

考試科目	Microeconomics	所別	Department of Public Finance	考試時間	5 月 22 日 (Sat.) 第 1 節
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Q1.) (20%) Consider an instructor who is planning a comprehensive exam in microeconomics for a class of Ph.D. students. The exam consists of two Sections: the General Theory Section and the Game Theory Section. Each Section has two Parts, and each Part has 25 points (so that the total number of points is 100). The General Theory Section consists of Production Theory Part and Consumer Theory Part; the Game Theory Section consists of Static Games Part and Dynamic Games Part. Let  $x_1$  be the hours of study for production theory,  $x_2$  be the hours of study for consumer theory,  $x_3$  be the hours of study for static games, and  $x_4$  be the hours of study for dynamic games. Accordingly, let  $w_i, i=1, 2, 3, 4$ , denote the "costs" of study for each subject, and the cost vector  $w = (w_1, w_2, w_3, w_4)$ . There are 2 types of students in the class: the cost vector of type-A students is  $w^A = (2, 1, 7, 8)$  and of type-B students is  $w^B = (4, 5, 5, 4)$ . (Type-A students are good at General Theory; type-B students are rather "average".) Assume further that for each hour a student spends, she receives 1 point in the exam. Students must receive 60 points to pass the exam. All the above are common knowledge.

- (a) Suppose the instructor wants the students to understand either general theory or game theory, and therefore, before the exam she made an announcement that the grades would be given according to:

$$\text{final score} = f(x_1, x_2, x_3, x_4) = 2 \times (\min\{x_1, x_2\} + \min\{x_3, x_4\}).$$

Calculate the minimum costs of passing the exam for each type of students.

- (b) Suppose the instructor wants the student to understand both general theory and game theory, and therefore, before the exam she made an announcement that the grades would be given according to:

$$\text{final score} = f(x_1, x_2, x_3, x_4) = 2 \times \min\{x_1 + x_2, x_3 + x_4\}.$$

Calculate the minimum costs of passing the exam for each type of students.

- (c) Which grading policy is better in terms of inducing the most efforts (hours for study) from both types of students?
- (d) Which grading policy is better in terms of minimizing both types of students' costs of passing?

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Q2.) (20%) Use the notation,  $v(\mathbf{p}, m)$ , for the indirect utility function, where  $\mathbf{p}$  denotes for the price vector,  $p_j, j = 1, \dots, n$ , for the individual price, and  $m$  for the constant income;  $e(\mathbf{p}, u)$ , for the expenditure function at price vector  $\mathbf{p}$  and utility  $u$ ;  $x_j(\mathbf{p}, m)$ , for demand function of the  $j^{\text{th}}$  good;  $h_j(\mathbf{p}, m)$ , for demand function of the  $j^{\text{th}}$  good. Answer the following questions:

- (a) Write down Roy's identity and show it holds.
- (b) Write down the Slutsky equation and show it holds.

A consumer has a CES utility function  $u(x_1, x_2) = (x_1^\rho + x_2^\rho)^{1/\rho}$ . Let the prices of goods  $x_1$  and  $x_2$  be  $p_1$  and  $p_2$  respectively.

- (c) Write down the expenditure minimization problem and derive the Hicksian demand functions, the expenditure function, the indirect utility function, and the Marshallian demand functions.



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Q3.) (30%) A firm has a sunk cost  $F > 0$  and marginal costs  $mc(y) = a + by$ , where  $y$  is output and  $a$  and  $b$  are both larger than zero.

(a) If the firm were a price-taker, what is the lowest price at which it would be prepared to produce  $y > 0$ ? If the competitive price were above this level, find the amount of output  $y^*$  that the firm would produce. [Hints: 1. The supply function of a competitive firm is the upward sloping part of the marginal cost curve that lies above the average variable cost curve. 2. A competitive firm produces at price equals marginal cost.]

(b) If the firm is actually a monopolist and the inverse demand function is:

$$P(y) = A - \frac{1}{2}By,$$

where  $A > a$  and  $B > 0$ . Find the price and output  $(p^m, y^m)$  that maximize the monopoly's profit. [Hint: a monopoly produces at: marginal revenue equals marginal cost.]

(c) With the inverse demand function in (b), what is the competitive price and output  $(p^*, y^*)$  in (a)? [Hint:  $(p^*, y^*)$  is determined at demand equals supply.]

(d) How does  $(p^*, y^*)$  in (c) compare to  $(p^m, y^m)$  in (b)?

(e) Suppose the government decides to regulate the monopoly in (b). The regulator has the power to control the price by setting a ceiling  $\bar{p}$ . What is the proper range of  $\bar{p}$  such that the monopoly will produce more than the  $y^m$  in (b)? Through the price ceiling policy, is there any chance that the monopoly would produce more than the competitive level at  $y^*$ ? [Hint: the question maybe easier to figure out if you would draw a graph.]

(f) Suppose the government decides to regulate the monopoly in (b) through tax or subsidy. What tax or subsidy rate per unit of output,  $t$ , would lead the monopoly to produce at the competitive level?

