0$ .

(iii) The firm maximizes the discounted value of profits.

a) (6%) Obtain the law of motion for  $q$  and  $k$  where  $q$  is the Tobin's  $q$ .

b) (6%) Draw the phase diagram for  $q$  and  $k$ .

c) (8%) Show the effects of a *permanent increase in the interest rate* in the short run and long run with the *phase diagram*.

5. (15%)

a) (7%) Perfect capital mobility in international financial markets implies that the uncovered interest parity condition should hold. Explain.

b) (8%) Explain intuitively the exchange rate overshooting in Dornbusch's model.



考試科目	計量經濟學	所別	經濟學系	考試時間	5 月 21 日(六) 第三節
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1. (70%) Consider a linear stochastic relationship as

$$Y_t = \mathbf{X}_t' \boldsymbol{\beta}_o + \varepsilon_t, \quad t = 1, \dots, n, \quad (1)$$

where we have  $n$  observations on the scalar dependent variable  $Y_t$  and the vector of explanatory variables  $\mathbf{X}_t = (X_{t1}, X_{t2}, \dots, X_{tk})'$ . The scalar stochastic disturbance  $\varepsilon_t$  is unobserved, and  $\boldsymbol{\beta}_o$  is an unknown  $k \times 1$  vector of coefficients.

- (1) Find the ordinary least squares (OLS) estimator,  $\hat{\boldsymbol{\beta}}_n$  say, for  $\boldsymbol{\beta}_o$ . (10%)
- (2) List the necessary conditions such that  $\hat{\boldsymbol{\beta}}_n$  will be an unbiased estimator for  $\boldsymbol{\beta}_o$ , and prove it. (10%)
- (3) List the necessary conditions such that  $\hat{\boldsymbol{\beta}}_n$  will be normally distributed, and prove it. (10%)
- (4) Given the observations of  $Y_t$  and  $\mathbf{X}_t, t = 1, \dots, n$ , we compute a 95% confidence interval for  $\boldsymbol{\beta}_o$ ,  $(b_l, b_u)$  say, based on the normality result in (3). What is the meaning of this computed interval  $(b_l, b_u)$ ? (5%)
- (5) Now suppose the data are generated as

$$Y_t = \mathbf{W}_t' \boldsymbol{\beta}_o + v_t, \quad \mathbb{E}[\mathbf{W}_t v_t] = 0,$$

but we measure  $\mathbf{W}_t$  subject to errors  $\eta_t$ , as  $\mathbf{X}_t = \mathbf{W}_t + \boldsymbol{\eta}_t, \mathbb{E}[\mathbf{W}_t \boldsymbol{\eta}_t'] = 0, \mathbb{E}[\boldsymbol{\eta}_t \boldsymbol{\eta}_t'] \neq 0, \mathbb{E}[\boldsymbol{\eta}_t v_t] = 0$ . (It means that  $\mathbf{W}_t$  is unobservable.)

- a. Show that the OLS estimator  $\hat{\boldsymbol{\beta}}_n$  computed from the model (1) would be biased. (10%)
  - b. How would you do if you observe another variable  $Z_t = \mathbf{W}_t + \boldsymbol{\xi}_t$ , where  $\boldsymbol{\xi}_t$  is uncorrelated with  $\boldsymbol{\eta}_t$  and  $v_t$ ? (10%)
- (6) Given the regularity conditions listed in (3), we consider the linear hypothesis:

$$H_0 : \mathbf{R} \boldsymbol{\beta}_o = \mathbf{r},$$

where  $\mathbf{R}$  is a  $q \times k$  non-stochastic matrix with  $\text{rank } q < k$ , and  $\mathbf{r}$  is a  $q \times 1$  vector of pre-specified, hypothetical values. Please show how to implement **Wald test**, **Lagrange multiplier test** and **Likelihood ratio test**. (15%)

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2. (20%) Consider a simple model for demand and supply functions:

$$\text{Demand: } Q = \alpha_1 P + \alpha_2 I + e_d$$

$$\text{Supply: } Q = \beta_1 P + e_s,$$

where  $Q$  is the quantity,  $P$  is the price and  $I$  is income. We assume  $I$  is exogenous,  $e_s$  and  $e_d$  are independent and  $\mathbb{E}[e_s] = \mathbb{E}[e_d] = 0$ ,  $\text{var}(e_s) = \sigma_s^2$ ,  $\text{var}(e_d) = \sigma_d^2$ .

- (1) Is demand function identified? How would you estimate the parameters  $\alpha_1$  and  $\alpha_2$ ? (10%)
  - (2) Is supply function identified? How would you estimate the parameter  $\beta_1$ ? (10%)
3. (10%) Explain the meaning of "spurious regression", and the concept of "cointegration".

