考試科目	數理統計	所別	統計研究所	考試時間	6月20日 星期二 上午第一節

1. Let $f(x;\theta) = \frac{1}{\pi[1+(x-\theta)^2]}$, $-\infty < x < \infty$. Consider testing $H_0 : \theta = 0$ v.s. $H_1 : \theta = 2$ using one observation. Let $A = f(x;0)/f(x;2)$.

20分 (a) Obtain the critical region based on A in terms of a region of x-values and determine the size of type I error.

10分 (b) Is the test $A < 1$ uniformly most powerful for H_0 against $\theta > 0$? Explain why or why not.

2. Prove or give a counter example.

10分 (a) Unbiased estimates are consistent.

10分 (b) Consistent estimates are unbiased.

3. Consider a random sample $(X_1, Y_1), \ldots, (X_n, Y_n)$ from the distribution whose density is

$$f(x, y, \theta) = e^{-\theta x - \frac{1}{\theta} y}, \quad x > 0, y > 0,$$

and $\theta > 0$ is a parameter.

12分 (a) Find the distributions for $2\theta \sum_{i=1}^{n} X_i$ and $\frac{2}{\theta} \sum_{i=1}^{n} Y_i$ respectively.

18分 (b) Find the MLE $\hat{\theta}$, and use a pivotal quantity based on $\hat{\theta}$ to construct a two sided confidence interval for θ.

4. The joint moment generating function of (X, Y) is given by

$$\mathrm{E}(e^{sX+tY}) = \frac{1}{4}(1+e^t)(1+e^{s+t}).$$

10分 (a) Find the correlation coefficient between X and Y. (Note: $\rho = \frac{\mathrm{E}(XY)-\mathrm{E}(X)\mathrm{E}(Y)}{\sqrt{\mathrm{V}(X)\mathrm{V}(Y)}}$.)

10分 (b) Show that X and Y X are independent.

考試科目	殘惜模式	所別	統計所	考試時間	6月20日 星期二 上午第二節

1. In simple linear regression model

$$Y_i = \beta_0 + \beta_1 x_i + \varepsilon_i, \quad i = 1, \cdots, n,$$

with $E(\varepsilon_i) = 0, Var(\varepsilon_i) = a_i^2 \sigma^2$, where the a_i are known constants, and the ε_i are independent normal random variables.

(a) Find the least squares estimators of β_0, β_1. (10)

(b) Find the maximum likelihood estimators of β_0, β_1. (10)

(c) Find the best linear unbiased estimators of β_0, β_1. (15)

(d) Find the minimum variance unbiased estimators of β_0, β_1. (15)

(e) If the ε_i's are not normally distributed, which one/ones of the above four is/are still true. Which one or ones will not be affected by the assumption of normality? (10)

2. Two objects of unknown weights β_1 and β_2 are weighted on an error-prone pan balance 4 times in the following four schemes:

I: object 1 is weighted by itself; object 2 is weighted by itself; the difference of the weights (the weight of object 1 minus the weight of object 2) is measured by placing the objects in different pans; the sum of the weights.

II: object 1 is weighted by itself twice; object 2 is weighted by itself twice.

III: weight the difference of the two weights four times.

IV: weight the sum of the two weights four times.

We want to estimate the true weights of the objects. The problem is to find which scheme will give us the best set of estimators.

(a) Set up a model, $\vec{Y} = X\vec{\beta} + \vec{\varepsilon}$. (10)

(b) Find the general form of the least squares estimators $\hat{\vec{\beta}}$ of β_1 and β_2, and $Cov(\hat{\vec{\beta}})$, respectively. (10)

In here, "best" means (i) minimize the trace of $Cov(\hat{\vec{\beta}})$; or (ii) minimize the determinant of $Cov(\hat{\vec{\beta}})$.

(c) Which scheme is the "best" one according to (i) and (ii), respectively. (20)