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Q4.) (30%) Consider a pure exchange economy where there are two persons with two goods (a typical Edgeworth box economy). Denote  $x_1$  and  $x_2$  for the consumption of goods 1 and 2 respectively. The preferences of person 1 can be represented by the following utility function:

$$\alpha \ln x_1 + (1 - \alpha) \ln x_2,$$

where:  $0 < \alpha < 1$ . The preferences of person 2 can be represented by the following utility function:

$$\beta \ln x_1 + (1 - \beta) \ln x_2,$$

where:  $0 < \beta < 1$ . Persons 1 and 2 are each endowed with  $(\omega_{11}, \omega_{21})$  and  $(\omega_{12}, \omega_{22})$  of goods 1 and 2. Note that  $\omega_{ik} \geq 0$  for  $i = 1, 2$  and  $k = 1, 2$ . Denote  $\omega_{11} + \omega_{12} = \bar{\omega}_1$  and  $\omega_{21} + \omega_{22} = \bar{\omega}_2$ . Let  $x_{ki}(\mathbf{p})$  be consumer  $i$ 's demand for good  $k$  at price vector  $\mathbf{p} = (p_1, p_2)$ .

(a) Write down the aggregate excess demand function,  $\mathbf{z}(\mathbf{p}) \equiv \begin{pmatrix} z_1(\mathbf{p}) \\ z_2(\mathbf{p}) \end{pmatrix}$ , where  $z_i(\mathbf{p})$  is the aggregate excess demand of good  $i$ ,  $i = 1, 2$ .

(b) Verify the value of the sum of aggregate excess demands is zero (i.e., the Walras' Law holds).

Suppose  $\bar{\omega}_1 = 10$ , and person 1 owns all of good 1;  $\bar{\omega}_2 = 20$ , and person 2 owns all of good 2. Let  $\alpha = 1/2$ .

In each of the following cases, find the competitive equilibrium price ratio  $\rho = p_1/p_2$ , and calculate the equilibrium allocations.

(c)  $\beta = 3/4$ .

(d) Change the utility function of person 2 to the following:

$$\beta x_1 + x_2,$$

and  $\beta = 3$ .

(e) Change the utility function of person 2 to the following:

$$\min\{\beta x_1, x_2\},$$

and  $\beta = 1$ .

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1. Consider a model economy in which total output is produced according to the following function:

$$Y_t = F(K_t, A_t L_t) = A_t K_t^\alpha L_t^{1-\alpha}, \quad 0 < \alpha < 1,$$

where  $Y_t$  is time- $t$  total output,  $K_t$  is time- $t$  total capital employed,  $A_t$  is time- $t$  technology level, and  $L_t$  is time- $t$  total labor employment. The population is equal to  $\bar{L}$  at the initial period and its growth rate is equal to  $n$ . The saving rate is constant in each period and equal to  $s$ . The government of this economy must finance its spending,  $gY_t$  ( $g$  is constant over time), by taxing total output at a rate of  $\tau$  (i.e.,  $g = \tau$ ) at  $t$ . Denote  $\dot{K}_t$  as the change of capital stock for a given time and assume that capital depreciates at a constant rate  $\delta$ ; thus,

$$\dot{K}_t = s(1 - \tau)Y_t - \delta K_t.$$

Answer the following questions.

- (a) Suppose that technology  $A_t$  grows exogenously at a constant rate  $\theta$ . Obtain the growth rates of per-labor output and total output, respectively, in the long-run for given  $s$  and  $\tau$ . (10%)
- (b) Suppose that the technology level  $A_t$  is related to capital stock; i.e.,  $A_t = K_t^{1-\alpha}$ . For simplicity,  $\bar{L} = 1$  and the population is constant over time. Obtain the growth rates of per-labor output and total output, respectively, in the long-run for given  $s$  and  $\tau$ . (10%)
- (c) Suppose that the technology level  $A_t$  is related to the government spending  $gY_t$  as  $A_t = (gY_t)^{1-\alpha}$ . For simplicity,  $\bar{L} = 1$  and the population is constant over time. Obtain the growth rate of per-labor output and total output for given  $s$  and  $\tau$ . (10%) Find the optimal share of the government spending that maximizes the growth rate of per-labor output. (10%)

(背面仍有試題)

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(d) "The economy with a higher saving rate  $s$  leads to a higher growth rate of per-labor output." Comment this statement based on your answer in (1), (2) and (3), respectively. Explain carefully. (10%)

(e) Suppose now that the technology level  $A_t$  is constant over time and the population growth rate is equal to  $n (> 0)$ . Find out the golden-rule level of the capital stock. (10%)

2. A student who intended to test the hypothesis of convergence has found the following regression result from 16 industrialized countries:

$$\ln \left[ \left( \frac{Y}{N} \right)_{i,1980} \right] - \ln \left[ \left( \frac{Y}{N} \right)_{i,1870} \right] = 6.952 - 0.899 \ln \left[ \left( \frac{Y}{N} \right)_{i,1870} \right]$$

(0.089)

$$R^2 = 0.82, \quad s.e.e. = 0.14$$

where  $\ln \left[ \left( \frac{Y}{N} \right)_{i,1980} \right]$  is log per-capita income in 1980 for country  $i$  and  $\ln \left[ \left( \frac{Y}{N} \right)_{i,1870} \right]$  is log per-capita income in the initial year 1870. Note also that the number in parentheses, 0.089, is the standard error of the regression coefficient.

Answer the following questions:

(a) Based on this regression result, which type of convergence, absolute convergence or conditional convergence, holds? (10%)

(b) Do you agree with this conclusion? Explain your answer clearly. (10%)

3. Explain and compare the following terms:

(a) Ricardian Equivalence vs. the Permanent-Income Hypothesis (10%)

(b) The Real-Business-Cycle Theory vs. Keynesian Theory (10%)